STV with Elimination of Discounted Contenders

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Abstract

Although it has many advantages, STV can occasionally yield perverse outcomes, because excluding the candidate with the fewest current votes can exclude a candidate who is better supported than others who remain in contention. STV with Elimination of Discounted Contenders (STV-EDC) substitutes a more sophisticated exclusion rule that ensures that the candidate selected for exclusion is not as well supported as those who remain in contention.

1 Introduction

Although STV is arguably the best electoral system in use for public elections today, like all systems it has its flaws. The STV rules applied in public elections differ to a greater or lesser extent in detail, but they all agree that when all surpluses have been transferred, the non-elected candidate who at that point is the first available preference on fewest votes is excluded, i.e., is not considered further in the course of the current count. A candidate who has much support but few first preferences can sometimes be excluded before the extent of that support has become apparent; this is illustrated by Election 3 below, in which two seats are contested and nobody is elected before the first exclusion. Conventional STV elects B and C despite E's bring ranked above them on more than two Droop quotas of votes in each case. I consider conventional STV's exclusion of E in this example to be perverse.

The position in which voter A ranks candidate X indicates A's desire not only for X to be elected rather than any candidate ranked below X, but also for X to be eliminated rather than any candidate ranked above X. It may be that A has no strong feelings about some of the candidates ranked above X and has expressed preferences for them simply to reduce the probability of X's election; if voting in an election in which British National Party candidates were standing, I personally would cast a preference for every non-BNP candidate but none for the BNP. If X has least support on the votes available to the contending candidates, it is reasonable to presume that a significant number of voters have voted against X and that effect should be given to their wishes.

I take it as axiomatic that in any version of STV a candidate who has attained a surplus when only originating surpluses and consequential surpluses arising from them have been transferred has an absolute right to a seat. When all the candidates who definitely should be elected have been elected, the emphasis should shift to identifying candidates who definitely should not be elected.

The aim of each round of STV with elimination of discounted candidates (STV-EDC) is to identify then eliminate the one *contending candidate* (ie, a candidate who is neither elected nor eliminated) who has less available support than any other; further rounds take place if there are seats not yet filled. STV-EDC is based on the fact that at least a Droop quota of the votes active at any given point in an election will not help to elect anyone; it awards those votes to a notional candidate and the common value of the contending candidates' votes is discounted to make up that notional candidate's quota. Thus every preference of every voter contributes to the tally of votes considered when a candidate is eliminated.

2 How STV-EDC works

An STV-EDC count is a series of rounds. Each round except the last has two stages, first an election stage then an elimination stage. Each round except the last culminates in the elimination of one candidate. In the election stage of the final round, all seats are filled before there is a need to exclude a candidate, and the election is over.

a. *The election stage*. The election stage is a conventional Meek count, except that contending candidates who attain the quota after the first exclusion are not classified as elected but remain contending; their surpluses are transferred in the normal way. When s candidates (where s is the number of seats being contested) have attained the quota, the election stage ends.

b. *The elimination stage*. Candidates excluded in the election stage are reclassified as contending. On every vote which bears a preference for *any* candidate who has not been eliminated in a previous STV-EDC, a preference for a notional candidate N is inserted immediately following the voter's final expressed preference. Each elimination stage is a quasi-Meek round using the final quota q inherited from the immediately preceding election stage. A candidate's *keep value* (kv) is the fraction of any incoming vote or part-vote that that candidate retains, passing the rest on to the next available preference, if any, otherwise to non-transferable. An initial kv of 1 is set for the notional candidate N and an initial common kv between 0 and 1 for the contending candidates; initial kvs between 0 and 1 are set also for the elected candidates (if any). The kvs of N and the elected candidates are adjusted upward or downward until they all have at least q votes; the common kv of all the contending candidates is adjusted upward or downward until the contending candidates collectively have fq votes or fewer where f is the number of seats yet to be filled. When it is known to be impossible for the *lowest candidate* (ie, the contending candidate with fewest votes) to get more votes than the lowest-but-one candidate, the lowest candidate is eliminated, preferences for N are deleted and the STV-EDC elimination stage ends. A suggested counting algorithm for the elimination stage is provided in the Appendix.

