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Voting matters

To advance the understanding of preferential voting systems

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Editorial

There are 4 items in this issue:

- The first paper, by Frank Bezzina and Anton Buhagiar, develops a version of the single transferable vote that, with a high probability, and particularly in a two party environment like that of Malta, achieves national proportionality for representatives elected from districts of five representatives each. Their method accomplishes this by selecting only four of the five representatives in the usual way, and then selecting the remaining representatives in a way that achieves national proportionality.
- In the second paper, Ross Hyman describes a way of adapting any of the wide variety of rounding methods that have been developed for proportional representation based on party lists, to count an election for multiple seats when every voter has provided an individual ranking of the candidates.
- In the third paper, Simon Gazeley develops a sophisticated new procedure for selecting the candidate who least deserves to remain in contention, when one candidate must be excluded.
- The fourth paper is a report by Michael Mernagh of his investigation into the question of whether Irish voters are more likely to elect candidates whose names are earlier in the alphabet and therefore higher on ballot papers.

Postscript

This is the first issue of *Voting matters* that I have edited. I plan to continue the practice, with which the journal started, of reproducing old documents that are relevant to preferential voting systems. Documents emerging from relevant committees of the Electoral Reform Society will also be considered for publication. But I expect that the preponderance of the items published will, like all four items in this issue, be new contributions from persons conducting their own inquiries. I also plan to continue publishing occasional reviews. Issues will continue to appear when there is sufficient material, and not according to a schedule.

Nicolaus Tideman

Readers are reminded that views expressed in ***Voting matters*** *by contributors do not necessarily reflect those of the McDougall Trust or its trustees.*

STV 4+: A Proportional System for Malta's Electoral Process

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Abstract

A recurring problem in Malta's Single Transferable Vote (STV) is the mismatch between the percentage of first preference votes a party wins nationwide and the corresponding percentage of seats it ends up with in parliament. In this paper we discuss how 'disproportionality' arises and propose the STV 4+ system, to restore proportionality whilst retaining STV as the basis for electing candidates. We describe how the divisor method can be used to implement an STV 4+ system, whereby a Maltese General Election is implemented conceptually as an Additional Member System (AMS). The Maltese General Election of 2008 is taken as an example. We believe that this fair system can be implemented in Malta and other countries that have experience with STV elections.

Keywords: Single Transferable Vote; Proportionality; Additional Member Systems; Malta

1 Introduction

In many types of electoral systems the number of parliamentary seats gained by a given party is not always proportional to the number of votes gained by that party. Some parties will receive disproportionately few seats compared to their votes while others will receive disproportionately many. The outcome of such an election is often accepted, yet the result may be considered unfair.

There is a debate in Malta at the moment as to whether the Single Transferable Vote (STV) system, used to elect the country's national legislatures, should be abandoned. The STV method currently employed in the island combines proportional representation and preferential voting. However, a recurring problem with this electoral system is the fact that it leads to "disproportionality". In fact, four out of the last six general elections held since 1981 resulted in a mismatch between the percentage of first count votes polled nationwide by a given party and the corresponding percentage of parliamentary seats gained by that party [14]. Although the constitution was amended twice to ensure that the party with the majority of first count votes would be given a majority of seats in Parliament, there is constant speculation that a repeat of the 1981 constitutional crisis, which will be discussed further below, is still possible. This in turn is casting doubts on the legitimacy of Malta's entire electoral system. For this reason, the present Prime Minister of Malta proposed that the electoral law should be reviewed and amended.

2 The Aim of This Study

The aim of this study is to provide a feasible solution that would overcome the 'disproportionality' problem that STV is facing in Maltese general elections. In this paper we discuss how this lack of proportionality arises and we propose the STV 4+ method. This electoral system falls into the category of additional member systems, which are used in many countries and have advantages that are well known to political analysts around the world. We illustrate how the STV 4+ system can be adopted in Malta to achieve nationwide representation without making changes to the existing STV ballot structure that was adopted in 1921.

3 Background

In this section we will provide background information about Malta and its electoral system.

3.1 Malta

The Republic of Malta lies at the centre of the Mediterranean Sea, 93 km South of Sicily and 288 km North of Africa, and has area of 316 km². It consists of an archipelago: Malta, Gozo, Comino and three other smaller, uninhabited islands. The climate is Mediterranean, with hot/dry summers and mild winters. Malta currently has 395,742 inhabitants [15], thus making it the second most densely populated country in Europe with 1,309 persons per km². Most residents of Malta are Maltese citizens, Catholic and can speak both Maltese and English. Malta receives around 1.2 million tourists per year, coming mainly from Europe. Malta became a member of the European Union (EU) in 2004.

3.2 The Electoral System in Malta

An electoral system is part of the electoral law which specifically determines "the means by which votes are translated into seats in the process of electing politicians into office." [9].

According to the Maltese constitution, the parliament consists of the President of the Republic and the House of Representatives. The House of Representatives should consist of 65 members of parliament (MP's) elected by means of the STV system, and a parliamentary term should not exceed five years. Currently, Malta is divided into 13 electoral districts and each district elects five MPs. However, when the party with the largest number of first count nationwide votes fails to obtain the largest number of seats, the Constitutional Amendments (CAs) of 1987 and/or 2007 can be invoked in order to restore the election to proportionality. In this case, the legislature will consist of more than 65 seats.

To eliminate any possible effect due to district size, the number of voters in each district should be within ± 5% of the average district size. However, a recent CA provides an exception to this rule for the island of Gozo, which is allowed to be a single district. Malta has the highest non-compulsory voting turnout in the world, and with a 93.3% turnout in the last election (2008) it ranks fifth in the world when including countries that are bound by law to vote [13]. Therefore, not voting in Malta can be considered a form of strong affirmative political action [11].

STV is a proportional representation electoral system conducive to multi-party politics. It has been used in all the 22 general elections held in Malta since 1921, but since 1964 it has evolved into a two-party system. The two dominating parties are the right-of-centre Nationalist Party (PN) and the moderately leftist Malta Labour Party (MLP) [1]. Since 1993, STV has been used to elect the members of the Local Councils and as from 2004 to elect the Maltese representatives in the EU parliament [7]. Apart from Malta, other countries that use STV include the Republic of Ireland (parliamentary elections, European elections, local government elections), Northern Ireland (regional assembly elections, European elections, local government elections), Scotland (for local government elections), Australia (where two forms of STV are used – the Hare-Clark Proportional Representation in the Tasmania House of Representatives and in the Australian Capital Territory Legislative Assembly and the Group Voting or Ticket-Voting Proportional Representation in the Australian Senate and in the legislative councils of Victoria, Western Australia, South Australia and New South Wales), United States (e.g., in city council and school committee elections as in Cambridge, Massachusetts) and New Zealand (e.g., in local government elections in Dunedin and the Wellington local health board elections).

In the STV ballot, Maltese voters can rank as many candidates as they wish in order of personal preference (1^{st}, 2^{nd}, 3^{rd}, 4^{th}, etc.). Voters are also free to float from one party to another and can select candidates for reasons other than party affiliation. However, since in Malta party influence is very strong, there are very few vote transfers between candidates of different parties (approximately, 1%).

An important quantity in STV is the Droop quota (also known as the Hagenbach-Bischoff quota), which is the number of votes required for a candidate to be elected. This is defined as follows:

Droop quota = 1 + integer part of
[(no. of valid votes cast) ÷ (no. of seats + 1)]

Additionally, the Droop quota can easily give the number of seats due to a party on the basis of the votes gained. If one assumes, as is almost always the case in Malta, that votes are always transferred to candidates of the same party, then within any district:

No. of seats gained by a party = integer part of
[(no. of votes polled by party) ÷ (droop quota)]

In the counting process, the first preference (FP) votes are examined first and the candidates who reach a quota of FP votes are elected. If a candidate exceeds the quota, each surplus vote is transferred at full value to the candidate indicated on the ballot as the voter's next ranked choice. A second count is made to see whether other candidates reached the quota through surplus votes. If not, the candidate with the lowest number of votes is eliminated and his or her votes are transferred to other candidates, provided the voters indicated their next preferred candidate on the ballot papers. This counting process continues until 5 candidates have been elected in each of the 13 districts. Since candidates can contest the elections in two districts, those who are elected in both districts must resign from one of the districts and a casual election is held. In this election, "the winner is determined by applying the STV procedures to the ballot papers credited to the vacating candidate in the general election" [12].

This system encourages parties to include many candidates in their rosters since a greater variety of candidates may help them attract more votes. After all, the trend is that the votes of the least popular candidates are not lost but transferred to other candidates of the same party. This results in severe competition among the candidates of the same party within each district.

Although theoretically the STV method should produce more proportional results than other electoral systems and is designed to minimise "wasted" votes, the mismatch between the percentage of FP votes a party wins nationwide and the corresponding percentage of seats it ends up with in parliament has been a recurring problem in Malta. In a detailed analysis of election results between 1921 and 2008, Lane argues that this "disproportionality" is not particularly surprising and is mainly due to the rule of five seats per district. He argues that "the two major parties usually divide the seats in a 3 to 2 ratio and the vote totals never quite produce a corresponding 60% to 40% ratio" [14]. This is indeed true as in Malta, there are no landslide victories in general elections; when the PN and the MLP obtained 51.8% and 46.5% of the first count votes respectively in the 1992 general election, it was considered by many to be a massive victory for the PN.

The most striking anomalous result in Maltese general elections occurred in 1981. In fact, the MLP won the election since it obtained a majority of seats in parliament [FP votes = 105,854 (49.1%), seats = 34]. However, it was the PN that obtained the majority of FP votes [FP votes = 114,132 (50.9%), seats = 31]. Lane attributes this discrepancy to the Droop quota, since "when all candidates in a constituency have been declared elected, there will remain some candidate(s) with accumulated votes who will not be elected and whose votes cannot be transferred" [14]. Buhagiar also reported that the vote-to-seat ratio in this election was 3235 for the MLP and 3682 for the PN. So, on average, the PN required 450 more votes than the MLP to elect a seat [4].

A constitutional crisis emerged when the PN refused to accept the outcome of the 1981 election and walked out of parliament. They claimed that this was a result of gerrymandering (the practice of drawing constituency boundaries with the intention of producing an inflated number of seats, generally for the governing party) by the MLP government and to make sure this never happened again, a CA was approved in 1987 such that the party with an absolute majority (over 50%) of first preference votes is topped-up with bonus seats to ensure a majority of seats in parliament. These provisions were in fact invoked in the elections of 1987 and 1996. However, the constitution was again amended in 2007 because it was believed that with the emergence of the third party, the green party known as *Alternattiva Demokratika* (AD), neither of the two major parties would obtain the absolute majority. So, this time, the new CA allowed top-up seats to be given to the party with the majority of FP votes, provided that only two parties secure seats in Parliament. This prediction materialised in the 2008 election as neither party obtained an absolute majority. The PN (FP votes = 143,468 (49.3%), seats = 31) obtained more FP votes but a smaller number of seats than the MLP (FP votes = 141,888 (48.8%), seats = 34), while the AD (FP votes = 3,810 (1.3%) FP votes, 0 seats) obtained no seats in Parliament. So the provisions of the 2007 CA were invoked and the PN was awarded 4 bonus seats for a total of 35 seats. During the election, a lot of tension was building up as regards to a possible situation in District 10, where an AD candidate had a possible chance of winning a seat after inheriting votes, even though the AD got a small number of FP votes nationwide. If this occurred, the 2007 CA could not be invoked and the party with the majority of seats and a smaller number of FP votes (i.e., the MLP) would have gained control of parliament, and the 1981 constitutional crisis would have repeated itself.

Now more than ever, smaller political parties such as the AD are complaining that the system is biased against them, particularly due to the fact that the major parties tell their supporters that it is 'dangerous' to vote for a smaller party or to transfer preferences from one party to another. There is also the general feeling among voters that improper practices and gerrymandering are still at play. Once again, in the last election, the vote percentages did not mirror the number of parliamentary seats and substantial discrepancies are still evident, particularly when one examines the results within the various districts [14]. Since another constitutional crisis is still a possibility, there is growing consensus among the political class and the public that the electoral law should be amended whilst at the same time retaining STV as the basis for electing candidates. This was one of the briefs of the Commission on Electoral Reform in 1994 [10]. One must acknowledge that STV is a sophisticated instrument which combines diverse preferences of voters and which transforms their preferences into parliamentary representation. In STV, it is entirely up to the public rather than the political parties to decide who has a seat in Parliament and who does not [12]. Besides, anomalous results are not limited to Malta or STV only, but happen quite frequently in any electoral system in any part of the world [9]. For this reason it would be unwise to scrap STV. Instead we need to find ways of averting another constitutional crisis, while preserving the good qualities of the STV method. In previous studies, the party-wise method [2] and the priority queue [3, 5] were proposed and although both these methods restore proportionality between parliamentary seats and FP votes, they assume no voter will ever mix parties. This might be unfair to a party that happens to have its votes in a district split among two or more hopeful candidates. In this paper we propose the STV 4+ system, whereby a Maltese general election is implemented conceptually as a new type of Additional Member System (AMS).

4 The Proposed Solution

In this section we describe a divisor method for assigning seats to parties or candidates and illustrate how this method can be used to achieve proportionality in our elections. We also describe how the divisor method can be used to implement an STV 4+ system whereby a Maltese general election is implemented conceptually as a new type of AMS.

4.1 The D'Hondt Divisor Method

This procedure for allocating seats to candidates in party-list proportional representation elections was invented by the Belgian mathematician Victor D'Hondt in 1878. It is widely used in Europe (e.g. Austria, Finland, Netherlands and Spain) and is based on the principle of the highest average. Once all votes are counted, the number of votes obtained by each party is divided successively by a series of divisors (1, 2, 3…) and the party with the highest quotient is allocated the next seat until the total number of seats available is consumed. The D'Hondt set of divisors is known to favour large parties while other popular sets such as the St. Laguë set (1, 3, 5…) make it progressively easier for the smaller parties to gain seats [17]. Since elections in Malta are dominated by two major parties, it seems more appropriate to adopt the D'Hondt divisor method. Furthermore, this method is relatively easier to follow.

In some countries, a threshold (e.g. 5% on a national or regional basis) is set such that a party that does not reach that threshold will not be allocated any seats even if it has enough votes to secure a seat. Those in favour of such thresholds generally argue that this promotes stable governments by preventing splinter parties from getting into parliament. However, others claim that the threshold is undemocratic; on one hand we say that the electorate is sovereign and on the other hand, with a 5% threshold in place, we tell 4.99% of the electorate that their vote is not as valid and hence they do not have the same voting rights as other members of the electorate [16]. A threshold also sends the message that major parties do not want anyone to try to compete with them in the election. However, thresholds are set in many electoral systems worldwide (e.g., Belgium 5%, Israel 2%, Russia 7%). In fact, in some systems, small parties combine their lists together to form a cartel in an attempt to overcome the election threshold. In other systems, cartels are assigned a separate threshold, while some parties may decide to avoid this cartel threshold by forming coalitions before the election.

4.2 The Additional Member System (AMS) and Other Variants of STV

Many voting systems such as STV or First-Past-the-Post (FPTP) often give a result in which the number of seats a party gains in parliament is not proportional to the percentage of the party vote nationwide. For this reason, methods have been implemented to restore such an election to proportionality. One such electoral process that has been receiving particular attention in electoral methods is the AMS.

Although several variants of the AMS have been proposed, they are basically combinations of the FPTP system and party list voting. The purpose is to retain the best features of FPTP, while introducing proportionality between parties through party list voting. Each voter casts two votes; one vote for a single candidate via FPTP, and one for a regional or national party list. Half the seats or more are allocated to the single-member constituencies and the rest to the party list. The percentage of votes obtained by the parties in the party list vote determines their overall number of representatives. The party lists are used to top up the FPTP seats gained by the party to the required number. So if a party has won one seat in the constituencies, but in proportion to its votes should have three, the first two candidates on its list are elected in addition. The AMS has been adopted in many countries, including Germany, Italy, Scotland, Wales and New Zealand.

In Britain, in 1998, The Independent Commission on the Voting System, chaired by Lord Jenkins, proposed an AMS to replace the FPTP in the UK General Elections. The new system was called the Alternative Vote Top-up (AV+). AV+ is a mixed system composed of two elements: a constituency element and a top-up. Voters have two votes, one to elect a candidate from a district and the other to select the party of choice. The constituency MPs are elected by the Alternative Vote (AV), whilst the 'top-up' MPs are elected on a corrective basis from open party lists [18]. The proposed AV+ has not yet been implemented in Britain or anywhere else.

Just as an AMS can be used to make the FPTP or the AV more proportional, an AMS can be used to improve proportionality in the local STV election. After all, the AV method is a special case of the STV method.

Wichmann [19] suggested another variant of STV which retains single member constituencies whilst at the same time avoiding the non-proportionality of AV. The election proceeds in two stages. In the first stage, each constituency is considered individually using STV (which degenerates to AV), and in the second stage those votes which have not been used to elect a candidate are forwarded to the "county vote" (the aggregate of about six districts). The county vote eliminates those candidates elected at the first stage and the votes used to elect them, and then uses STV on the remaining votes from all districts in the county to fill the county seats.

The retention of single member constituencies is considered to be very important in any reform of the electoral system in the UK, since the British political class and the general public are strongly in favour of such constituencies. Conversely in Malta, constituencies elect at least five candidates. For this reason, there is no need for such a two-tier system in Malta, and the STV4+ method proposed in this paper would be more appropriate in the Maltese context.

4.3 Implementing the STV 4+ System

Conceptually, the STV 4+ system (an AMS) can be implemented quite easily in Malta, since it is a natural extension of the current STV process. The electorate can vote in the usual manner, with a Droop quota based on five candidates per district. However, instead of electing five MPs from each of the 13 districts, we propose that four MPs are elected from each district (by using the quota for electing five) while the remaining 13 MPs are topped up from a list of unelected candidates who gained the highest proportion of higher count votes. Certain provisions are added in order to guarantee that there will always be five representatives per district. More details on the eight steps required to implement the STV 4+ method are provided below.

STEP 1: Determine the number of seats each party should be assigned from a total of 65 seats by using the D'Hondt divisor method on the basis of their first count vote throughout the whole country. This is the nationwide distribution and so it gives the total number of seats a party should get in the election.

STEP 2: In each district, apply the usual STV procedures, but instead of electing five representatives, elect four with the quota for five.

STEP 3: Determine the number of top-up seats each party should be assigned. The number of top-up seats represents the difference between the nationwide calculation of seats and the districtwise results of seats for each party.

STEP 4: Select the top-up seats from either (a) closed party lists, (b) open party lists, or (c) the district candidates who are as yet unelected. In Malta, we propose the adoption of the last option since it can be implemented quite easily.

STEP 5: In each district, continue as in an STV election, but with the provision that no one else will get elected. Make sure that all surpluses of the four elected candidates get distributed, continue eliminations and vote transfers as long as there are candidates with less than half a quota, eliminate those with even larger quotas unless they are the last candidates for their parties, and stop when there is no more than one unelected candidate standing for each party in each district.

STEP 6: Declare elected every candidate who remains the only standing contestant for the fifth seat in the district. Once the fifth candidate is elected, the candidates from all the other parties from the same district are eliminated. Before progressing, move to the central level. Examine the seat allocation and see if there is one or more party that does not have enough standing candidates to get its allocated seats. If so, reallocate seats to other parties. Alternatively, one could allocate from party lists (which would cause some districts to have six representatives while others would end up with four) or else prevent a party's last candidate from being eliminated at the district level, no matter how few votes the candidate had.

STEP 7: Determine the proportion of higher count votes polled by the standing candidates in the districts they contested, to avoid any bias associated with unequal district sizes or voting rates. Sort these proportions in descending order in order to establish priority for the assignment of the top-up seats. To guarantee that there will always be five representatives per district, once a fifth candidate is selected from a district, the candidates from all other parties from that district are eliminated. Furthermore, as soon as the number of a party's candidates in the final reckoning is down to the number the party was assigned, all are elected.

STEP 8: Since candidates in Malta can contest the elections in two districts, those who are elected in both districts must resign from one of the districts and a casual election is held. In this election, the winner is determined by applying the STV procedures to the ballot papers credited to the vacating candidate in the general election.

The principles of the system can be applied when the number of seats in a district is some number other than 5. A polity that has S seats per district rather than 5 would elect S – 1 in each district and elect one per district from the top unelected candidates.

4.4 Applying the STV 4+ System

To illustrate the proposed STV 4+ system, we consider the Maltese General Election of 2008(see [7] for election results). In Table 1 (see Appendix), the D'Hondt divisor is used to determine how the 65 seats available are assigned to the various parties on the basis of their nationwide FP votes. No threshold is assumed in this case and to elect 65 seats, the largest 65 numbers are chosen from the columns. These 65 numbers are marked with an asterisk and the smallest of these numbers represents the quota for the divisor method. Here, it is equal to 4348, and corresponds to the 33rd seat of the PN. All the other numbers represent seats not won. So, the number of seats assigned to candidates in the nationwide distribution should be 33 seats for the PN and 32 seats for the MLP. Similarly, if 52 seats are to be allocated, then Table 1 shows that 26 seats are to be allocated to the PN and 26 seats to the MLP.

In each district, four candidates are elected, using the quota for electing five. Table 2 (see Appendix) exhibits the names of the first 52 elected candidates, the political party they represent and the total number of seats won by each party in each district and nationwide. The first 52 seats were distributed as follows: 24 seats for the PN and 28 seats for the MLP.

After assigning 52 seats from the 65 available seats, the next step is to assign the remaining 13 'top-up' seats. The difference between the nationwide calculation and districtwise results indicates that the PN should be provided with nine top-up seats (33 – 24 = 9) and the MLP with four top-up seats (32 – 28 = 4).

Table 3 (see Appendix) shows that in the final count, 18 candidates (with not more than one hopeful candidate for each party per district) were left standing, and eight particular districts produced a single standing contestant for the fifth seat. So, these eight sole contestants (five from the PN and three from the MLP) were automatically elected and these districts were declared closed. Since both parties had enough standing candidates to get their allocated seats, no reallocation of seats was necessary.

The remaining 10 hopeful candidates were placed in order of the percentage of higher count votes polled in their respective district. This priority queue list for these remaining standing candidates is presented in Table 4 (see Appendix). So, at this stage, we need to allocate four (9 – 5 = 4) top-up seats to the PN and one (4 – 3 = 1) top-up seat to the MLP.

With eight top-up seats already assigned, the ninth top-up seat was awarded to Fredrick Azzopardi (PN) who obtained 21.22% of the higher count votes in District XIII and who was first in the priority queue list of the remaining hopeful candidates. Thus, the other candidate in this district (Joseph Cordina, MLP) was automatically eliminated. The second candidate in the priority queue list was Jesmond Mugliett (PN) who obtained 20.19% of the higher count votes in District IV. He was elected while the other hopeful candidate contesting this district (Owen Bonnici, MLP) was eliminated. This process continued until the fifth candidate for each remaining district was determined. A summary of the elected candidates together with the district they were elected from is presented in Table 4 (see Appendix).

The final distribution of seats obtained by the PN and MLP in each district is presented in Table 5 (see Appendix). This table also shows that in the actual 2008 Maltese General Election, *via* STV, the MLP obtained 34 seats while the PN obtained 31 seats. Since the PN had a majority of FP votes (PN = 49.33%, MLP = 48.90%), the CA of 2007 was invoked and so the PN was awarded 4 additional seats (31 + 4 = 35) to enable it to have an absolute majority of seats in parliament. All these issues would have been resolved if STV 4+ was implemented on the same election results.

Table 6 provides the results of the STV 4+ system implemented on the Maltese General Elections since 1981. This table also reveals that in the actual election results, the CAs were invoked three times – in the elections of 1987, 1996 and 2008. It is therefore clear that the STV 4+ system addresses the discrepancy between a party's FP votes and its parliamentary seats assigned and hence avoids the lack of proportionality which is so persistent in Maltese general elections. An interesting feature in Table 6 is that in 1992, according to the nationwide calculation of seats, the AD would have earned their first seat in Parliament, since they obtained 1.69% of the FP votes (N = 4186). However, the highest number of FP votes obtained by the AD in the districts was 421 (District XI) and all the AD candidates were eliminated by the 11th count. Hence the AD did not have enough votes for one representative and so this seat would have been awarded to the MLP. This also happened in the actual election of 1992 under the STV system currently in use.

5. Countering Possible Objections and Problems

Electoral systems are required to satisfy a wide spectrum of desirable properties and it is often not possible to satisfy them all at the same time [6]. In this section we will discuss possible objections and problems the proposed STV 4+ system is likely to encounter from the electorate and/or by those who might decide to implement it.

5.1 What Changes Will Occur to the Districts and to the STV Ballots?

The districts remain the same as before, and the voters will be required to vote exactly as they did in the past. The change from the usual STV to STV 4+ should be transparent to the electorate.

5.2 Will the Top-Up Candidates be Attached to Districts or Not?

Top-up candidates are generally selected from closed party lists, open party lists or from district candidates who are still unelected. In this particular implementation, the top-up MPs are extracted from the unelected district candidates of the parties contesting the election. In this way, the general objection that the top-up candidates are not attached to any district does not hold.

5.3 The Electorate is Used to Having 5 MPs Elected from Each District. Will This Change?

Although in the STV method an equal number of seats (i.e., five seats) is assigned to each of the 13 districts, when CAs are invoked (as was the case in the elections of 1987, 1996 and 2008) and top-up seats are added to the party that has a minority of parliamentary seats but a majority of FP votes, then the number of seats assigned to each district does not remain the same. In fact, Table 5 shows that in the 2008 general election, Districts I, II, X and XI were assigned six seats instead of five after the 2007 CA was invoked. Thus STV, due to its enduring problem of disproportionality, produces an unequal number of elected candidates across districts, but each district is guaranteed five parliamentary seats.

In the STV 4+ method proposed in this paper, each district is guaranteed to have exactly five representatives, as confirmed in Table 5.

5.4 Will the Proposed Additional Member System Encourage the Proliferation of Small Parties and Favour Coalitions?

It is generally acknowledged that the presence or otherwise of small parties in a parliament depends more on the mood of the electorate rather than on the electoral system adopted. Small parties were quite popular in Malta under STV up to the election of 1962. In 1947, for instance, the Gozo and Jones parties, with their exclusive basis in one district, gained 5 seats with a minimal share of the nationwide vote (3.47% and 5.21% respectively). The Malta Workers Party also managed to elect a good number of MPs in the 1950's [14]. The STV did not preclude small parties from gaining seats if the electorate demanded it. After 1971, however, the small parties did not remain popular with the Maltese public. Since then, no small party managed to get more than 2% of the national vote or to gain a single seat in parliament.

At the end of the day, the electorate is sovereign and there is nothing to stop the electorate from voting for other political formations [8]. In other words, if the small parties were to regain their popularity with the Maltese public, it will be difficult to keep them out of Parliament, whatever the electoral process employed. Thus, coalition governments are indeed possible under all methods of election. Even so, a coalition government need not necessarily be weak just as a single-party government need not necessarily be strong. In Malta for instance, the single party governments of 1950 and 1996, as well as the coalition governments of 1951 and 1953 did not last longer than two years. On the other hand, most of the governments elected since 1945 in Germany were coalitions, but this, once again, depends to a large extent on the political mood of the electorate rather than on the electoral system employed. And once again, in the Republic of Ireland, all governments brought to power with the STV system as from 1989 onwards were coalitions. It is worth noting that both these countries have performed quite well economically under coalition governments.

5.5 What If a Party Obtains Seats Thanks to Second or Later Preferences?

A party with few or no first-count votes may inherit a substantial number of higher-count votes from other parties. A decision must be made as to whether such a party should retain its extra (or "overhang") seats. If the party is allowed to retain these extra seats, the number of seats attained by each party can only be "broadly" rather than "strictly" proportional to the first-count votes. Additionally, a third party in Malta could get 2% of the vote in each of the 13 districts and have enough votes for one representative. To us, this seems not enough in a system with districts. We think that top-up candidates should be limited to those candidates who have at least half a quota after four candidates have been elected and candidates with smaller quotas have been eliminated.

6 Conclusion

Malta has been using STV to elect its national legislatures since 1921 and to abandon this electoral system now would be a severe blow to this method of conducting elections. In this study we proposed the STV 4+ system in an attempt to maintain the STV ballot structure and to increase the proportionality between the percentage of a party's FP votes and its percentage of parliamentary seats. The STV 4+ system provides a swift indication of which party or coalition of parties will lead the parliament (since delaying the outcome would increase unnecessary political tension during a general election) and it eliminates the gain from gerrymandering. Based on the above, we believe that the proposed STV 4+ is a fair system that can be adopted in Malta as well as in other countries that already have experience with STV elections.

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Appendix

Table 1: Nationwide distribution of seats using the D'Hondt divisor (Election of 2008)

	PN	MLP	AD	Others
Divide by	143,468	141,888	3,810	1,633
1	143,468*	141,888*	3,810	1,633
2	71,734*	70,944*	1905	817
3	47,823*	47,296*	1270	544
4	35,867*	35,472*	953	408
5	28,6934*	28,378*	762	327
6	23,911*	23,648*		
7	20,495*	20,270*		
8	17,934*	17,736*		
9	15,941*	15,765*		
10	14,347*	14,189*		
11	13,043*	12,899*		
12	11,956*	11,824*		
13	11,036*	10,914*		
14	10,248*	10,135*		
15	9,565*	9,459*		
16	8,967*	8,868*		
17	8,439*	8,346*		
18	7,970*	7,883*		
19	7,551*	7,468*		
20	7,173*	7,094*		
21	6,832*	6,757*		
22	6,521*	6,449*		
23	6,238*	6,169*		
24	5,978*	5,912*		
25	5,739*	5,676*		
26	5,518*	5,457*		
27	5,314*	5,255*		
28	5,124*	5,067*		
29	4,947*	4,893*		
30	4,782*	4,730*		
31	4,628*	4,577*		
32	4,483*	4,434*		
33	4,348*	4,300		
34	4,220	4,173		

Table 2: First Four Elected Representatives (Election of 2008)

District	PN Candidates	MLP Candidates	PN Seats	MLP Seats
I	Marco De Marco Austin Gatt	Alfred Sant Jose Herrera	2	2
II	Lawrence Gonzi	Michael Falzon Joseph Mizzi Stefano Buontempo	1	3
III	Mario Galea Carmelo Mifsud Bonnici	George Vella Carmelo Abela	2	2
IV	Jason Azzopardi	Silvio Parnis Karl J. Chircop Charles Mangion	1	3
V	Anthony (Ninu) Zammit Franco Debono	Karmenu Vella Marlene Pullicino	2	2
VI	John Dalli	Marie Louise Coleiro Charles Mangion Roderick Galdes	1	3
VII	Jeffrey Pullicino Orlando Joe Cassar	Noel Farrugia Anthony Zammit	2	2
VIII	Antonio Fenech Tonio Borg Beppe Fenech Adami	Alfred Sant	3	1
IX	Lawrence Gonzi Dolores Cristina	Leo Brincat Adrian Vassallo	2	2
X	Robert Arrigo Dolores Cristina	Michael Falzon Evarist Bartolo	2	2
XI	Jeffrey Pullicino Orlando Anthony Borg	Anthony Agius Decelis Angelo Farrugia	2	2
XII	Michael Gonzi Tonio Fenech	Evarist Bartolo Michael Farrugia	2	2
XIII	Giovanna Debono Christopher Said	Anton Refalo Justyne Caruana	2	2
Total			24	28

Table 3: Sole Contestants for 5th seat in District (Election of 2008)

Candidate Names	Party	District	Higher Count Votes %	Overall Top-Up Seat No.	PN Top-up Seat No.	MLP Top-up Seat No.
Helena Dalli	MLP	III	16.27	1		1
George Pullicino	PN	X	15.94	2	1	
Clyde Puli	PN	VI	15.78	3	2	
George Vella	MLP	V	14.96	4		2
David Agius	PN	XI	14.5	5	3	
Christian Cardona	MLP	VIII	14.23	6		3
Louis Deguara	PN	XII	13.94	7	4	
Robert Arrigo	PN	IX	13.88	8	5	

Table 4: Priority Queue list and assignment of the remaining nine top-up seats (Election of 2008)

Hopeful Candidate Name	Party	District	% Votes	Top-Up Seat No	PN Top-up No	MLP Top-up No
Fredrick Azzopardi	PN	XIII	21.22	9	6	
Jesmond Mugliett	PN	IV	20.19	10	7	
Helena Dalli	MLP	II	16.94	11		4
Charles Buhagiar	MLP	VII	16.67	*		
Luciano Busuttil	MLP	I	16.23	*		
Jean Pierre Farrugia	PN	I	15.94	12	8	
Francis Agius	PN	II	14.80	**		
Philip Mifsud	PN	VII	13.79	13	9	
Owen Bonnici	MLP	IV	11.65	**		
Joseph Cordina	MLP	XIII	9.99	**		

* district seat is available but the MLP used up the six assigned top-up seats. Candidate is eliminated

** district seat is unavailable. Candidate is eliminated.

Table 5: Actual Results and STV 4+ Results for MP seats by party and district (Election of 2008)

Election 2008	Proposed STV 4+ Results			Actual Election Results (STV)		
District	PN	MLP	Total	PN	MLP	Total
I	3	2	5	2+1*	3	5+1*
II	1	4	5	1+1*	4	5+1*
III	2	3	5	2	3	5
IV	2	3	5	2	3	5
V	2	3	5	2	3	5
VI	2	3	5	2	3	5
VII	3	2	5	2	3	5
VIII	3	2	5	3	2	5
IX	3	2	5	3	2	5
X	3	2	5	3+1*	2	5+1*
XI	3	2	5	3+1*	2	5+1*
XII	3	2	5	3	2	5
XIII	3	2	5	3	2	5
Total	33	32	65	31 + 4*	34	65+4*

*Constitutional Amendment of 2007 was invoked

Table 6: Distribution of Seats *via* STV 4+ and in actual elections

Election	First Preference Votes (%)			STV 4+ Seats Assigned			Actual Election STV Result CA top-up Total		
	MLP	PN	AD	MLP	PN	AD	MLP	PN	AD
1981	109,990 (49.07%)	114,132 (50.92%)	NA	32	33	NA	34 * 34	31 * 31	NA
1987	114,936 (48.87%)	119,721 (50.91%)	NA	32	33	NA	34 0 34	31 4 35	NA
1992	114,911 (46.5%)	127,932 (51.77%)	4186 (1.69%)	31	34	0**	31 NA 31	34 NA 34	0 NA 0
1996	132,497 (50.72%)	124,864 (47.80%)	3820 (1.46%)	33	32	0	31 4 35	34 0 34	0 0 0
1998	124,220 (46.97%)	137,037 (51.81%)	3208 (1.21%)	31	34	0	31 NA 31	34 NA 34	0 NA 0
2003	134,092 (47.51%)	146,172 (51.79%)	1929 (0.68%)	31	34	0	30 NA 30	35 NA 35	0 NA 0
2008	141,888 (48.79%)	143,468 (49.34%)	3810 (1.31%)	32	33	0	34 0 34	31 4 35	0 0 0

* Constitutional Amendment was not invoked in 1981 as it was as yet unavailable.
** AD lost the seat based on FP votes to the MLP since the AD candidates did not have enough votes for one representative. All the AD candidates were eliminated by Count 11.

Divisor Method Proportional Representation in Preference-Ballot Elections

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Abstract

This paper describes a preference-ballot voting procedure that satisfies proportionality conditions consistent with allocation rules for divisor-method party-list elections such as d'Hondt, Sainte-Laguë, or Huntington-Hill. The procedure generalizes Douglas R. Woodall's Quota Preferential by Quotient procedure, which proportionally assigns candidates to seats in accordance with the d'Hondt divisor method. Variations of the procedure consistent with party-list elections but violating the later-no-harm/help criterion are also presented.

Keywords: Divisor method, Proportional representation, Preference ballot

1 Introduction

Proportional representation in multi-candidate elections is achieved through two different mechanisms in common use today: party-list elections and single transferable vote (STV) preference-ballot elections. This paper describes a preference-ballot voting procedure that is similar to STV but with a proportionality condition satisfied by divisor-method party-list elections instead of the proportionality condition satisfied by STV elections.

The procedure described in this paper can be used in national party-list elections, such as those in Scandinavian countries, to determine the number of seats that each party is awarded to national parliaments. The procedure allows voters to rank parties instead of just voting for one, while retaining the divisor methods that the countries are currently using. The procedure can also be used wherever STV elections can be used.

Party-list proportional representation elections are used in many countries for multi-seat elections to parliaments. In party list elections, the electorate votes for parties not candidates.[1] Seats are awarded to each party in proportion to its vote total, and candidates are elected on the basis of their rankings on party lists that are published before the election.

Perfect proportionality between awarded seats and votes is unachievable. There are two common classes of methods for assigning seats in party-list elections in an approximately proportional way [1, 5, 11]. They are the divisor method (with d'Hondt [2, 3, 10] rounding, or Sainte-Laguë [14] rounding, or Huntington-Hill [8, 9] rounding, etc.) and the largest remainder method (with the Hare [7] quota, or the Droop [4] quota, etc.). Both classes of methods are described in Section 2 of this paper.

In STV [6, 7] elections, which are also used for multi-seat elections to parliaments, voters construct their own ranked list of preferred candidates instead of choosing amongst ready-made party lists. STV uses a quota (Hare, Droop, etc.) to assign seats in an approximately proportional way, and it satisfies a quota-based proportionality condition.

In the October 2003 issue of *Voting matters*, Douglas R. Woodall [18] introduced a preferential voting procedure based upon the divisor method with d'Hondt rounding. The procedure is based on an idea of Olli Salmi [15, 16] to add an elimination procedure to the d'Hondt-

[1] In "open" elections of this type, voters are able to vote for one or more candidates as well, which can reorder the candidates on the party list.

Phragmén method proposed by Lars Edvard Phragmén in 1895 [12, 13].

In this paper, Woodall's procedure is generalized, so that other rounding rules such as Sainte-Laguë or Huntington-Hill can be used to proportionally assign seats in preference-ballot elections. The generalized procedure, like Woodall's d'Hondt procedure and STV, satisfies the later-no-harm/help election criterion. Unlike STV, it satisfies a divisor-method proportionality condition instead of a quota-based proportionality condition.

In addition, Woodall's single-round procedure is modified so that ballot seat values can never decrease.

If each voter is a party loyalist and ranks all of the candidates from their party on their preference ballot in party order, and ranks no other candidates on their ballot, the procedure will *not* in general elect the same number of candidates from each party as the equivalent divisor-method party-list election. (The procedure does still satisfy the divisor-method proportionality condition, which is a less restrictive condition than perfect agreement with party list elections.) Alternative procedures are presented that agree with party list elections when voters vote only party lists, but at the expense of not satisfying the later-no-harm/help election criterion.