Consider the following election for one seat:

Election 1 (1 seat)

35	AD
34	BD
31	CD

In the election stage, D, C and B are excluded in that order and the final quota is 34.5. The initial quota is 50; A does not attain it before the first exclusion and so is not elected. In the first iteration of the elimination stage with the common keep value t set to 0.15910 and N's kv to 1.0, effective votes are:

A	5.56863
B	5.40952
C	4.93221
D	13.37896
N	70.71068

The common kv t for A, B, C and D has to be recalculated so that their collective total of votes is nearer to 34.5; this is 0.18741 (to 5 decimal places – the calculations in this example are actually performed to 13 decimal places). Effective votes are now:

A	6.55930
B	6.37190
C	5.80967
D	15.22867
N	66.03046

In the next iteration N's kv is set to 0.53052, making N's votes 35.03046, but leaving the other candidates' votes unchanged. The total surplus (ie, the difference between N's votes and the total of the votes of A, B, C and D) is 1.06092. In the next iteration, t is reset to 0.19034 and votes are:

A	6.66173
B	6.47140
C	5.90039
D	15.41077
N	34.77859

The total surplus is 0.33430, less than the difference between the votes of B and C, so C is eliminated; this ends the first round. In like fashion, B is eliminated in the second count. At the election stage of the third round A gets 35 votes and D gets 65, so D is elected.

The Condorcet winner (if any) will usually, but not always, be elected in an STV-EDC count for one seat. This is because, as seen above, the Condorcet winner will usually not be the lowest candidate in an STV-EDC round and will thus escape being eliminated.

3 Discussion

Any counting system worthy of consideration should observe Woodall's Droop Proportionality Criterion (DPC) [5], which he stated thus:

> *If, for some whole numbers* k *and* m *satisfying 0* < k ≤ m, *more than* k *Droop quotas of voters put the same* m *candidates (not necessarily in the same order) as the top* m *candidates in their preference listings, then at least* k *of those* m *candidates should be elected.*

STV-EDC possesses this property. A group of voters who prefer every candidate within a set to any candidate outside it are said to *solidly support* that set.

> *Proof: Let there be in an STV-EDC election a set of* m *candidates whom* k *(where* m ≤ k < m + 1*) Droop quotas of voters solidly support. All these candidates would be elected at the election stage of a count because all surpluses would be transferred to other members of the set before being transferred to non-members. If the set instead contained* m + n *candidates where* n ≥ 1, *but with the same number of voters solidly supporting it, then if one member of the set were eliminated it would still contain at least* m *candidates.*

The contending candidate who ultimately gets fewest votes in an elimination stage has less available support than any other and for that reason has a worse claim to a seat than any other contending candidate.

4 STV-EDC or Sequential STV?

Sequential STV was originally devised by David Hill [1]; he withdrew the original version in favour of a revised version on which I collaborated with him [2, 3]. Its aim was to identify a set of *s* candidates which, when tested against all the other candidates one at a time, was the most appropriate set to be elected. A problem with Sequential is that special measures are needed to break paradoxes; barring ties, STV-EDC needs no such measures. I believe the systems to be broadly comparable in terms of outcomes and computer time.

Consider how STV-EDC treats Elections 2 and 3, which have been used to test Sequential STV. These are presented side by side so that the differences between the two can be seen more easily.

Election 2 (2 seats)		Election 3 (2 seats)	
104	ABCD	104	AEBCD
103	BCDA	103	BECDA
102	CDBA	102	CEDBA
101	DBCA	101	DEBCA
3	EABCD	3	EABCD
3	EBCDA	3	EBCDA
3	ECDBA	3	ECDBA
3	EDCBA	3	EDCBA

422 votes are cast. Election 3 differs from Election 2 only in that the voters whose first preferences were A, B, C or D have inserted E between their first and former second preferences. Meek elects B and C in both, but Sequential elects BC in 2 and BE in 3. STV-EDC endorses Sequential.