Section 2 of this paper introduces largest remainder and divisor methods for assigning seats in party list proportional representation elections. Section 3 introduces the divisor method in priority form, the form needed for preference voting. It also makes the case that Huntington-Hill divisor methods are the only divisor methods that are unbiased between large and small parties. Section 2 and Section 3 can be skipped by those already familiar with divisor methods. In Section 4, the divisor method preference voting procedure satisfying later-no-harm/help is described and demonstrated. Section 5 demonstrates properties of the election procedure including the proportionality condition. Section 6 presents variations of the procedure to reproduce party list elections at the cost of not satisfying later-no-harm/help. Section 7 concludes the paper.

2 Approximately Proportional Methods for Party List Elections

This section introduces largest remainder and divisor methods for proportionally assigning seats in party list elections. In party list elections, seats are awarded to parties in proportion to their vote totals. The numbers of seats, S_i, apportioned to parties are perfectly proportional to votes, V_i, if there is a single quota Q such that $S_i = V_i/Q$ for each party. If votes for each party only came in multiples of the quota, then a party would be assigned one seat for each quota of votes. For example, if 500 voters vote in a party list election for 5 seats and 200 voters choose the Red Party, 200 Voters choose the Green Party, and 100 voters choose the Blue party, dividing each total party vote by 100 assigns 2 seats to the Red Party, 2 seats to the Green Party and 1 seat to the Blue Party.

Since total party votes are generally not integer multiples of the desired quota and seats must be assigned in integer units, perfect proportionally is generally unattainable and rounding is not guaranteed to produce the desired number of total seats. For example, if 500 voters vote in a party list election for 5 seats and 222 voters choose the Red Party, 149 voters choose the Green Party, and 129 voters choose the Blue Party, dividing each total party vote by 100 assigns 2.22 seats to the Red Party, 1.49 seats to the Green Party and 1.29 seats to the Blue Party for a total of five seats. Conventional rounding assigns 2 seats to the Red Party, 1 seat to the Green Party, and 1 seat to the Blue party for a total of only 4 seats.

Approximate proportionality that assigns the desired total number of seats can be achieved through largest remainder or divisor methods. For the largest remainder method, a quota is fixed and the rounding rule is adjusted so that the desired number of candidates is elected. In the above example, 5 seats are assigned if rounding up occurs not at 0.5 but at any number greater than 0.29 but less than or equal to 0.49. This adjusted rounding rule assigns 2 seats to the Red party, 2 seats to the Green party, and 1 seat to the Blue Party, for a total of 5 seats. The largest remainder method is so-called because it is equivalent to rounding up the party seat assignments in decreasing order from the largest fractional remainder to the smallest, until the desired number of seats is assigned. STV is a largest remainder method.[2]

For divisor methods, a rounding rule is fixed and the quota is adjusted so that the desired number of candidates is elected. In the above example, 5 seats are assigned for conventional rounding if party votes were divided not by 100 but by any number greater than 88.8 but less than or equal to 99 ⅓. For example, dividing party votes by 99 instead of 100 assigns 2.242 seats to the Red Party, which rounds down to 2 seats, 1.505 seats to the Green Party, which rounds up to 2 seats, and 1.303 seats to the Blue Party, which rounds down to 1 seat, for a total of 5 seats. This paper presents a procedure for applying divisor methods to preference voting.

In Section 3, the priority formulation of the divisor method, which is needed for preference voting, is introduced.

3 Divisor Methods in Priority Form

This section develops and demonstrates the priority formulation of divisor methods, which will be applied to preference-ballot voting in Section 4. Also, several rounding rules in common use are described, and their bias for small or large parties is shown with an apportionment slide rule.

3.1 An Apportionment Slide Rule

Imagine two sliding rulers, one on top of the other, with logarithmic scaling on each.[3] Let the top ruler be the Votes Ruler and the bottom ruler be the Seats Ruler. For a given positioning of the two rulers, the number of seats awarded to a party is the number of seats on the Seats Ruler directly below the number of votes on the Votes Ruler that a party received. Each positioning of the Votes Ruler with respect to the Seats Ruler corresponds to a different perfect apportionment (before rounding) corresponding to a particular quota.

Different rounding rules can be visualized in the following way. For each integer N, a fixed-rounding mark, $\log(F_{N-1,N})$ is placed between $\log(N-1)$ and $\log(N)$ on the Seats Ruler. For each $\log(N)$, the rounding mark $\log(F_{N-1,N})$ is to its left and the rounding mark $\log(F_{N,N+1})$ is to its right. The segment of the seats ruler between consecutive rounding marks $\log(F_{N-1,N})$ and $\log(F_{N,N+1})$ is the integer seat region for N seats. When the Votes Ruler is positioned over the Seats Ruler so that $\log(V_i)$ is over any part of the N seat region, the i^{th} party is assigned N seats.

3.2 Rounding Rules

Two common rounding rules for party-list proportional representation elections are the Jefferson-d'Hondt rounding rule and the Modified Sainte-Laguë rounding rule. Jefferson-d'Hondt rounding is the same as rounding down. The seat region boundary marks are at $F_{N-1,N} = N$ and the segment from $\log(N)$ to $\log(N+1)$ is the Jefferson-d'Hondt region for N seats.

Modified Sainte-Laguë rounding is conventional rounding, except for $F_{0,1}$. The seat region boundary marks are at $F_{N-1,N} = N-0.5$ and the segment from $\log(N-0.5)$ to $\log(N+0.5)$ is the Sainte-Laguë region for N seats. Modified Sainte-Laguë, sets $F_{0,1} = 0.7$, instead of the unmodified 0.5, making it harder for a small party to gain a seat. We will see below that all values of $F_{0,1} \le F_{1,2}/2 = 0.75$ are admissible for preference voting. Because the Sainte-Laguë rounding marks are closer to the rightmost integer than the leftmost integer on a logarithmic scale, more seats will be rounded down than rounded up.

On a logarithmic scale, the distance between consecutive integers decreases as the integers increase. Because of this, when the number of seats apportioned to a party is rounded to an integer, the shift away from perfect proportionality is greater for a party with a small number of votes than it is for a party with a large number of votes. For this reason, a rounding rule that rounds down more than it rounds up (such as d'Hondt[4] or SainteLaguë) is biased against

[2] For Meek's method and for some other STV systems, the quota is recalculated when ballots become inactive.

[3] On a logarithmic scale the distance between two numbers is proportional to their ratio.

[4] d'Hondt rounding's bias in favor of large parties is often counted as a point in its favor since

small parties compared to large parties and a rounding rule that rounds up more than it rounds down is biased in favour of small parties compared to large parties.

The only rounding rule that isn't systematically biased on a logarithmic scale is one with rounding marks placed exactly between the integers on such a scale.[5] The Huntington-Hill rounding rule, which is used in the United States to apportion the seats of the House of Representatives to the states, is defined in this way. The Huntington-Hill rounding mark between $\log(N-1)$ and $\log(N)$ is half way between them:[6] that is,

$$\log(F_{N-1,\,N}) = ½ (\log(N-1) + \log(N)),$$

so

$$F_{N-1,\,N} = \sqrt{(N(N-1))},$$

the geometric mean. This assigns the integer region for N seats to the region of the slide rule closest to $\log(N)$.

Without modification, Huntington-Hill awards a seat to any candidate getting just one first choice vote, since $F_{0,\,1} = 0$. An increased $F_{0,\,1}$ above zero makes Huntington-Hill viable for proportional representation elections. We will see below that all values of $F_{0,\,1} \le F_{1,\,2}/2 = \sqrt{½}$ are permissible for preference voting. Modified Huntington-Hill with $F_{0,\,1} = \sqrt{½}$ agrees with Jefferson-d'Hondt when all parties receive 2 or fewer seats. Since $\sqrt{½}$ is approximately 0.7, and $\sqrt{(N(N-1))}$ is approximately $N-0.5$ for large N, modified Huntington-Hill with $F_{0,\,1} = \sqrt{½}$ is similar to modified Sainte-Laguë.[7]

3.3 Priority/Load Formalism

If the Votes Ruler is positioned over the Seats Ruler such that $\log(V)$ votes on the Votes Ruler is positioned directly over $\log(S)$ seats on Seats Ruler then the quota is V/S and the fraction of seats that each ballot accounts for is S/V. Due to the magic of logarithms, every $\log(V)$ and every $\log(S)$ that are positioned directly over each other on the two rulers have the same V/S ratio for a given positioning of the two rulers.

The slide rule can systematically assign seats to parties by placing the Votes Ruler to the left of the Seats Ruler and moving it to the right, which decreases the quota and increases the seat fraction per ballot. Each time the vote mark for the i^{th} party crosses a rounding mark, the i^{th} party acquires an additional seat.

When $\log(V_i)$ on the Votes Ruler is directly over $\log(F_{N-1,\,N})$ on the Seats Ruler, the i^{th} party crosses from the $N-1$ seat region to the N seat region and acquires its N^{th} seat. The quota for when this occurs is $V_i/F_{N-1,\,N}$. This is the *priority* or *quotient* for the i^{th} party to have N seats. The inverse priority, $F_{N-1,\,N}/V_i$, which without rounding is the average number of seats per ballot, is the *load* [16] for the i^{th} party to have N seats.[8] One calculates priority quotients or loads for parties to acquire seats and assigns seats to the parties in order from highest priority to lowest, or lowest load to highest, stopping when the appropriate number of seats has been reached. For party-list elections the priority formalism is commonly used. Phragmén invoked the load formalism for his preference-ballot procedure [12, 13, 16]. A priority tends to be a large number divided by a small number while a load tends to be a small number divided by a large number. We will find that the load

it discourages party splits and encourages party mergers. Only d'Hondt rounding guarantees that a majority of voters will be awarded a majority of seats. In Sainte-Laguë and Huntington-Hill rounding, a majority could have its seats rounded down while a minority has its seats rounded up, resulting in a majority rule violation.

[5] Balinsky and Young argue that Huntington-Hill is more biased than Sainte-Lague. However, they did not use a logarithmic scale in defining their bias criteria.

[6] This is why Huntington called his method "Equal Proportions."

[7] The choice $F_{0,\,1} = \sqrt{½}$ is also motivated by allowing inverse integer seat regions, $1/N$, between 0 and 1, which are the mirror images, on a logarithmic scale, of the integer seat regions between 1 and infinity. For these additional regions, $F_{1/(N+1),\,1/N} = \sqrt{((1/(N+1))1/N)}$, and in particular $F_{1/2,\,1} = \sqrt{½}$ is the rounding mark between the ½ seat region and the 1 seat region. Since seats can only be assigned in integer units, parties that would receive fractional seats are excluded.

[8] Phragmén uses the term belastnig, which Olli Salmi translates as load, for what we call seat value [13].

formalism is more natural for preference voting when presented in the abstract but that priorities have the advantage over loads, when concrete examples are presented, of being easier to calculate and compare magnitudes by hand.

For an example of the priority (load) formulation, consider the party-list election for 5 seats in which 222 voters choose the Red Party, 149 voters choose the Green Party, and 129 voters choose the Blue Party. For simplicity, Jefferson-d'Hondt rounding is used. The five highest priorities (lowest loads) are the Red Party's priority for one seat, 222/1=222 (1/222 = 0.0045), the Green Party's priority for one seat, 149/1=149 (1/149 = 0.0067) the Blue Party's priority for one seat, 129/1=129 (1/129 = 0.0078), the Red Party's priority for two seats, 222/2 = 111 (2/222 = 0.009), and the Green Party's priority for two seats, 129/2 = 64.5 (2/129 = 0.016). All other priorities are lower than these. The Red Party is apportioned two seats, the Green Party is apportioned two seats, and the Blue Party is apportioned one seat.

The priority/load formulation of fixed-rounding is generalized to preference-ballot elections in Section 4.

4 Proportional Preference-Ballot Voting by the Divisor Method

This section develops a divisor method for electing candidates in preference-ballot elections. The method is a generalization of the priority/load formalism for divisor method party-list elections described above, and of Woodall's Quota Preferential by Quotient procedure for d'Hondt rounding. The procedure described in this section satisfies the later-no-harm/help criteria but is not guaranteed to agree with the results of a party-list election if each voter votes a party list.

The single-round, d'Hondt version of the election procedure described in this section differs from Woodall's single-round procedure slightly in that seat values are guaranteed never to decrease. The multi-round, d'Hondt version of the election procedure is identical to Woodall's multi-round procedure.

The procedure can be visualized by imagining a Votes Ruler, as before, but now also many Seats Rulers, one for each ballot. Since each ballot counts for one vote, the Votes Ruler has a mark at $V = 1$ and nowhere else. The values of the seat regions on the Seats Rulers can be any number, not just integers, and their values and rounding marks can be different for each ballot and are determined as the election count proceeds.

We begin with a series of definitions.

Elected, hopeful, and excluded candidates

Following Woodall, each candidate is in one of three states, designated as elected, hopeful and excluded. At the start of the first stage, every candidate is hopeful. As the count proceeds hopeful candidates are reclassified as elected or excluded.

Active and inactive ballots

Following Woodall, a ballot is active when it ranks at least one hopeful candidate. It is inactive when it ranks no hopeful candidate.

The seat value of a ballot

Following Woodall, each ballot is assigned a seat value that corresponds to the fractional number of candidates that the ballot can be said to have elected. The seat value of ballots cannot decrease (the exception is in the multi-round version of the procedure, when the count is restarted and all seat values are reset to zero). The sum of seat values over all ballots is the current number of elected candidates.

The candidate election load

In the priority/load formalism for divisor-method party-list elections, the Votes Ruler is shifted to the right and a new seat is acquired by the i^{th} party each time the Vote mark for the i^{th} party crosses the next rounding mark on the seat ruler. We will perform the same procedure for preference voting, except that seat regions and rounding marks are not fixed beforehand. Instead, as we shift the Votes Ruler to the right, a trial rounding mark on each ballot's Seat Ruler directly follows underneath the $V = 1$ mark on the Votes ruler. At any particular positioning of the Votes Ruler with respect to the Seat Rulers, the value of the trial rounding mark, f, and the ballot's seat value s, determine the seat value of the trial seat region to its right according to the formula $s' = g(s, f)$, where $g(s, f)$ is a function that depends on which divisor method rounding rule is being used. At some point, as the Votes Ruler and trial rounding mark moves to the right, the sum of the trial seat values, s', for all of the ballots with candidate c as the topmost hopeful candidate will equal the sum of the current seat values for those ballots + 1. The value of the trial rounding mark at that point is f_c, the load to elect candidate c. The priority to elect candidate c is $p_c = 1/f_c$. These definitions are consistent with the party-list definitions for the load and priority of a candidate to be elected. The load to elect candidate c satisfies

$$\sum_{cballots}\left(g\left(s, f_c\right) - s\right) = 1,$$

where the sum is taken over all ballots with topmost active hopeful candidate c. In the above and all subsequent ballot sums, the ballot index for each seat value has been suppressed. It is important to keep in mind that seat values are for ballots and can be different for each ballot in the sum, while a load is for a candidate (or a group of candidates, as we will see below) and is a constant in the ballot sum.

4.1 Properties of $g(s, f)$

A ballot with seat value s has its seat value increased to $s' = g(s, f)$ when its topmost hopeful candidate is elected with load f. The function $g(s,f)$ must satisfy the following conditions:

a) $g(s,f) \geq f$ for $f > s$,

b) $g(K-1, F_{K-1,K}) = K$,

c) $ag(s,f) = g(as, af)$,

d) $g(s,f) = s$ when $s \geq f$,

e) $g(s,f)$ is monotonically decreasing in s for $s \leq f$, that is $g(s,f)$ does not increase when s increases with f held fixed, and

f) $g(s,f)$ is strictly increasing in f for $f \geq s$ and s held fixed.

Condition a) guarantees that a ballot's seat value cannot decrease. Conditions b) and c) together guarantee that $g((K-1)/V, F_{K-1,K}/V) = K/V$ for any V, which is required for consistency with divisor method rounding rules. Condition d) allows ballot sums to include ballots with seat values larger than the load. Condition e) is required to guarantee that electable candidates remain electable. Condition f) guarantees that there is one and only one f_c for each candidate c. Condition f) is not an independent condition. It is a consequence of condition a), condition c), and condition e) which together guarantee that $\partial g/\partial f \geq 1$ for $f \geq s$.

4.2 Rounding Rules

All of the following rounding rules have $g(s, f)$ functions that satisfy the above conditions. They are determined by inverting the rounding mark formulas $f = f(s, s')$. For d'Hondt, $g(s, f) = \max(s, f)$. For unmodified Sainte-Laguë $g(s, f) = \max(s, 2f - s)$. For unmodified Huntington-Hill $g(s, f) = \max(s, f^2/s)$. Modified Huntington-Hill and modified Sainte-Laguë rounding have $F_{0,1}$ above their unmodified values. Condition c) guarantees that $g(s, f) = f\,h(s/f)$ where $h(x)$ is a function of one variable. Applying condition b) we have $h(0) = 1/F_{0,1}$ and $h(1/F_{1,2}) = 2/F_{1,2}$. There are many ways to extrapolate $h(x)$ between these points that satisfy the rounding rule conditions. A linear extrapolation leads to

$$h(x) = 1/F_{0,1} + (2 - F_{1,2}/F_{0,1})x$$

for $x \leq 1/F_{1,2}$, with unmodified $h(x)$ for $x \geq 1/F_{1,2}$. Hence, for $f \geq F_{1,2}$,

$$g(s, f) = \frac{f}{F_{0,1}} - \left(F_{1,2} - 2F_{0,1}\right)\frac{s}{F_{0,1}},$$

with unmodified $g(s,f)$ for $f \leq F_{1,2}s$. Condition e) requires that $F_{0,1} \leq F_{1,2}/2$. For modified Huntington-Hill, in which $F_{0,1} = F_{1,2}/2 = (\sqrt{2})/2$, $g(s, f) = (\sqrt{2})f$ for $f \geq (\sqrt{2})s$, and $g(s, f) = \max(s, f^2/s)$ otherwise. For modified Sainte-Laguë with $F_{0,1} = F_{1,2}/2 = 0.75$, $g(s, f) = 4f/3$ for $f \geq 1.5s$ and $g(s, f) = \max(s, 2f - s)$ otherwise. For modified Sainte-Laguë with $F_{0,1} = 0.7$, $g(s, f) = (10f - s)/7$ for $f \geq 1.5s$ and $g(s, f) = \max(s, 2f - s)$ otherwise.

4.3 The Electability Load

The fact that candidate c has the lowest election load does not mean that candidate c should necessarily be elected. It could be that all the voters who voted for candidates other than c command enough votes to fill all of the remaining seats with candidates other than c, and at lower loads than the load to elect c, if only they had voted more strategically. The lowest possible load to fill the remaining seats with non-c candidates, f_{notc}, satisfies

$$\sum_{notcballots}\left(g\left(s, f_{notc}\right)-s\right)=R ,$$

where the sum is over all active ballots in which c is not the topmost active hopeful candidate, and R is the remaining number of seats to be filled.[9] Hopeful candidate c is electable when $f_c < f_{notc}$

It is not necessary to calculate f_{notc} to determine whether c is electable. The electability load, f_{elect}, satisfying

$$\sum_{active}\left(g\left(s, f_{elect}\right)-s\right)=R+1 ,$$

where the sum is over all active ballots, is always between f_c and f_{notc} and therefore can be used as an alternative electability criteria for c. Hopeful candidate c is electable when $f_c < f_{elect}$

Proof: If f_{elect} were less than f_c and f_{notc} then the sum on the LHS would be less than $R + 1$. If f_{elect} were greater than f_c and f_{notc} then the sum on the LHS would be greater than $R + 1$.

One consequence of this fact is that if f_c is the lowest election load and $f_{notc} \le f_c$ so that c is not electable, then no hopeful candidate is electable. The electability load serves a similar purpose to the quota in STV elections, of determining whether a hopeful candidate is electable. In divisor methods, at any stage, the quota is the range of values with a maximum equal to the election priority of the electable candidate with lowest election priority and with a minimum that is just greater than the election priority for the unelectable candidate with highest election priority. The electability priority $Q = 1/f_{elect}$ always falls in this range, so it is a valid quota. This is the generalization of the quota, Q, used in Woodall's paper.

4.4 Explicit Load Formulas

The election loads and electability load are determined from implicit formulas of the form

$$\sum_{s}\left(g\left(s, f\right)-s\right)-M=0 ,$$

differing only in which ballots are summed and the value of M. Since $g(s, f)$ is strictly increasing in f for $f > s$, the election load and electability load equations have unique solutions. Inverting load equations is complicated by the fact that $g(s, f)$ is piecewise continuous, with different formulas when s is less than or greater than f and for modified rules, when s is less than or greater than $f/F_{1,2}$ An iterative method to find f in such equations is as follows.

Step 1. All ballots that are included in the sum are placed into groups in increasing seat value order: $s_1 < s_2 < s_3$ etc. The number of ballots in the k^{th} group is V_k.

Step 2. Calculate the next iteration of f from one of the following formulas. For the first iteration include all ballots in the following sum and for the previous value of f, choose infinity. For later iterations, include only those ballots with seat values less than the previous value of f.

For unmodified rules and modified rules in which every seat value that is less than f is larger than $f/F_{1,2}$ use $f =$

$$\frac{F_{0,1}M+\sum_{k=1}^{r}V_k s_k}{\sum_{m=1}^{r}V_m}$$

for d'Hondt ($F_{0,1} = 1$) and unmodified Sainte-Laguë ($F_{0,1} = 0.5$), and $f =$

$$\sqrt{\frac{M+\sum_{k=1}^{r}V_k s_k}{\sum_{m=1}^{r}\frac{V_m}{s_m}}}$$

for unmodified Huntington-Hill.

For modified rules in which every seat value that is less than f is larger than $f/F_{1,2}$ use $f =$

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[9] This distribution of non-c candidates is not necessarily attainable since it requires each voter to split their ballot into R equal pieces and vote for one of R non-c candidates on each piece, each split ballot counting for $1/R$ of a vote. However, the attainability of the distribution is not as important as the fact that the electability criterion leads to the desired proportionality condition, as will be proved below.

$$\frac{F_{0,1}M + \left(F_{1,2} - F_{0,1}\right)\sum_{k=1}^{r} V_k s_k}{\sum_{m=1}^{r} V_m}.$$

Otherwise use f =

$$\frac{\sqrt{2\left(\sum_{k=1}^{p} V_k\right)^2 + 4\left(\sum_{k=p+1}^{r} \frac{V_k}{s_k}\right)\left(M + \sum_{k=1}^{r} V_k s_k\right)} - \sqrt{2}\sum_{k=1}^{p} V_k}{2\sum_{k=p+1}^{r} \frac{V_k}{s_k}}$$

for modified Huntington-Hill and f =

$$\frac{F_{0,1}M + \left(1.5 - 2F_{0,1}\right)\sum_{k=1}^{p} V_k s_k + F_{0,1}\sum_{k=p+1}^{r} V_k s_k + F_{0,1}\sum_{k=1}^{r} V_k s_k}{\sum_{k=1}^{p} V_k + 2F_{0,1}\sum_{k=p+1}^{r} V_k}$$

for modified Sainte-Laguë. In the above expressions, s_p is the largest seat value that is less than the current value of $f/F_{1,2}$ and s_r is the largest seat value less than the current value of f.

Step 3. Repeat Step 2 until an f has been found such that s_p and s_r are unchanged. That value of f is the correct load.

In the following election procedure Woodall's d'Hondt single-round procedure [18] is generalized to other divisor methods.

4.5 Election Procedure 1

The following is a single-round election procedure for N seats that satisfies both later-no-harm/ help and a divisor-method proportionality condition.

Step 1. At the start of the first stage every candidate is hopeful and the seat value of every ballot is zero. The remaining number of seats to be filled, R, is set to N, the total number of seats to be filled.

Step 2. The election load f_c for each hopeful candidate c that is the topmost hopeful candidate on at least one ballot is determined from

$$\sum_{cballots}\left(g\left(s, f_c\right) - s\right) = 1,$$

where the sum is taken over all ballots where c is the topmost hopeful candidate and the electability load is determined from

$$\sum_{active}\left(g\left(s, f_{elect}\right) - s\right) = R + 1,$$

where the sum is taken over all active ballots. If at least one hopeful candidate is electable, that is, $f_c < f_{elect}$, go to step 3a. If no candidate is electable, go to step 3b.

Step 3a. The electable candidate with the lowest election load is elected. (If the total number of elected candidates is N, the count can be ended since no more candidates will be elected). R is reduced by 1. If candidate c is elected, the seat value for each ballot with seat value s that contributed to electing c is increased to $g(s, f_c)$. Proceed to Step 2.

Step 3b. Exclude the candidate with the largest election load amongst those that are the topmost hopeful candidate on at least one ballot. Also exclude all hopeful candidates that do not appear as the topmost hopeful candidate on any ballot. (If the total number of elected plus hopeful candidates is N then all of the hopeful candidates can be elected and the count ended since they are all guaranteed to be elected.) Proceed to Step 2.

The single-round procedure with d'Hondt rounding differs from Woodall's single-round procedure in the calculation of loads/priorities, so that seat values cannot decrease. This is demonstrated with Election 1 from Woodall's paper. Loads rather than priorities will be presented to be consistent with the formulism used throughout this paper. However, all loads will be presented as the inverse of priorities for easy comparison with Woodall's examples and the priority formalism.

Election 1 (3 seats, d'Hondt)

16	AB
12	B
12	C
12	D
8	EB

Stage 1: The election and electability loads are f_A = 1/16, f_B = 1/12, f_C = 1/12, f_D = 1/12, f_E = 1/8, and f_{elect} = 4/60 = 1/15. The lowest election load is f_A. It is lower than f_{elect}. Candidate A is elected. Each of the seat values for the 16 ballots ranking candidate A first is increased from zero to 1/16. Stage 2: Candidate B's election load is decreased to f_B = 2/(12+16) = 1/14. The

other loads are unchanged. No candidate's election load is lower than the electability load so candidate E, who has the highest election load, is excluded. Candidate B's election load is again decreased, this time to 1/20. This is calculated by only including the ballots ranking B first (not 2/36= 1/18, as that incorrectly includes the 16 ballots with seat values of 1/16 which do not contribute to B's load since 1/16 is greater than 1/18). Candidate B is elected. Each of the seat values for the 20 ballots ranking B first is increased from zero to 1/20. Stage 3: $f_C = 1/12$, $f_D = 1/12$, $f_{elect} = 2/24 = 1/12$. Neither of the remaining candidates is electable. One must be excluded and the other is elected.

In the next section, properties that Woodall proved for d'Hondt rounding are proved for the general case. The section culminates in a divisor method proportionality condition.

5. Properties of the Election Procedure

The goal of this section is to prove that the election procedure described in the previous section satisfies a divisor method proportionality condition. This is done through a serious of steps following the logic Woodall used to demonstrate d'Hondt proportionality (which turns out to be the same as Droop proportionality). The first two proofs together combine to prove that an electable candidate remains electable. The next two proofs together combine to prove that all electable candidates will eventually be elected. From there the proportionality condition is proved by considering the worst case scenario in which a candidate is electable.

Election loads of hopeful candidates cannot increase

Electing and excluding candidates other than hopeful candidate c can increase but can never decrease the number of ballots in which c is the topmost hopeful candidate. More ballots means that more can contribute to

$$\sum_{cballots}\left(g\left(s, f_c\right) - s\right),$$

which from properties a), d), and f) cannot increase the f_c required to bring the sum to one.

The electability load cannot decrease

Excluding candidates does not change the seat values of ballots. It can cause some ballots to become inactive which can only increase f_{elect}. When a candidate is elected, the sum of seat values of all ballots increases by one and the remaining number of seats, R, decreases by one. Moving the seat value sum to the RHS in the defining equation for the electability load we have a new RHS that is unchanged after an election, provided no new ballots have become inactive:

$$\sum_{active} g\left(s, f_{elect}\right) = 1 + R + \sum_{active} s\ .$$

When no ballots become inactive the electability load cannot decease after an election of candidate c with load $f_c < f_{elect}$, since $g(g(s, f_c), f_{elect}) \le g(s, f_{elect})$ and $g(s, f)$ is monotonically increasing in f. The requirement $g(g(s, f_c), f_{elect}) \le g(s, f_{elect})$ is true for the case $g(s, f_c) \le f_{elect}$ since $g(s, f_c) \ge s$ and $g(s, f_{elect})$ is monotonically decreasing in s for $s \le f_{elect}$. It is true for the case $g(s, f_c) \ge f_{elect}$ since in that case $g(g(s, f_c), f_{elect}) = g(s, f_c) \le g(s, f_{elect})$ and $g(s, f)$ is monotonically increasing in f.

In addition to the above, at each stage some ballots can become inactive. Fewer active ballots means that fewer will contribute to

$$\sum_{active}\left(g\left(s, f_{elect}\right) - s\right),$$

which, from properties a), d), and f), cannot decrease the f_{elect} needed to bring the sum to R + 1.

If c is electable, it will remain electable

Since a candidate's election load can only decrease and f_{elect} can only increase, if c is electable at one stage it will remain electable at later stages.

At any stage, at most R hopeful candidates are electable

There is only a possibility of more than R electable candidates if there are more than R hopeful candidates. If there are more that R hopeful candidates, let f_{large} be the R + 1th smallest load of the more than R hopeful candidates. Call the hopeful candidates with rounding marks less than or equal to f_{large} the smaller candidates. It must be the case that

$$\sum_{active}\left(g\left(s, f_{large}\right)-s\right) = \sum_{hopefuls}\sum_{cballots}\left(g(s, f_{large})-s\right)$$
$$\geq \sum_{smalls}\sum_{cballots}\left(g(s, f_{large})-s\right) \geq \sum_{smalls} 1 \geq R+1,$$

which makes $f_{large} \geq f_{elect}$ and therefore all candidates with election loads greater than or equal to f_{large} are unelectable, so at most R are electable.

If there are R remaining hopeful candidates then at least one is electable

Let $f_{smallest}$ be the smallest load of the R hopeful candidates. It must be the case that

$$\sum_{active}\left(g\left(s, f_{smallest}\right)-s\right)$$
$$= \sum_{hopefuls}\sum_{cballots}\left(g(s, f_{smallest})-s\right) \leq \sum_{hopefuls} 1 = R,$$

which makes $f_{smallest} < f_{elect}$, so the hopeful candidate with the smallest election load is electable when there are R hopeful candidates remaining. (This argument can also be used to show that if there are R+1 remaining hopeful candidates then at least one is electable or all hopeful candidates are tied with election loads equal to the electability load.)

An electable candidate is guaranteed to be elected

Removing the premature stopping condition in parenthesis in Step 3a (that the procedure stops when N candidates are elected) can have no effect since after N candidates are elected, no additional candidates can be elected since none of the remaining hopeful candidates will be electable. Likewise, removing the premature stopping condition in parenthesis in Step 3b (that the procedure elects the remaining hopeful candidates and stops when the total number of elected plus hopeful candidates equals N) also can have no effect since after the total number of elected plus hopeful candidates equals N, the procedure will still elect all of the remaining hopeful candidates, since at least one will always be electable. Since the premature stopping conditions can be removed without changing which candidates are elected, and without the premature stopping conditions the election procedure ends when all candidates are either elected or excluded, and electable candidates cannot be excluded, an electable candidate is guaranteed to be elected.

Proportionality condition

The conditions above insure that the count cannot end before all electable candidates are elected. Therefore demonstrating that a candidate is electable is equivalent to proving that it will be elected. From this we can prove the following proportionality condition.

If there are N seats to be filled and V_T total valid ballots and V ballots all rank the same L candidates higher than all other candidates then at least $K \leq L$ of those candidates will be elected if

$$\frac{V}{V_T} > \frac{\widetilde{F}_{K-1,\,K}}{(N-K+1)F_{0,\,1} + \widetilde{F}_{K-1,\,K}} \ ,$$

where $\widetilde{F}_{K-1,\,K} = F_{K-1,\,K}$ for d'Hondt, unmodified Sainte-Laguë, and unmodified Huntington-Hill and $\widetilde{F}_{K-1,\,K} = K-1+F_{0,\,1}$ for modified Sainte-Laguë for $0.75 \geq F_{0,\,1} \geq 0.5$ and modified Huntington-Hill for $\sqrt{½} \geq F_{0,\,1} \geq 1/2$.

Proof: Consider the worst case scenario to elect K candidates, which is that the K candidates appear only on the V ballots and not on any others. Assume that K – 1 of the candidates have already been elected. The load to elect the K^{th} candidate is determined by

$$\sum_{vballots} g(s, f) = K \ ,$$

where the sum is over the V ballots and the seat values satisfy

$$\sum_{vballots} s = K-1 \ .$$

The maximum value of f is found by minimizing

$$\sum_{vballots} g(s, f)$$

with respect to s with the above seat value constraint and then increasing f until

$$\sum_{vballots} g(s, f) = K \ .$$

The minimum for convex functions is $s = (K-1)/V$ for each seat value in the sum, from which

$$K = Vg\left(\frac{K-1}{V}, f\right) = g\left(K-1, Vf\right),$$

where $ag(s, f) = g(as, af)$ has been used. The solution, using

$$K = g\left(K-1, F_{K-1,\,K}\right)$$

is

$$f = \frac{F_{K-1,\,K}}{V}.$$

This is the maximum load to elect K candidates for d'Hondt, unmodified Sainte-Laguë and unmodified Huntington-Hill which all have a $g(s, f)$ that is convex in s. Modified Sainte-Laguë for $0.75 \geq F_{0,1} \geq 0.5$ and modified Huntington-Hill for $\sqrt{½} \geq F_{0,1} \geq 1/2$ have non-convex $g(s, f)$ in which a straight line connecting $g(0, f)$ to $g(f, f)$ is lower than $g(s, f)$ at every point along the line.[10] Therefore, for these rounding rules,

$$\sum_{vballots} g(s, f)$$

is minimized by V_2 ballots with $s = 0$ and V_1 ballots with $s = f = (K-1)/V_1$, so that

$$K = V_2 g\left(0, \frac{K-1}{V_1}\right) + V_1 g\left(\frac{K-1}{V_1}, \frac{K-1}{V_1}\right)$$
$$= \frac{V_2}{F_{0,1}} \frac{K-1}{V_1} + K - 1,$$

where $g(0, f) = f/F_{0,1}$ (required by properties b and c), and $g(x, x) = x$ (property d) have been used. The solution is $f = (K-1)/V_1 = F_{0,1}/V_2$. Solving for f in terms of $V = V_1 + V_2$ produces $f = (K - 1 + F_{0,1})/V$.

The load to elect $N - K + 1$ candidates other than the K^{th} candidate so that the K^{th} candidate cannot be elected satisfies

$$\sum_{notvballots} \left(g\left(s, f_{notv}\right) - s\right) = N - K + 1.$$

These candidates occur as topmost hopeful candidates only on the $V_T - V$ ballots. The lowest possible load is found when the seats values are as small as possible, which is s = 0. The minimum load to elect $N - K + 1$ candidates is determined by

$$N - K + 1 = \left(V_T - V\right) g(0, f_{notv}) = \frac{\left(V_T - V\right) f_{notv}}{F_{0,1}}$$

The solution is

$$f_{notv} = \frac{\left(N - K + 1\right) F_{0,1}}{V_T - V}.$$

The K^{th} candidate is electable if $f < f_{notv}$. When rearranged, this is the proportionality condition given above.

A consequence of the proportionality condition is that only d'Hondt rounding guarantees that a majority of voters will be awarded a majority of seats. The d'Hondt proportionality condition guarantees that if the number of seats is $2m + 1$ and there is a voting block that commands more than half of the ballots, then at least $m + 1$ of the seats will be awarded to that voting block. This is not guaranteed for other rounding rules. This is true for party-list elections and preference-ballot elections. The other rounding rules give greater weight to the first ranked candidate, so for majority rule to be violated the majority must rank their candidates mostly the same while the minority distributes the first ranked position more equally amongst their preferred candidates. For party-list elections the number of parties is typically much less than the number of seats, so the extreme circumstances required for majority rule to be violated are much less likely to occur.

6 Variations of the Procedure

In this section, variations of the election procedure are presented that satisfy different voting system criteria.

6.1 Election Procedure 2: A Single-Round Procedure Agreeing with Party-List Elections

The election procedure described in Section 4 can fail to reproduce the result of a party-list election when each voter votes a party-list. This problem exists for STV elections too and is caused by incorrectly excluding candidates because of an artificially small electability load (an artificially large quota) caused by the presence of ballots that become inactive later in the count. Election 2 is an example of this failure.

Election 2 (2 seats, d'Hondt)

90 A1 A2 …
44 B1 B2 …
43 C1 C2 …
41 D1 D2 …
36 E1 E2 …
20 F1 F2 …

[10] In classical thermodynamics this is called "Maxwell's construction" for minimizing non-convex free energy functions.

The d'Hondt divisor method applied to the equivalent party-list election elects A1 and A2. The preference-ballot procedure with permanent exclusions excludes A2 and elects A1 and B1. However, candidates A1 and A2 are elected if the preference-ballot election procedure is altered so that all excluded candidates are recalled to hopeful status every time a candidate is elected and the premature stopping condition in Step 3b is removed. The altered procedure proceeds as follows. Stage 1: f_{A1} = 1/90, f_{B1} = 1/44, f_{C1} = 1/43, f_{D1} = 1/41, f_{E1} = 1/36, f_{F1} = 1/20, f_{elect} = 3/274 = 1/91.33. The election loads of other candidates are not calculated as they are not the topmost hopeful candidates on any ballots. No candidate's election load is less than the electability load, so candidate F1, with the largest election load, is excluded. Also excluded are all candidates that are not the topmost hopeful candidate on any ballot, including A2. The electability load increases to f_{elect} = 3/254 = 1/84.67. The election loads of the remaining hopeful candidates are unchanged. Candidate A1 is elected. The 90 ballots ranking A1 first are assigned seat value 1/90. Stage 2: All excluded candidates are recalled to hopeful status. The loads are f_{A2} = 2/90 =1/45, f_{B1} = 1/44, f_{C1} = 1/43, f_{D1} = 1/41, f_{E1} = 1/36, f_{F1} = 1/20, f_{elect} = 2/184 = 1/92 (not 1/91.33 as the 90 ballots with seat value 1/90 don't contribute). The election loads of other candidates are not calculated as they are not the topmost hopeful candidates on any ballots. No candidate's election load is less than the electability load, so candidate F1, with the largest election load, is excluded. Also excluded are all candidates that are not the topmost hopeful candidate on any ballot. The electability load is increased to f_{elect} = 1/84.67. No candidate's election load is less than the electability load, so candidate E1 is excluded. The procedure continues with, D1 and C1 successively excluded, at which point the electability load is increased to f_{elect} = 3/134 = 44.67, and candidate A2 is elected.