The following example devised by Douglas Woodall shows that Sequential does not always elect the set of *s* candidates that beats every other candidate in contests of *s* + 1; Sequential elects C and D, but AB is the set that beats all comers. However, it is arguable that AB is not the best set of candidates to elect.

Election 4 (2 seats) – Woodall's Torpedo

11	AC
9	ADEF
10	BC
9	BDEF
10	CA
10	CB
10	EFDA
11	FDEB

STV-EDC elects C and F. Owing to a paradox involving D, E and F, this outcome is as acceptable as that of Sequential. Neither Sequential nor STV-EDC achieves Sequential's stated objective in this case.

It would be interesting to analyse differences in outcomes between Sequential, STV-EDC and Nicolaus Tideman's STV with comparisons of pairs of outcomes (CPO-STV) [4]. CPO-STV compares each possible set of *s* candidates with every other; the set that gets more support than any other is elected. The sets compared are those that contain all the candidates who would have been elected before the first exclusion in a conventional Meek count; if there are no such candidates, all sets are compared. The different approach of CPO-STV self-evidently avoids the problem of premature exclusion but at the price of sometimes having to select one of the competing potentially winning sets of s candidates by a separate process.

I conclude that there is little or nothing to choose between Sequential STV and STV-EDC in terms of outcomes, but this conclusion must be provisional until much research which I am unable to do has been completed. The different approach of CPO-STV is likely to produce different outcomes in some circumstances; whether the outcomes of CPO-STV are better or worse than those of STV-EDC with the same voting profiles again cannot be determined without much research. STV-EDC has the advantage that (barring ties) it gives one definitive outcome in every case.

I believe that STV-EDC offers a workable and robust solution to the problem of premature exclusion arising from the exclude-the-lowest rule in conventional STV.

7 Acknowledgement

I am grateful to David Hill who rescued me from a fatal error in an earlier version of this paper.

8 References

[1] Hill, I. D. (1994) Sequential STV, *Voting matters* 2, 5-7.

[2] Hill, I. D. and Gazeley, Simon (2002) Sequential STV: A New Version, *Voting matters* 15, 13-16.

[3] Hill, I. D. and Gazeley, Simon (2005) Sequential STV: A Further Modification, *Voting matters* 20, 6-8.

[4] Tideman, T. Nicolaus and Richardson, Daniel (2000) Better Voting Methods Through Technology: The Refinement-Manageability Trade-off in the Single Transferable Vote, *Public Choice* 103, 13-34.

[5] Woodall, D. R. (1994) Properties of Preferential Election Rules, *Voting matters* 3, 8-15.

About the Author

Simon Gazeley is a retired civil servant. He has served on the Council of the Electoral Reform Society on three occasions since 1992 and was a member of the Technical Committee.

Appendix: A counting algorithm for the elimination stage of STV-EDC

At any given point:

c is the total of the votes credited to the contending candidates;
d is the total of the votes credited to the elected candidates and N;
e is the number of candidates elected so far;
f is the number of vacant seats;
n is the number of non-eliminated candidates.

1. On every vote which bears a preference for any non-eliminated candidate, insert a preference for a notional candidate N immediately following the voter's final expressed preference. Set N's initial kv to 1; set the common kv t of the contending candidates and the initial kvs of the elected candidates to $1 - \sqrt[n]{(1/(s + 1))}$. Set q to the final quota in the immediately preceding election stage. Set the iteration count to 0.

2. Increase the iteration count by 1. Distribute the votes, then:

a. If the iteration count is odd, recalculate t as follows:

If c < fq, *multiply* t *by* fq/c, *otherwise by an iota less than* $(\text{fq/c})^2$.

b. If the iteration count is even, recalculate the kv of N and of each elected candidate by multiplying the present kv by

$$(q(s + 1) - c)/(e + 1).$$

If the new kv > 1, reset it to 1.

If $fq > c$ or if N or any elected candidate has fewer than q votes, go to 2.

3. Calculate the total surplus $x = d - (e + 1)c/f$. If x exceeds the difference between the two lowest candidates' votes, go to 2. Otherwise, eliminate the lowest candidate, delete preferences for N and end the elimination stage.