Temporarily rather than permanently excluding candidates can violate later-no-harm/help. Elections 3 and 4 are examples of this violation.

Election 3 (2 seats, d'Hondt)

18 A
15 AB
24 C
23 D
20 BA

Stage 1: f_A = 1/33, f_B = 1/20, f_C = 1/24, f_D = 1/23, f_{elect} = 1/33.33. Candidate B is excluded. Candidate A's load decreases to f_A = 1/53. Other loads are unchanged. Candidate A is elected. Each ballot electing candidate A is assigned a seat value of 1/53. Stage 2: Candidate B is recalled to hopeful status. Candidate B's election load decreases to f_B = (35/53 +1)/35 = 1/21.08. Candidate C has the lowest election load and is eventually elected.

Had the 20 voters ranking candidate B before candidate A been aware that candidate A would be elected without their help, these voters could have left candidate A off their ballots to increase the chance of their favoured candidate, B, winning the second seat. This is demonstrated in Election 4.

Election 4 (2 seats, d'Hondt)

18 A
15 AB
24 C
23 D
20 B

Stage 1: f_A = 1/33, f_B = 1/20, f_C = 1/24, f_D = 1/23, f_{elect} = 1/33.33. Candidate B is excluded. The election load is increased to f_{elect} = 3/80 = 1/26.67. Other loads are unchanged. Candidate A is elected. Each ballot electing candidate A is assigned a seat value of 1/33. Stage 2: Candidate B is recalled to hopeful status. Candidate B's election load decreases to f_B = (15/33 +1)/35 = 1/24.06. Candidate B has the lowest election load and is eventually elected.

This is a violation of later-no-harm/help since candidate B's election was achieved by removing candidates ranked below B on ballots. The example demonstrates that the procedure encourages free riding, which is the same tactical voting procedure encouraged by all proportional multi-seat preference voting systems, including those that satisfy later-no-harm/help. It is advantageous for some voters to be free riders by not ranking very popular candidates, so that more of their vote will count for their favoured unpopular candidates. But the temptation to be a free rider is tempered by the knowledge that if all voters acted in that way, the popular candidates would lose. It is unclear how the incentive to be a free rider under the procedure violating later-no-harm/help compares to the incentive under the procedure that satisfies later-no-harm/help.

6.2 Election Procedure 3: A Multi-Round Procedure Satisfying Later-No-Harm/ Help and Providing No Benefit to Woodall Free-Riding

Woodall proposed a multi-round version of his election procedure in which the election is rerun after each exclusion, which has the effect of reassigning seat values on ballots to what they would be if the excluded candidate had never run. The multi-round procedure prevents any benefit from what Markus Schulze [17] refers to as Woodall free riding, in which a voter ranks an unpopular candidate she is confident will be excluded above a popular candidate she is confident will be elected so that more of her vote will be counted for lower ranked candidates. It is easily generalized to Election Procedure 1, by recalling all elected candidates to hopeful status, setting all seat values to zero, and setting $R = N$ at the end of Step 3b in Procedure 1 before proceeding to Step 2. However, this procedure, like Election Procedure 1, will not in general reproduce the results of a party-list election when each voter votes a party list.

6.3 Election Procedure 4: Proportionality without an Electability Test

The simplest procedure that agrees with party-list elections when voters vote a party list as in Election Procedure 2, and provides no benefit to Woodall free riding as in Election Procedure 3, is presented below as Election Procedure 4. It satisfies divisor-method proportionality while not requiring that the electability load ever be calculated. However, its violation of later-no-harm/help is more severe than that of Election Procedure 2. It does not reduce to the Alternative Vote for the case of one seat.

If there are M candidates, the procedure first calculates which candidates would be elected in an $(M-1)$-seat election. The $M-1$ winners are entered in an election for $M-2$ seats and the one non-elected candidate is excluded and is assigned a final election load. The $M-2$ winners are entered in an election for $M-3$ seats and the one non-elected candidate is excluded and is assigned a final election load, etc. For an N-seat election, the count can stop when N hopeful candidates remain. Alternatively the count can be continued until all candidates have been assigned a final election load. In that case, candidates with the N lowest final election loads are the elected candidates in an N-seat election.

The method does not require the calculation of the electability load since it is guaranteed that for an election for $X-1$ seats for X candidates, the candidate with the lowest load is electable.[11] The proof that the method satisfies the divisor-method proportionality condition is as follows: The multi-round Election Procedure 3 for N seats will elect the same N candidates as Election Procedure 4 if Procedure 3 is modified so that the candidate chosen for exclusion when no candidate is electable is not the hopeful candidate with the largest election load, but instead the hopeful candidate with the largest final election load as produced from Procedure 4. Agreement with the divisor method proportionality condition follows since the condition does not depend on which candidate is excluded when no candidate is electable.

Step 1. At the start of the first round every candidate is hopeful and the seat value of every ballot is zero.

Step 2. Election load f_c for each hopeful candidate c is determined from

$$\sum_{cballots}\left(g\left(s,f_c\right)-s\right)=1\,,$$

where the sum is taken over all ballots where c is the topmost hopeful candidate. If there is more than one hopeful candidate, go to Step 3a. If there is just one hopeful candidate, go to Step 3b.

Step 3a. If there is more than one hopeful candidate, elect the candidate with the lowest

[11] When all remaining candidates are tied with loads equal to the electability load none are electable and a tiebreaking procedure is needed to elect one of the candidates. But it is still the case that one does not need to calculate the electability load in this situation.

load. If candidate c has just been elected, the seat value for each ballot with seat value s that contributed to electing c is increased to $g(s, f_c)$. Proceed to Step 2 to begin the next stage.

Step 3b. If there is just one hopeful candidate, exclude it. Its election load becomes its final election load. If all candidates are excluded (and therefore all have been assigned final election loads), the candidates with the N lowest final election loads are the elected candidates in an N-seat election and the count is concluded. Otherwise, if there is one or more elected candidate, set all elected candidates to hopeful. Set all seat values to zero. Proceed to Step 2 to begin the next round.

For this procedure, changing the number of seats without changing ballots has no effect on final election loads. Therefore, elected candidates remain elected if the count is rerun for a larger number of seats with ballots unchanged. Also, if all voters are party loyalists so that they only rank candidates from their party, although in any order and not necessarily ranking every candidate from their party, the final election loads produced by counting each party's ballots separately will not change if the ballots are all counted together. Therefore, elected candidates remain elected if the count is rerun with ballots added for a new party and the number of seats increased until the total number of seats awarded to the previous parties is at least as large as it was previously. Lastly, if all voters are party loyalists so that they only rank candidates from their party, although in any order, and they rank *all* of the members of their party, then for d'Hondt rounding only,[12] the final election loads for a party that receives v votes will be $1/v$, $2/v$, $3/v$, etc. An increase (decrease) in a party's votes will decrease (increase) the party's final loads without changing the loads for other parties. Therefore, for fixed number of seats, an increase in a party's votes cannot decrease the number of seats awarded to that party and a decrease to a party's votes cannot increase the number of seats awarded to that party. However, monotonicity for the individual candidates is not guaranteed since the rank of candidates within a party can change non-monotonically as party ballots are added or removed. These properties are demonstrated by Elections 5 and 6.

Election 5 (2 seats, d'Hondt)

35 ACB
33 BAC
32 CBA

The final election loads are $f_A = 1/100$, $f_B = 3/100 = 1/33.33$, and $f_C = 2/100 = 1/50$, so that candidate A is elected to a one-seat election and candidates A and C are elected to a two-seat election. For d'Hondt rounding only, for any set of 100 ballots where each voter ranked all three candidates, the final loads are guaranteed to be 1/100, 2/100, and 3/100, although which loads candidates are assigned will depend on the ballots.

The consequences of having an additional candidate, D, with 33 votes and with ballots otherwise unchanged, can be seen in Election 6.

Election 6 (2 seats, d'Hondt)

35 ACB
33 BAC
32 CBA
33 D

The final election loads for candidates A, B, and C are unchanged. Candidate D's final election load is $f_D = 1/33$. Candidate A still wins a one-seat election and candidates A and C still win a two-seat election. For STV and all of the other election procedures described in this paper, candidates A and C are elected to a two-seat election when D voters don't vote but candidates A and B are elected when the D voters vote.

A demonstration of Election Procedure 4's violation of later-no-harm/help is provided by Election 7.

Election 7 (1 seat, d'Hondt)

35 A
33 BC
32 CA

Procedure 4 elects candidate A. However if the voters who ranked candidate A first also ranked C second, the procedure would have instead elected candidate C. This shows that voters can be harmed by ranking an additional

[12] Only for d'Hondt rounding is $NF_{0,1} = F_{N-1,N}$ for all N, which is required for any distribution of party ballots to produce the same final election loads.

candidate. Election 8 shows that they can also be helped.

Election 8 (1 seat, d'Hondt)

35 A
33 BC
32 CA

Procedure 4 elects candidate B. However if the voters who ranked candidate A first had ranked C second, the procedure would have elected candidate A.

6.4: Election Procedure 5: Reproducing Party-List Elections, Providing No Benefit to Woodall Free-Riding, and Reducing to Alternative Vote for One Seat.

The final election procedure presented in this paper combines: excluding candidates with the largest election load as in Procedures 1-3 to provide agreement with Alternative Vote in one-seat elections, the recalling of excluded candidates after an election as in Procedure 2 to provide agreement with party list elections when voters vote a party list, and the reassigning of seat values after an exclusion as in Procedure 3 to provide no benefit from Woodall free-riding Its violation of later-no-harm/help when the election is for more than one seat is no more severe than that of Election Procedure 2. However it does not have the properties of party separability and monotonicity with respect to the number of seats, of Election Procedure 4. The procedure temporarily re-excludes all previously excluded hopeful candidates while seat values are being reassigned.

Step 1. At the start of the first stage every candidate is hopeful and the seat value of every ballot is zero. The remaining number of seats to be filled, R, is set to N, the total number of seats to be filled. Proceed to Step 4.

Step 2. Set all elected candidates to previously elected hopeful status. Set all previously excluded hopeful candidates to temporarily excluded status. Set all seat values to zero.

Step 3. Election load f_c for each previously elected hopeful candidate c is determined from

$$\sum_{cballots}\left(g\left(s, f_c\right) - s\right) = 1$$

where the sum is taken over all ballots where c is the topmost hopeful candidate. Re-elect the candidate with the lowest load. If previously elected candidate c is re-elected, the seat value for each ballot with seat value s that contributed to re-electing c is increased to $g(s, f_c)$. If not all previously elected candidates have been re-elected, proceed to Step 3 for the next re-election. Otherwise recall all temporarily excluded candidates to previously excluded hopeful status.

Step 4. The election load f_c for each hopeful candidate c is determined from

$$\sum_{cballots}\left(g\left(s, f_c\right) - s\right) = 1\,,$$

where the sum is taken over all ballots on which c is the topmost hopeful candidate and the electability load is determined from

$$\sum_{active}\left(g\left(s, f_{elect}\right) - s\right) = R + 1\,,$$

where the sum is taken over all active ballots. If at least one hopeful candidate is electable go to step 5a. If no candidates are electable, go to step 5b.

Step 5a. Set the electable candidate with the lowest election load to elected. (The count can be stopped if N candidates are elected). The next stage begins. R is reduced by 1. Set all excluded candidates to previously excluded hopeful status. Proceed to Step 2.

Step 5b. Exclude the candidate with the largest election load amongst those that are the topmost hopeful candidate on at least one ballot. Also exclude all hopeful candidates that that do not appear as the topmost hopeful candidate on any ballot. If all of the candidates excluded in this step have been previously excluded proceed to Step 4. Otherwise, proceed to Step 2. Election 9 demonstrates the procedure.

Election 9, (2 seats, d'Hondt)

13 AB
8 AC
4 DAC

Stage 1: f_A = 1/21, f_D = 1/4, f_{elect} = 3/25=1/8.33. Candidates B and C are not the topmost hopeful candidate on any ballot so their loads are not calculated. The lowest election load, f_A, is lower than the electability load so candidate A is elected. Stage 2: Each of the twenty one ballots that contributed to candidate A's election are assigned a seat value of 1/21. F_B = (13/21+1)/13 = 1/8.0, f_C = (8/21 +1)/8 = 1/5.8, f_D = 1/4, f_{elect} = 1/8.33. No candidate's election load is less than the electability load. Candidate D has the largest election load, is excluded. Reweighing: With D excluded, candidate A's load is decreased to f_D =1/25. Candidate A is re-elected. Each of the twenty five ballots that contributed to candidate A's re-election is assigned a seat value of 1/25. The loads are now f_B = (13/25 + 1)/13 =1/8.55, f_C = (12/25 + 1)/12 = 1/8.11, f_{elect} = 1/8.33. Candidate B is elected. In the single round procedure, candidate C is elected even though 13 voters wanted A and B and only 12 wanted A and C.

7 Conclusion

In this paper, a generalization of Woodall's QPQ procedure has been presented for assigning seats from preference ballots in multi-candidate elections, using divisor methods (d'Hondt, Sainte-Laguë. Huntington-Hill, etc.) commonly used in party-list proportional representation elections. The procedure satisfies a proportionality condition that, in general, is different from Droop proportionality. Versions of the procedure can satisfy later-no-harm/help criteria or reproduce the results of party-list elections when each voter votes a party list, but not both at the same time. I gratefully acknowledge Douglas Woodall for his very helpful comments and suggestions. All errors are my own.

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STV with Elimination of Discounted Contenders

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Abstract

Although it has many advantages, STV can occasionally yield perverse outcomes, because excluding the candidate with the fewest current votes can exclude a candidate who is better supported than others who remain in contention. STV with Elimination of Discounted Contenders (STV-EDC) substitutes a more sophisticated exclusion rule that ensures that the candidate selected for exclusion is not as well supported as those who remain in contention.

1 Introduction

Although STV is arguably the best electoral system in use for public elections today, like all systems it has its flaws. The STV rules applied in public elections differ to a greater or lesser extent in detail, but they all agree that when all surpluses have been transferred, the non-elected candidate who at that point is the first available preference on fewest votes is excluded, i.e., is not considered further in the course of the current count. A candidate who has much support but few first preferences can sometimes be excluded before the extent of that support has become apparent; this is illustrated by Election 3 below, in which two seats are contested and nobody is elected before the first exclusion. Conventional STV elects B and C despite E's bring ranked above them on more than two Droop quotas of votes in each case. I consider conventional STV's exclusion of E in this example to be perverse.

The position in which voter A ranks candidate X indicates A's desire not only for X to be elected rather than any candidate ranked below X, but also for X to be eliminated rather than any candidate ranked above X. It may be that A has no strong feelings about some of the candidates ranked above X and has expressed preferences for them simply to reduce the probability of X's election; if voting in an election in which British National Party candidates were standing, I personally would cast a preference for every non-BNP candidate but none for the BNP. If X has least support on the votes available to the contending candidates, it is reasonable to presume that a significant number of voters have voted against X and that effect should be given to their wishes.

I take it as axiomatic that in any version of STV a candidate who has attained a surplus when only originating surpluses and consequential surpluses arising from them have been transferred has an absolute right to a seat. When all the candidates who definitely should be elected have been elected, the emphasis should shift to identifying candidates who definitely should not be elected.

The aim of each round of STV with elimination of discounted candidates (STV-EDC) is to identify then eliminate the one *contending candidate* (ie, a candidate who is neither elected nor eliminated) who has less available support than any other; further rounds take place if there are seats not yet filled. STV-EDC is based on the fact that at least a Droop quota of the votes active at any given point in an election will not help to elect anyone; it awards those votes to a notional candidate and the common value of the contending candidates' votes is discounted to make up that notional candidate's quota. Thus every preference of every voter contributes to the tally of votes considered when a candidate is eliminated.

2 How STV-EDC works

An STV-EDC count is a series of rounds. Each round except the last has two stages, first an election stage then an elimination stage. Each round except the last culminates in the elimination of one candidate. In the election stage of the final round, all seats are filled before there is a need to exclude a candidate, and the election is over.

a. *The election stage*. The election stage is a conventional Meek count, except that contending candidates who attain the quota after the first exclusion are not classified as elected but remain contending; their surpluses are transferred in the normal way. When s candidates (where s is the number of seats being contested) have attained the quota, the election stage ends.

b. *The elimination stage*. Candidates excluded in the election stage are reclassified as contending. On every vote which bears a preference for *any* candidate who has not been eliminated in a previous STV-EDC, a preference for a notional candidate N is inserted immediately following the voter's final expressed preference. Each elimination stage is a quasi-Meek round using the final quota q inherited from the immediately preceding election stage. A candidate's *keep value* (kv) is the fraction of any incoming vote or part-vote that that candidate retains, passing the rest on to the next available preference, if any, otherwise to non-transferable. An initial kv of 1 is set for the notional candidate N and an initial common kv between 0 and 1 for the contending candidates; initial kvs between 0 and 1 are set also for the elected candidates (if any). The kvs of N and the elected candidates are adjusted upward or downward until they all have at least q votes; the common kv of all the contending candidates is adjusted upward or downward until the contending candidates collectively have fq votes or fewer where f is the number of seats yet to be filled. When it is known to be impossible for the *lowest candidate* (ie, the contending candidate with fewest votes) to get more votes than the lowest-but-one candidate, the lowest candidate is eliminated, preferences for N are deleted and the STV-EDC elimination stage ends. A suggested counting algorithm for the elimination stage is provided in the Appendix.

Consider the following election for one seat:

Election 1 (1 seat)

35	AD
34	BD
31	CD

In the election stage, D, C and B are excluded in that order and the final quota is 34.5. The initial quota is 50; A does not attain it before the first exclusion and so is not elected. In the first iteration of the elimination stage with the common keep value t set to 0.15910 and N's kv to 1.0, effective votes are:

A	5.56863
B	5.40952
C	4.93221
D	13.37896
N	70.71068

The common kv t for A, B, C and D has to be recalculated so that their collective total of votes is nearer to 34.5; this is 0.18741 (to 5 decimal places – the calculations in this example are actually performed to 13 decimal places). Effective votes are now:

A	6.55930
B	6.37190
C	5.80967
D	15.22867
N	66.03046

In the next iteration N's kv is set to 0.53052, making N's votes 35.03046, but leaving the other candidates' votes unchanged. The total surplus (ie, the difference between N's votes and the total of the votes of A, B, C and D) is 1.06092. In the next iteration, t is reset to 0.19034 and votes are:

A	6.66173
B	6.47140
C	5.90039
D	15.41077
N	34.77859

The total surplus is 0.33430, less than the difference between the votes of B and C, so C is eliminated; this ends the first round. In like fashion, B is eliminated in the second count. At the election stage of the third round A gets 35 votes and D gets 65, so D is elected.

The Condorcet winner (if any) will usually, but not always, be elected in an STV-EDC count for one seat. This is because, as seen above, the Condorcet winner will usually not be the lowest candidate in an STV-EDC round and will thus escape being eliminated.

3 Discussion

Any counting system worthy of consideration should observe Woodall's Droop Proportionality Criterion (DPC) [5], which he stated thus:

> *If, for some whole numbers* k *and* m *satisfying 0* < k ≤ m*, more than* k *Droop quotas of voters put the same* m *candidates (not necessarily in the same order) as the top* m *candidates in their preference listings, then at least* k *of those* m *candidates should be elected.*

STV-EDC possesses this property. A group of voters who prefer every candidate within a set to any candidate outside it are said to *solidly support* that set.

> *Proof: Let there be in an STV-EDC election a set of* m *candidates whom* k *(where* m ≤ k < m + 1*) Droop quotas of voters solidly support. All these candidates would be elected at the election stage of a count because all surpluses would be transferred to other members of the set before being transferred to non-members. If the set instead contained* m + n *candidates where* n ≥ 1*, but with the same number of voters solidly supporting it, then if one member of the set were eliminated it would still contain at least* m *candidates.*

The contending candidate who ultimately gets fewest votes in an elimination stage has less available support than any other and for that reason has a worse claim to a seat than any other contending candidate.

4 STV-EDC or Sequential STV?

Sequential STV was originally devised by David Hill [1]; he withdrew the original version in favour of a revised version on which I collaborated with him [2, 3]. Its aim was to identify a set of *s* candidates which, when tested against all the other candidates one at a time, was the most appropriate set to be elected. A problem with Sequential is that special measures are needed to break paradoxes; barring ties, STV-EDC needs no such measures. I believe the systems to be broadly comparable in terms of outcomes and computer time.

Consider how STV-EDC treats Elections 2 and 3, which have been used to test Sequential STV. These are presented side by side so that the differences between the two can be seen more easily.

Election 2 (2 seats)		Election 3 (2 seats)	
104	ABCD	104	AEBCD
103	BCDA	103	BECDA
102	CDBA	102	CEDBA
101	DBCA	101	DEBCA
3	EABCD	3	EABCD
3	EBCDA	3	EBCDA
3	ECDBA	3	ECDBA
3	EDCBA	3	EDCBA

422 votes are cast. Election 3 differs from Election 2 only in that the voters whose first preferences were A, B, C or D have inserted E between their first and former second preferences. Meek elects B and C in both, but Sequential elects BC in 2 and BE in 3. STV-EDC endorses Sequential.

The following example devised by Douglas Woodall shows that Sequential does not always elect the set of *s* candidates that beats every other candidate in contests of *s* + 1; Sequential elects C and D, but AB is the set that beats all comers. However, it is arguable that AB is not the best set of candidates to elect.

Election 4 (2 seats) – Woodall's Torpedo

11	AC
9	ADEF
10	BC
9	BDEF
10	CA
10	CB
10	EFDA
11	FDEB

STV-EDC elects C and F. Owing to a paradox involving D, E and F, this outcome is as acceptable as that of Sequential. Neither Sequential nor STV-EDC achieves Sequential's stated objective in this case.

It would be interesting to analyse differences in outcomes between Sequential, STV-EDC and Nicolaus Tideman's STV with comparisons of pairs of outcomes (CPO-STV) [4]. CPO-STV compares each possible set of *s* candidates with every other; the set that gets more support than any other is elected. The sets compared are those that contain all the candidates who would have been elected before the first exclusion in a conventional Meek count; if there are no such candidates, all sets are compared. The different approach of CPO-STV self-evidently avoids the problem of premature exclusion but at the price of sometimes having to select one of the competing potentially winning sets of *s* candidates by a separate process.

I conclude that there is little or nothing to choose between Sequential STV and STV-EDC in terms of outcomes, but this conclusion must be provisional until much research which I am unable to do has been completed. The different approach of CPO-STV is likely to produce different outcomes in some circumstances; whether the outcomes of CPO-STV are better or worse than those of STV-EDC with the same voting profiles again cannot be determined without much research. STV-EDC has the advantage that (barring ties) it gives one definitive outcome in every case.

I believe that STV-EDC offers a workable and robust solution to the problem of premature exclusion arising from the exclude-the-lowest rule in conventional STV.

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I am grateful to David Hill who rescued me from a fatal error in an earlier version of this paper.

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About the Author

Simon Gazeley is a retired civil servant. He has served on the Council of the Electoral Reform Society on three occasions since 1992 and was a member of the Technical Committee.

Appendix: A counting algorithm for the elimination stage of STV-EDC

At any given point:

c is the total of the votes credited to the contending candidates;

d is the total of the votes credited to the elected candidates and N;

e is the number of candidates elected so far;

f is the number of vacant seats;

n is the number of non-eliminated candidates.

1. On every vote which bears a preference for any non-eliminated candidate, insert a preference for a notional candidate N immediately following the voter's final expressed preference. Set N's initial kv to 1; set the common kv t of the contending candidates and the initial kvs of the elected candidates to $1 - \sqrt[n]{1/(s+1)}$. Set q to the final quota in the immediately preceding election stage. Set the iteration count to 0.

2. Increase the iteration count by 1. Distribute the votes, then:

a. If the iteration count is odd, recalculate t as follows:

If c < fq*, multiply* t *by* fq/c*, otherwise by an iota less than* $(\mathrm{fq/c})^2$.

b. If the iteration count is even, recalculate the kv of N and of each elected candidate by multiplying the present kv by

$$(q(s+1) - c)/(e+1).$$

If the new kv > 1, reset it to 1.

If $fq > c$ or if N or any elected candidate has fewer than q votes, go to 2.

3. Calculate the total surplus $x = d - (e+1)c/f$. If x exceeds the difference between the two lowest candidates' votes, go to 2. Otherwise, eliminate the lowest candidate, delete preferences for N and end the elimination stage.

What’s in a Name? A Political Myth?

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Abstract

This paper first reviews three methodologies for deriving a data frame that represents all of the electorate, to permit a comparison of the alphabetic distribution of TDs (“Teachtaí Dála”) who are members of the “Dáil” (the lower house of the Irish parliament) with the alphabetic distribution of the population. The paper then uses statistical graphs to assess how the Irish electorate have voted since 1922. (In Ireland, candidates’ names are listed alphabetically on ballot papers.) The paper concludes with an opinion on whether Ireland ought to retain alphabetical listing.

Keywords: ballot randomisation; donkey voter; Electoral Register; Benford’s law; Markov chain.

1 Definition

A *donkey voter* [11] or *top-to-bottom voter* [13] is someone who votes for candidates based solely upon the sequential order in which they appear on a ballot sheet, rather than taking the time to number the candidates in the voter's own thought-out order of preference.

In countries where voting is compulsory, apathetic voters sometimes cast donkey votes just to avoid a fine.

2 Finding an Adequate Comparison Frame

The Central Statistics Office (CSO) in Ireland conducts total enumerations of the population in the quinquennial census. Nevertheless, the CSO neither compiles nor maintains databases of named persons. The CSO abides by the principle of using information for statistical purposes only. This reassures the public’s confidence in the CSO as an independent body.

3 The Electoral Register

The national Electoral Register records electors in ‘Polling Books’ for 3,400 District Electoral Divisions.

Random samples are over-represented in larger households, which contain more electors, and so have a higher probability of selection than addresses that contain a smaller number of electors. A sampling bias also arises from the non-listing of households in which no member appears on the Electoral Register.

In their 1973 paper on Alphabetical Voting, Robson and Walsh [10] used as a benchmark frame the alphabetical distribution of a random sample of 2,100 people from the national Electoral Register. They grouped those names into five sets: A–C, D–G, H–L, M–O and P–Z, with an average of 420 names in each set.

In a written answer to Parliamentary Question Number 484 on 17 Feb 2004, the Data Protection Commission [2] made it clear that it is no longer legal (since 2001) to use the “full” Electoral Register for anything other than electoral or statutory use, even if it were possible to get hold of it.

The “edited” register lists only persons who have indicated that they have no objection to their details being used for purposes other than electoral or other statutory uses.

Persons contacted for interview who had opted out of the edited register might well raise objections and seek to find out how their names had been obtained. If illegal uses of the register became widespread, then it could be brought into disrepute—perhaps to the point where some people might choose not to register to vote at all.

Apart from that, a bias would result from using the edited version, because it would be improbable that those who opt out are a random sample of the full version.

4 Matheson's Methodology

In 1894, Sir Robert E. Matheson [7] issued his *Special Report on Surnames in Ireland* as an Appendix to the *Twenty-ninth Report of the Registrar-General of Marriages, Births and Deaths in Ireland*. It was the first detailed official work on surnames in Ireland

Matheson's methodology was to list all surnames (including their variations) that accounted for five or more births in the year 1890. He tabulated some 2600 names with the total number of births for each of those names.

He then multiplied the numbers of births in 1890 by 44.8 (which was the overall Birth Rate per 1,000 at that time) and rounded the result to the nearest hundred to estimate the population size of each surname stratum.

He listed the 100 most numerous names in the country at large with an estimate of the numerical frequency of each surname. These estimates are available at Freepages [3].

Matheson's printed opus is in two parts. The first part is a long table of statistics based on the registration of births in all of Ireland in 1890. For each name, there are six columns: the total registered, number registered in each of the four provinces, and notes on the counties in which each name was principally found.

The second part presents a list and index of names with variant forms and includes four fascinating, and sometimes amusing, chapters on spelling, contractions, interchangeable names, and English and Gaelic forms.

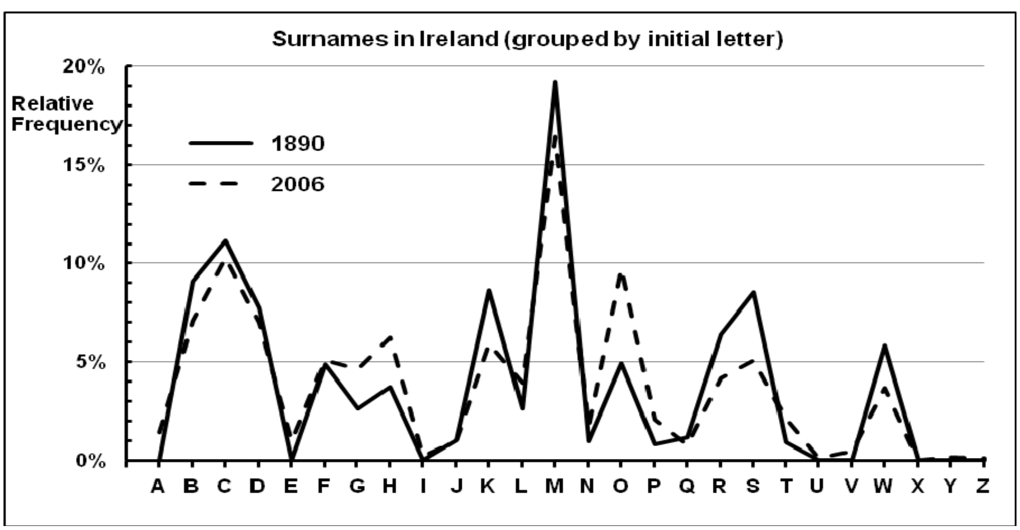
Matheson's ingenious method gave a rational approximation to the frequencies of surnames in Ireland in 1890. Incidentally, the roundings in the arithmetic generated relatively small margins of error that are best estimated using Chebyshev's inequality.

Madison's frequency table of surnames covered the island of Ireland. This is a disadvantage if used to compare the distribution of surnames of candidates successful in general elections in the Republic of Ireland with the corresponding distribution among the electorate.

5 Telephone Directories

I adopted a different approach to derive similar data. I comprehensively sampled all six residential telephone directories that covered the Republic of Ireland for the year 2009. I counted the numbers of pages devoted to each surname categorised by initial letter. I multiplied those numbers by the average number of entries per page. Then, I aggregated and rounded the results to the nearest 100 to estimate the total number of residential phone users. Lastly, I multiplied those data by a grossing factor to reflect the total population according the census of 2006.

Figure 1



6 Comparison of Methodologies

Although the compilation methodologies differ, the sets of data for 1890 and 2006 are highly correlated.

The most noticeable change is the decrease in the relative frequency of surnames that begin with the letter "M" and the partial switch of frequencies from the reduction of names beginning with "S" to increase the number of names beginning with "O".

For his analysis, Matheson used the most commonly found forms of surnames. Thus, he listed Shea rather than O'Shea and Sullivan rather than O'Sullivan. After 1890, there was a tendency to resume the "O" in names that had previously dropped it. Surnames beginning with "O" are found to cluster in southwestern Ireland.

Analysis of each regional telephone directory shows that surnames beginning with "M" are relatively more numerous in the northern half of the island of Ireland. The decrease with respect to "M" is explained by the fact that the compilation for 1890 included Northern Ireland (where names beginning with "Mac" or "Mc" predominate), whereas the compilation for 2006 covered only the Republic of Ireland. The 2006 data correspond to the area represented by TDs in the Dáil.

Figure 1 compares the data derived from the telephone directories with the result of the classic research by Matheson.

In passing, it may be observed that an inverse square root transformation of the rank order of surnames (grouped by initial letter), linearises the relationship of rank order with surname initial letter frequencies.

For the 1890 data, the relationship between the rank order of a name and the frequency of the name can be linearised, as shown in Figure 2.

A similar linearised relationship exists between the rank order of letters and their relative frequencies in the 2006 data, as shown in Figure 3.

7 An Adaptation of Benford's "Law"

It might be expected, prima facie, that roughly the same number of surnames would begin with each letter of the alphabet and that the proportion of surnames beginning with any given letter would be roughly uniformly 1/26.

However, for many kinds of alphabetic data, the distribution of initials is highly skewed. A precise mathematical relationship, (known as *Benford's law* for numeric data) seems to hold (when adapted to model alphabetic data). See Plus maths [9].

This law does not work for truly random sets of data. It works best for data that are neither completely random nor overly constrained, but rather lie somewhere in between. These data can be wide ranging, and are typically the result of several processes, with many influences.

Figure 2

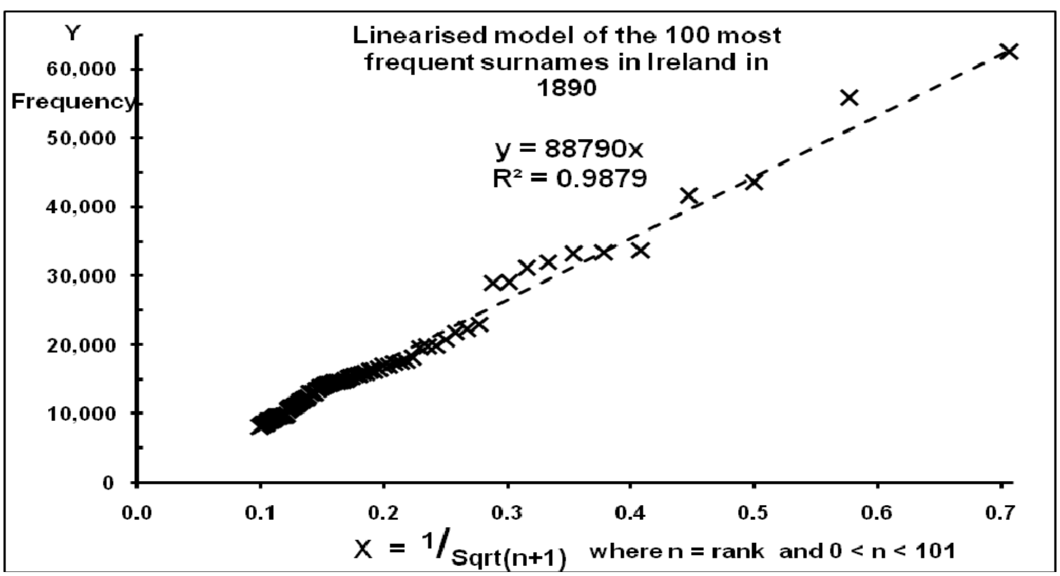
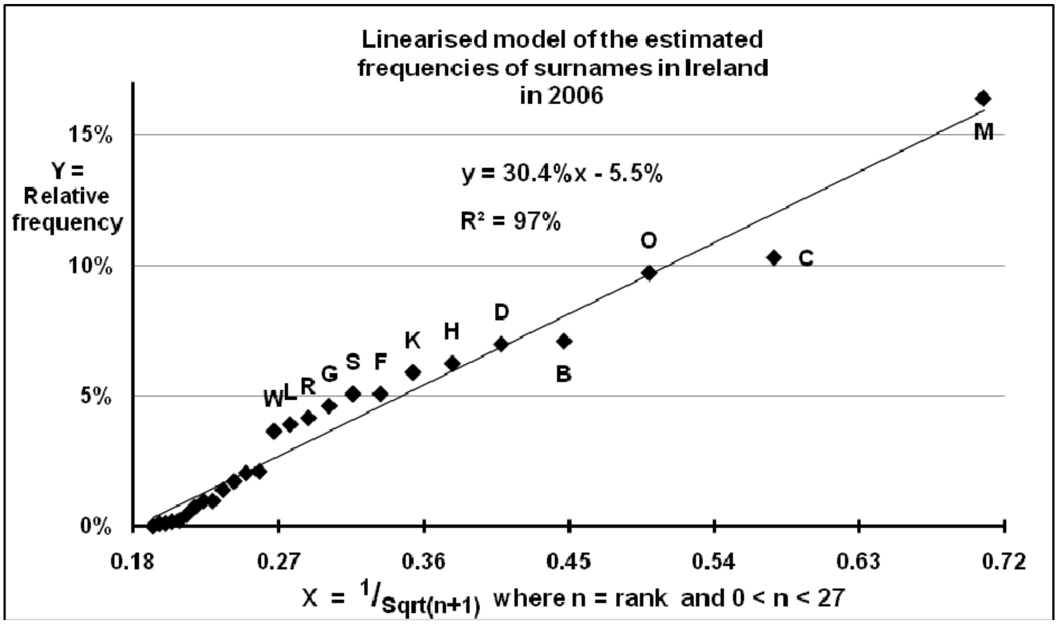


Figure 3



The expected proportion of surnames beginning with any letter is

$$\log_{27}[(n+1)/n],$$

where $0 < n < 27$ is the frequency rank of the letter. The cumulative function of

$$y = \log_{27}\{(n+1)/n\}$$

is

$$\Sigma y = \log_{27}(n+1).$$

8 Review of All Results in the 28 General Elections from 1922 to 2007

The analysis in this section is based on a historical list of TDs provided by Wikipedia [12].

The R^2 statistics measure how well the alphabetic distributions of TDs' surnames correspond with the comparison population.

A pattern can be observed in the graph of the R^2 values over time.

Figure 4 shows that from 1922 to 1965, the Irish electorate tended more to favour candidates who had a higher alphabetic ranking on the ballot papers.

The lowest R^2 value was in 1969.

The 1960s saw a surge of economic growth in modern Ireland. The establishment of a national television station in 1961 broadened political debate among the electorate. Free secondary education was introduced in 1966 for all social classes in both urban and rural areas. In 1972, the voting age was reduced from 21 to 18. Ireland joined the European Economic Community in 1973.

From 1973 to 2007, the trend has been away from alphabetical voting. In 2002, the R^2 value had regained the high level recorded in 1922.

Figure 5 shows the randomness of the spreads of the relative frequencies of surnames from 1922 to 2007.

Figure 6 compares the frequency of surname initials among TDs with the frequency of surname initials in the population. There are two outliers in Figure 6. They indicate that on average, historically, candidates whose surnames began with *B* had an advantage over candidates whose surnames began with *M*.

9 Analysis of All TDs Elected in 2007

The analysis of all TDs elected in 2007 is based on a report of the Government of Ireland [5]. Figure 7 illustrates the divergences of the frequencies of TDs' surnames from the corresponding population frequencies. The divergences are randomly scattered.

Figure 8 shows that the distribution of the surnames of TDs elected in 2007 does not conform to "Benford's Law". *The inference to be*

Figure 4

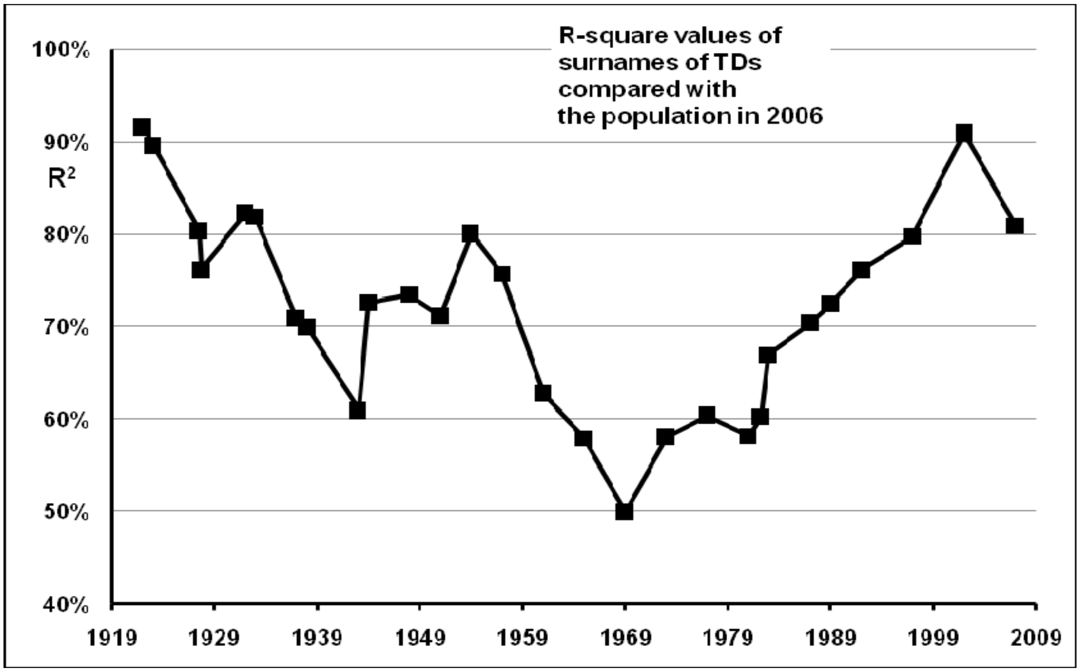


Figure 5

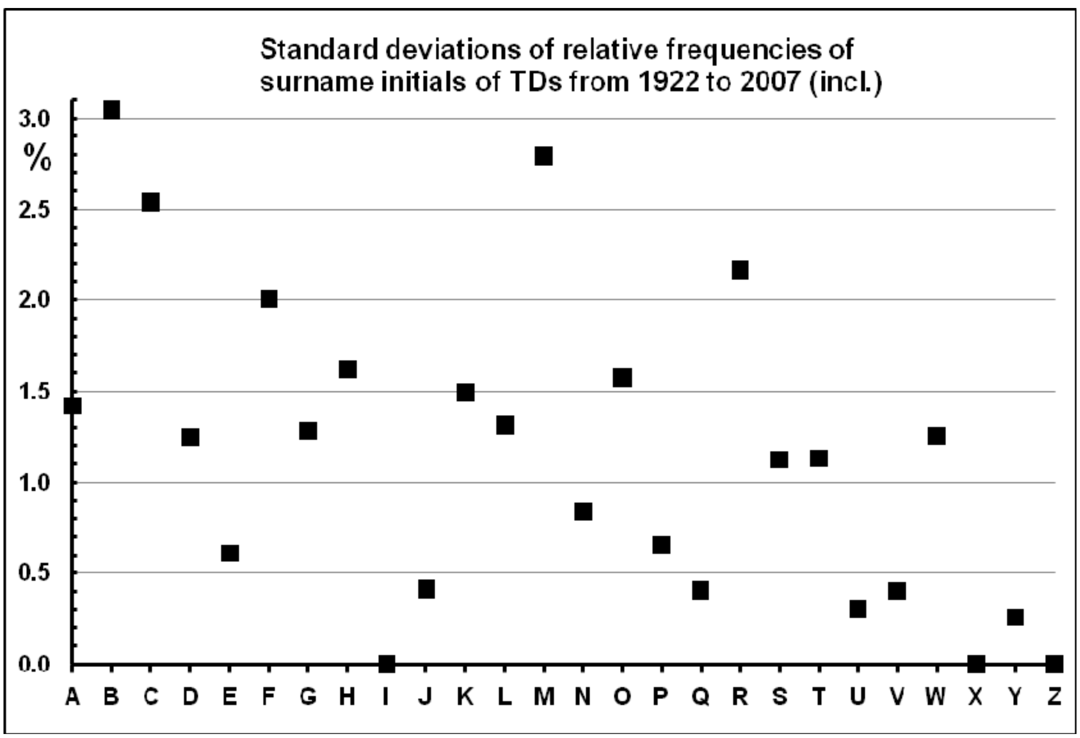
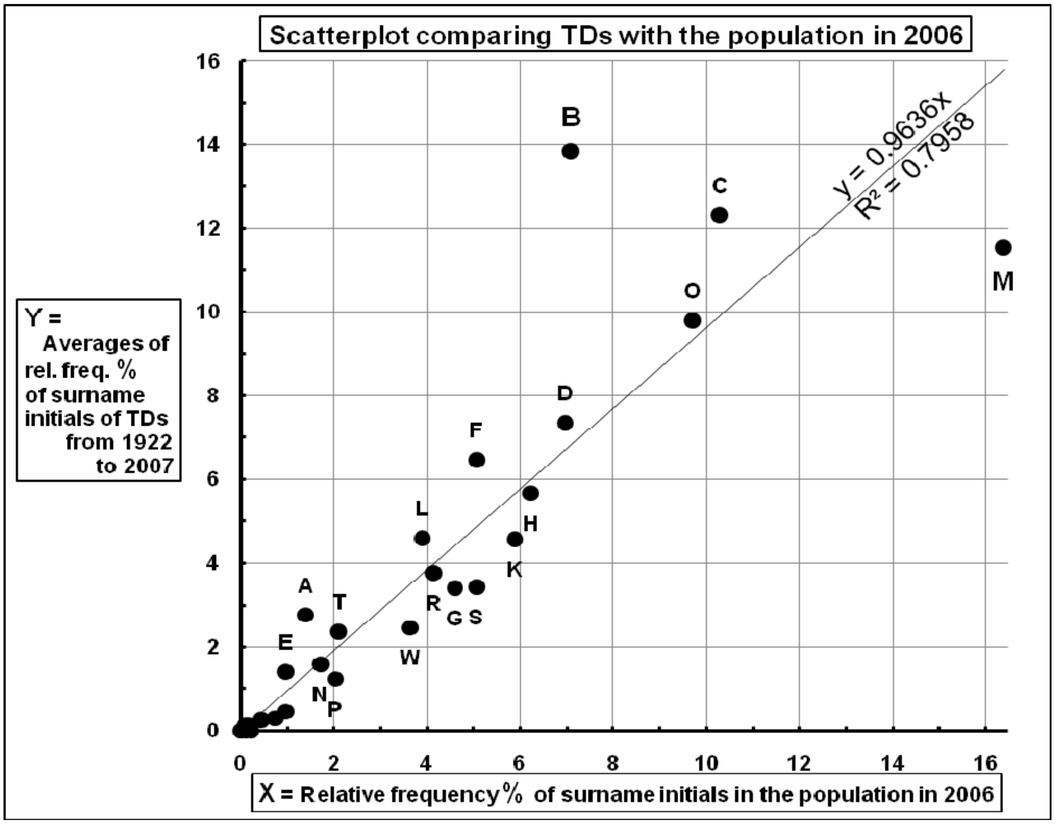


Figure 6



made is that the electorate did not tend to vote in accordance with the order of appearance of candidates' names on a ballot paper.

Table 1 summarises the average perpendicular distance of the cumulative datasets from both the cumulative Uniform and cumulative Benford distributions.

Table 1

Variable	Standard deviation of variable from	
	Uniform	Benford
1890 Population	6.3%	12.4%
2006 Population	7.0%	12.5%
TDs (2007)	6.2%	10.6%

On average, both population and TDs' datasets are closer to the uniform distribution than they are to Benford's.

10 Applying Markov Chain Theory to Derive Long-Run Results

What will be the distribution of TDs' surnames in the long-term? Construct a matrix of the relative frequencies of surnames beginning with the 23 letters of the alphabet in columns (excluding *I*, *X* and *Z*) by the 23 Dáils in rows from 1933 (the 8th Dáil) to 2007 (the 30th Dáil), inclusive.

The total of the elements in each row is unity (100%).

Calculations using Markov chains then indicate that in the future the distribution of surnames in the 34th Dáil, sometime in the future, will be as shown in Figures 9 and 10.

Since the lifetime of a Dáil can last for up to five years, this could be a 20-year prediction.

It is clear from Figure 9 that the predicted over-representation of surnames beginning with *B* will be matched by the under-representation of surnames beginning with *M*. Excluding *A* and *B*, there is a general oscillation between

Figure 7

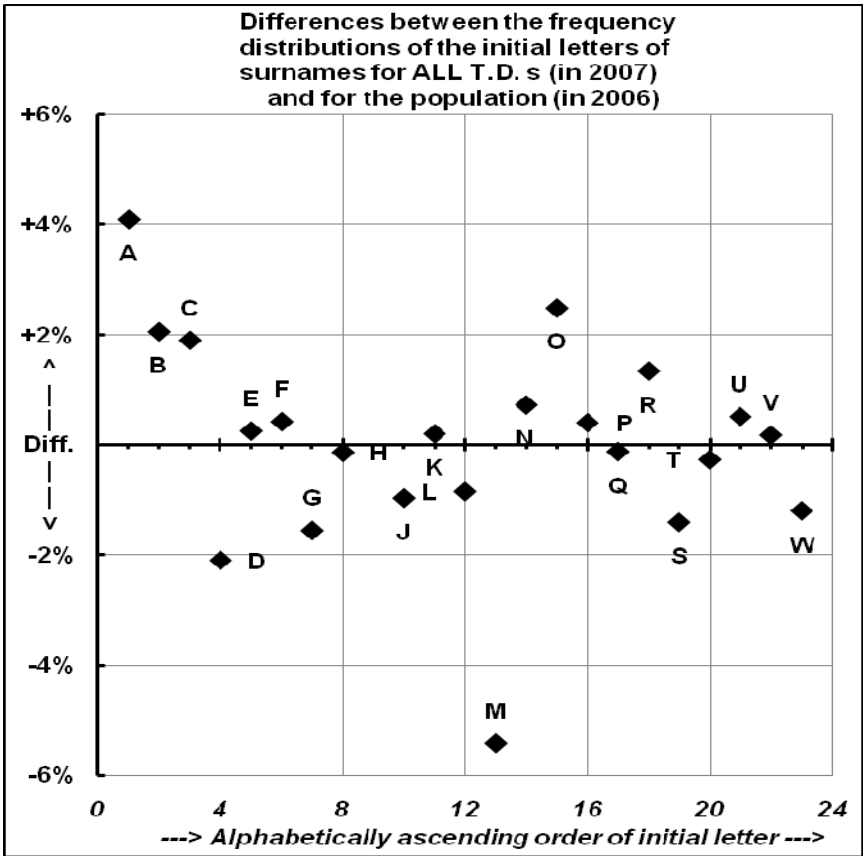


Figure 8

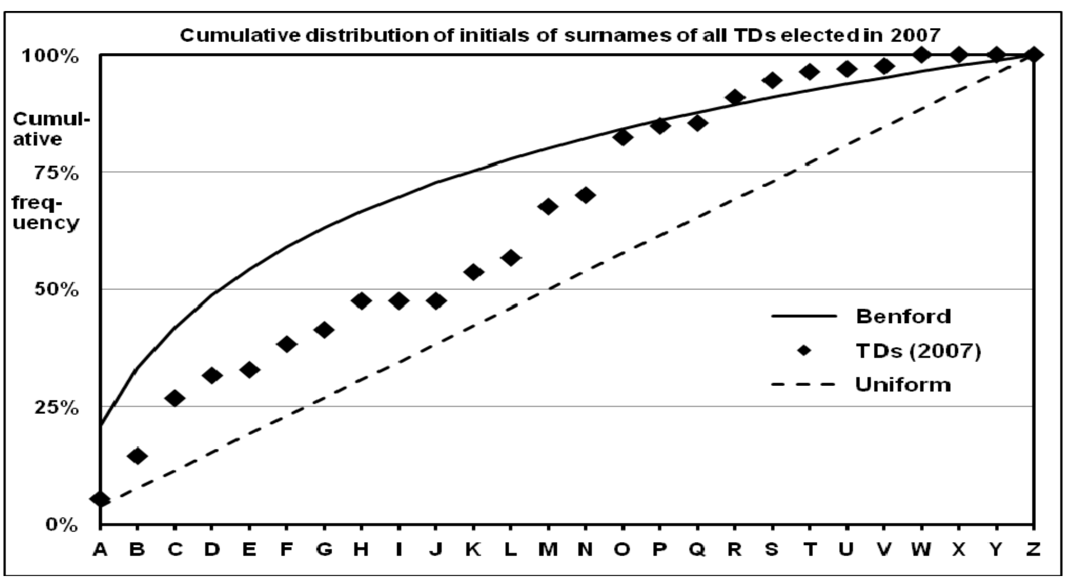
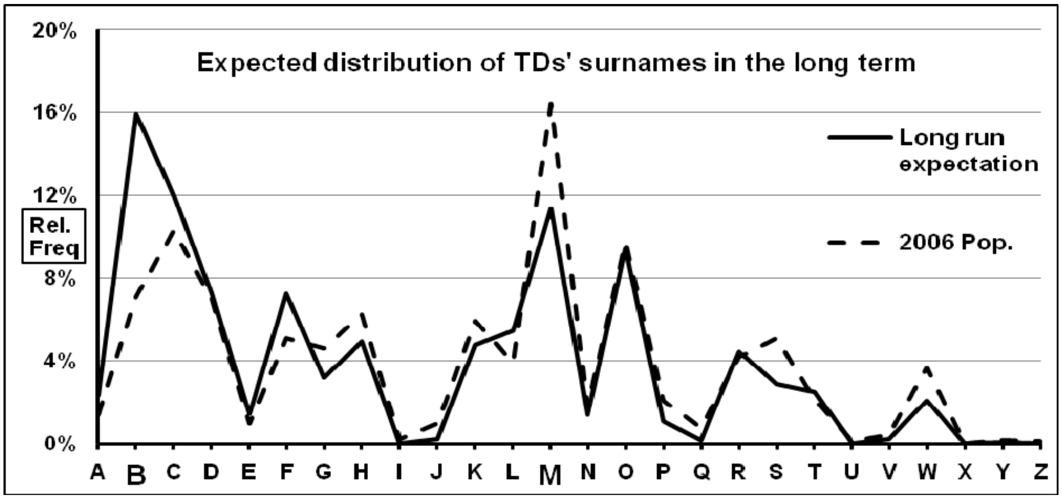


Figure 9



successive letters from *C* to *W* in the relative frequencies of surnames that begin with those letters.

11 Representation of Women in the Dáil

Figure 11 shows that at least one woman has always been elected to the Dáil. From 1922 to 1969, the average number of women TDs was 3.65. Since 1969, that number has grown exponentially, reaching 23 in 2002.

From independence to the present day, 73 women have won 223 seats (an average of 3.05 election victories per woman). Figure 12 shows that frequency distribution.

Figure 13 shows the resemblance between the alphabetic distribution of surnames in the population and the corresponding distribution for the 73 women who were elected down through the years.

12 Political Dynasties

For many years, a feature of Irish political life has been the tendency for sons and daughters to "inherit" the parliamentary seats previously held by their parents. This tends to happen most often in by-elections, where a bereaved candidate often attracts a significant "sympathy vote".

A sine wave curve accounts for nearly two-thirds of the variation in the dataset illustrated in Figure 4. It implies that the trend in the "inheritance" pattern in political history repeats itself every 125 years (which span five

Figure 10

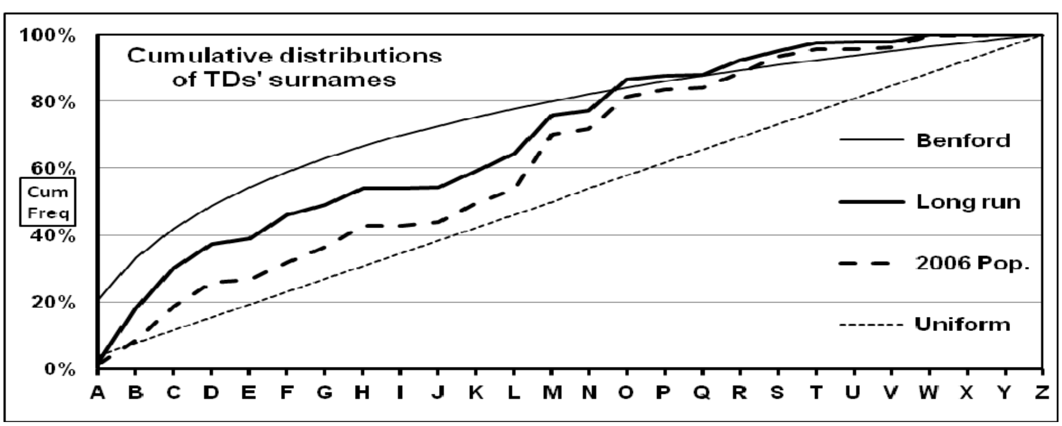
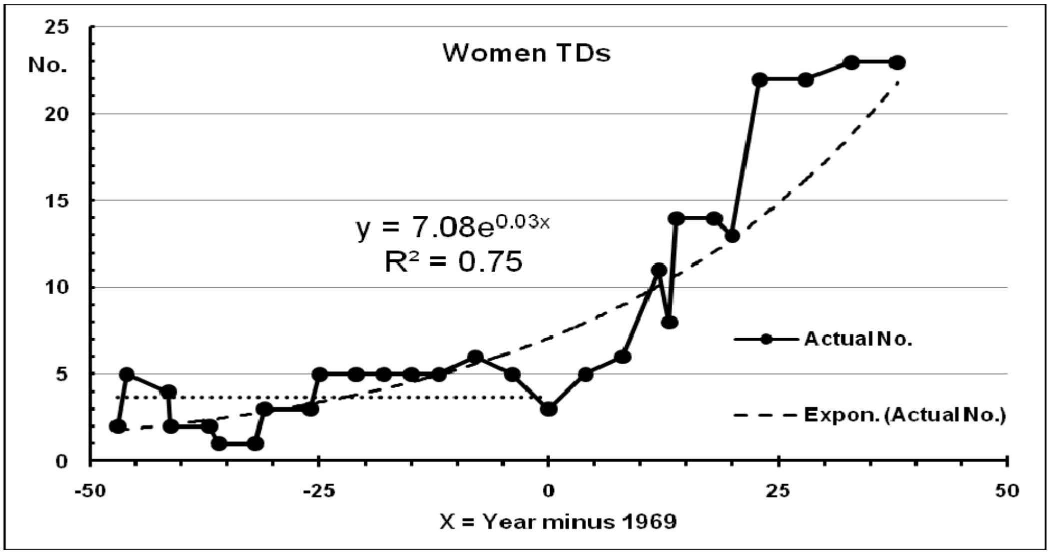


Figure 11



generations from great-great-grandparent to grandchild). See Figure 14.

13 Data Correlations

Table 2 shows the R^2 matrix of the datasets studied in this paper up to this point. Taking the variables pairwise, each R^2 value quantifies the amount of variation within one of those variables that can be accounted for by a linear model of the other variable.

The correlation of the TDs in 2007 with the population in 2006 is stronger than the correlation of the TDs in 2007 with the disaggregated cumulative Benford distribution.

14 Comparisons of "First-Preference" and "First-Past-The-Post" Voting Systems

The analysis in this section is based on a report of the Government of Ireland on the election of 2002 [4]. Figure 15 shows that the estimated frequencies of initial letters of surnames in the population of Ireland in 2006 explain 86% of the variation of first preference votes for successful candidates in the 2002 general election. This percentage rises to 92% for 2002 if all first preference votes for both successful and unsuccessful candidates are taken into account (see Figure 16).

Figure 12

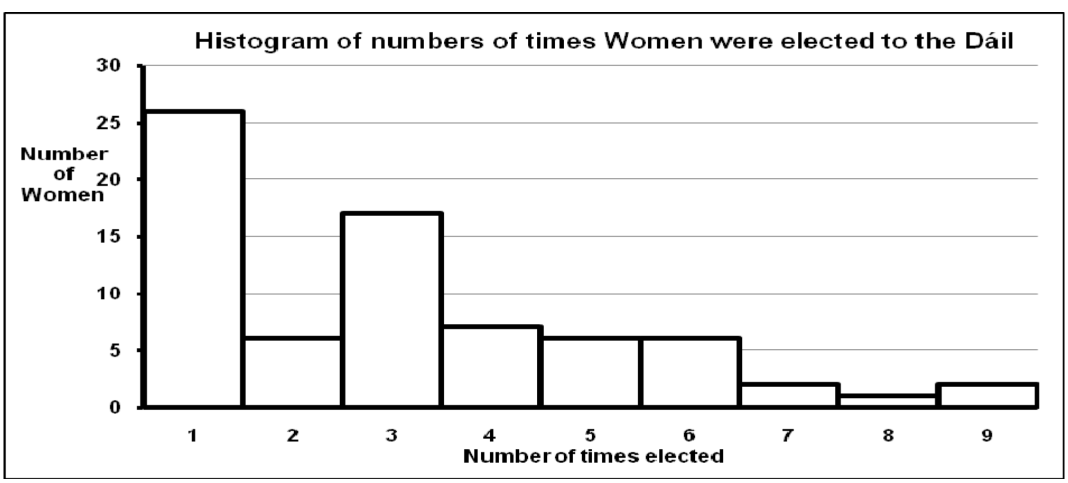


Table 2

R^2	I	II	III	IV	V	VI	VII	VIII
I	100%							
II	85%	100%						
III	1%	2%	100%					
IV	0%	0%	0%	100%				
V	62%	73%	14%	0%	100%			
VI	61%	81%	14%	0%	81%	100%		
VII	66%	80%	14%	0%	98%	87%	100%	
VIII	57%	77%	9%	0%	87%	88%	90%	100%

KEY to Table 2					
I	=	**1890 Population**	**V**	=	**Long run expectation**
II	=	**2006 Population**	**VI**	=	**ALL TDs (2007)**
III	=	**Benford Distribution**	**VII**	=	**ALL TDs (ALL years)**
IV	=	**Uniform Distribution**	**VIII**	=	**Women TDs (ALL years)**

The proportional representation system of voting does not necessarily always elect the candidates with the highest number of first preference votes.

One might wonder if the results under the "first-past-the-post" system would reflect the distribution of surnames in the population better, but the R^2 statistic for such a system is only 81%. Thus, in 2002 proportional representation reflected the general population better than the "first-past-the-post" system.

Similarly, based on a report of the Government of Ireland on the election of 2007 [5], Figure 17 shows that the estimated frequencies of surnames explain 79% of the variation of first preference votes for successful candidates in that election. This percentage rises to 89% for 2007 if all first preference votes for both

Figure 13

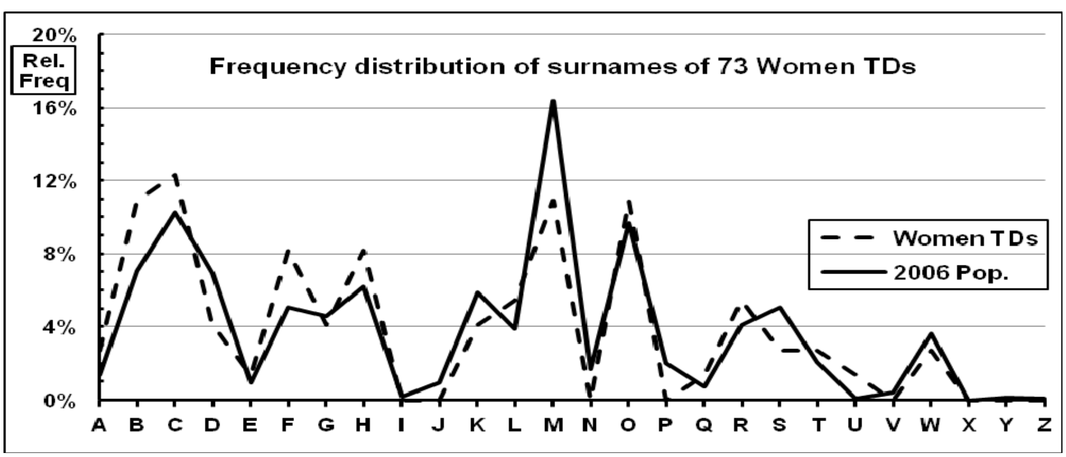
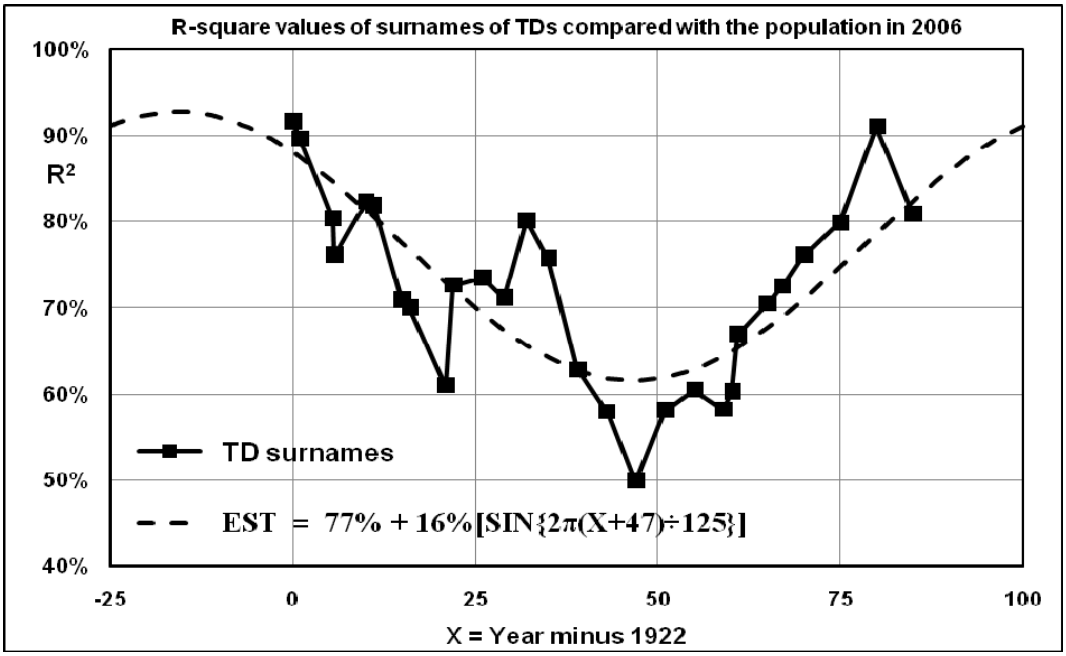


Figure 14



successful and unsuccessful candidates are taken into account (see Figure 18).

For 2007, the results under the "first-past-the-post" system do not reflect the distribution of surnames in the population any better (the R^2 statistic is also 79%).

The surplus votes from candidate with a very large number of first preference votes can, when transferred, benefit another candidate who did not poll so well. A system of "vote management" has developed, where the constituency is divided into canvassing areas based on such candidates' home bases.

Figure 15

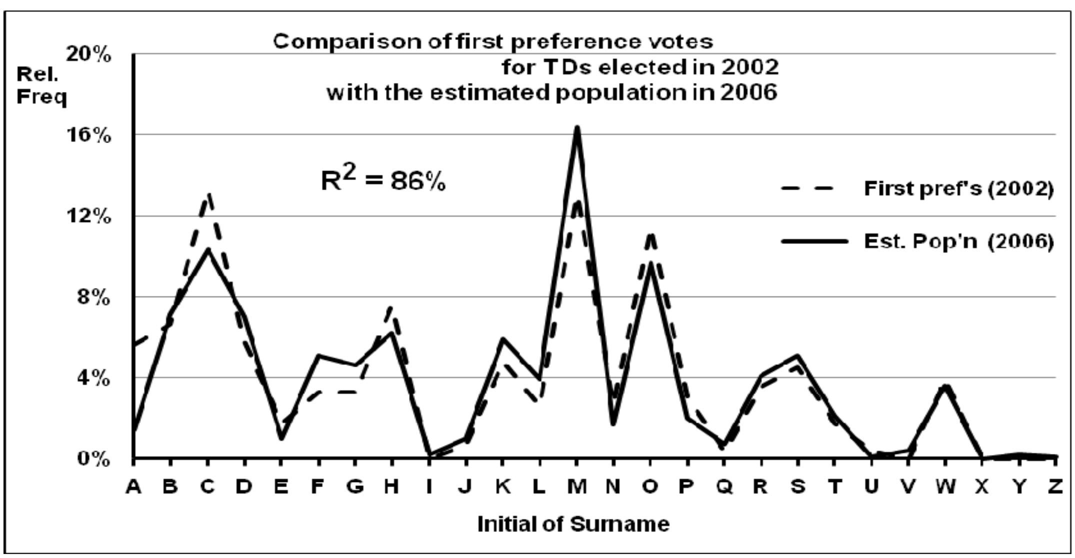
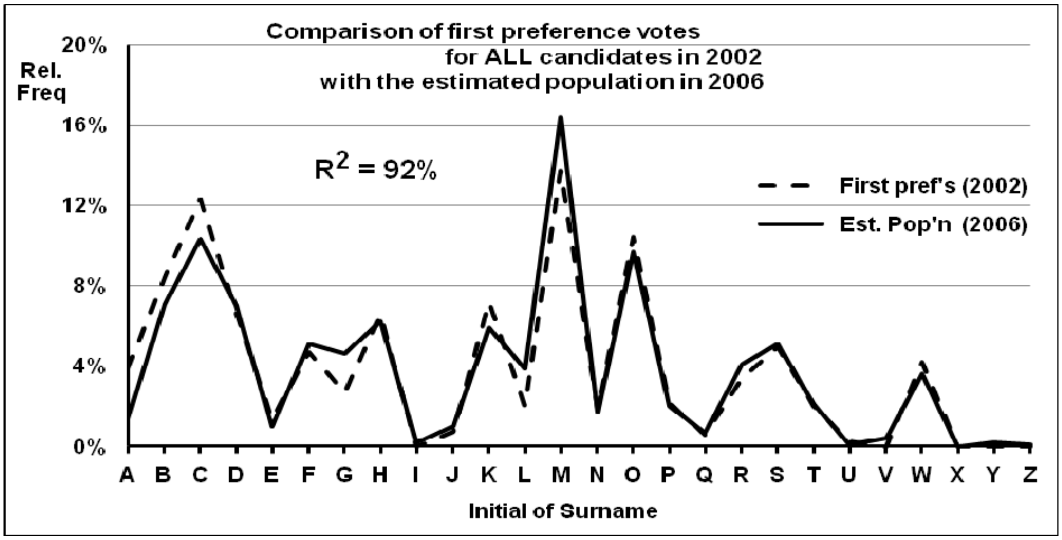


Figure 16



15 Summary

1. Matheson's data for 1890 are highly correlated with the set of data for 2006 extracted from the telephone directories.
2. The alphabetic distribution of TDs' surnames corresponds well with the general population.
3. From 1922 to 1969, the Irish electorate tended more and more to favour candidates who had a higher alphabetic ranking on the ballot papers, but this trend was reversed from 1969 onwards.
4. The divergences of the frequencies of TDs' surnames from the corresponding population frequencies are randomly scattered.

Figure 17

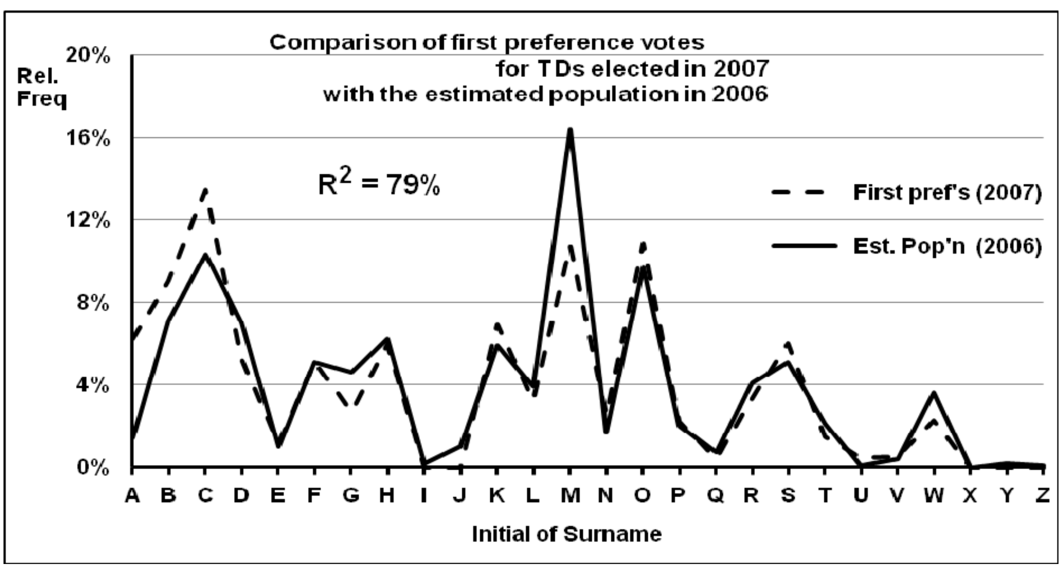
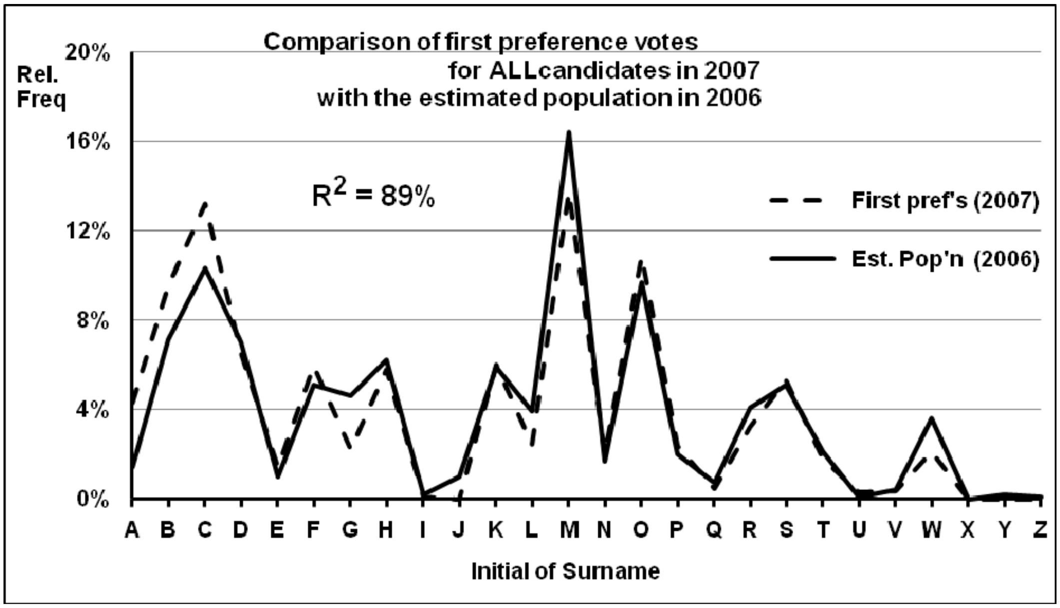


Figure 18



5. The divergences of the frequencies of TDs' surnames from the corresponding population frequencies are randomly scattered.
6. Based on past results, in the long-term future, the over-representation of surnames beginning with *B* will be matched by an under-representation of surnames beginning with *M*. Surnames of TDs beginning with letters other than *B* and *M* will mirror their distribution in the general population.
7. Since 1969, the number of women elected to the Dáil continues to grow.
8. The distribution of the surnames of women TDs conforms closely to the population's distribution.
9. Political dynasties are a fact of Irish political history.
10. In general, results under the "first-past-the-post" system do not reflect the distribution of surnames in the population better than the proportional representation system.
11. In 2007, the electorate did not vote strictly in accordance with the order of appearance of candidates' names on the ballot papers.
12. In the 2002 and 2007 Irish general elections, candidates did not gain a significant advantage exclusively because of their alphabetical order of appearance on ballot papers.

16 Conclusion: Should Ireland Retain Alphabetical Listing?

In the 1986 case of *O'Reilly v Minister for Environment*, the Irish High Court upheld the constitutional validity of alphabetical listing against an equality-rights challenge. The court noted that despite its faults, A to Z does have the advantage of making it easy to find candidates on the ballot-paper.

Since 1965, the political party to which a candidate belongs is printed beside the candidate's name. (See Marsh [6].) Candidates' photographs and their party logos have appeared on ballot papers since 2002 (Office of the Attorney General [8]). These measures partially offset any perceived advantage arising from the listing of surnames alphabetically.

Furthermore, many Irish voters have strong political opinions and vote for the party for which they have always voted. Intelligent voters go to the polls with their minds made up beforehand on how they choose to vote.

Usually, there are only between eight and 15 candidates on most ballot paper. Voters are not bewildered by a multitude of names and so are less likely to take the easy option to vote like donkeys, particularly as they are not obliged to list their preferences for all candidates.

Since 1979, Australia has used a system called Robson Rotation. Each ballot paper contains a different permutation of candidates. Each candidate's name appears a certain proportion of times at every position on the paper. This disperses the donkey votes equally and nullifies their impact on the result.

Computerised randomisation could be incorporated into the modern processes for printing ballot papers. The order of the candidates could also be rotated so that if there were ten candidates, each would head the ballot on one-tenth of the papers. Such a system was used in New York City from 1937 to 1947.

Electronic voting machines costing €52 million were tested in three constituencies in Ireland in the general election of 2002. Subsequently, much debate and serious doubt arose about the accuracy of the software (Coyle et al. [1]). The system was abandoned. The annual cost of insurance and storage for those idle machines is about €800,000. In the interests of openness, transparency and accountability, any randomisation software for ballot papers would have to pass stringent tests to satisfy public confidence.

We should never under-estimate the collective intelligence of the electorate. They are not such donkeys as is commonly supposed. When all of the votes have been cast and counted, Democracy is the only guarantor of the least evil. People elect governments whom they think will do the least harm, but do it very well. In the final analysis, we are governed by the best of a bad lot.

The only thing as bad as a 'donkey-vote' is a 'reverse donkey-vote' (or a 'bottom-to-top vote').

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About the Author

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