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Voting matters

To advance the understanding of preferential voting systems

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The Trust's work includes the maintenance and development of the Lakeman Library for Electoral Studies, a unique research resource, the production and publication of Representation: The Journal of Representative Democracy, and, of course, this publication **Voting matters**, that examines the technical issues of the single transferable vote and related electoral systems.
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Editorial

There are 3 items in this issue:

- The paper by Joseph Durham and Peter Lindener 'Moderated Differential Pairwise Tallying' considers a method of electing candidates using a preferential ballot which is quite unlike STV. The paper details the rationale behind the method, using Borda scores and Condorcet as starting points.

  As with all such methods, it is difficult to convince people to use a new system, even given a detailed analysis of its effects. How do voters react to knowing that later preferences can affect the earlier ones?

- In the second paper, David Hill considers the problem of filling a casual vacancy when an election uses the Meek algorithm for STV.

- The last paper, by Philip Kestelman, presents a detailed analysis of the proportionality attained when using the Alternative Vote. This is particularly relevant at the moment since the use of AV has been proposed for the House of Commons.

New Zealand election rules

It has recently been noted that the formulation of the Meek rules given in the New Zealand regulations is incorrect in significant ways. However, this is not as worrying as it might appear, since it is thought that the actual computer software used to implement the rules actually implements the Meek algorithm correctly.

Postscript

This is the last issue of *Voting matters* which I will edit. An announcement of the new editor is expected shortly.

Voting matters originally started by reproducing documents which were hard to obtain, like Meek's description of his method. Subsequently, articles were often papers presented to the Electoral Reform Society's Technical Committee.

After some initial difficulties, a strategy was developed which used one anonymous referee for each submitted paper. The referee was almost always, but not exclusively, an author of a *Voting matters* paper. I would like to thank all those who acted as a referee, since they performed a vital role and it was essential for me to trust their conclusions.

A review of *Voting matters* undertaken by the Trust resulted in a change to the wording to ensure that the scope was preferential voting systems, not just Single Transferable Vote.

Having a free journal published via the Internet has obvious advantages; there need be no page limit and issues can appear when there is material, not just on a certain date. Hence, although the dates of issues are somewhat erratic, we have never had a significant backlog.

There are two people I need to thank by name: Paul Wilder for the website provision and the reproduction of some copies, and David Hill who has carefully proof-read each issue.

Moderated Differential Pairwise Tallying: A Voter Specified Hybrid of Ranking by Pairwise Comparisons and Cardinal Utility Sums

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1 Introduction

In our increasingly interactive world, elections and other forms of collaborative group decision-making are becoming ever more important. Many voting systems have been proposed, from common plurality-based methods to the historical approaches of Condorcet and Borda and, more recently, Single Transferable Vote, Range Voting, and Alternative or Instant Runoff Voting ([10], [12], [14], [19]). There is increasing awareness of how the mathematical properties of voting systems affect not just the election outcome but also which options are really considered and the content of pre-election debate. An understanding of the tradeoffs of various approaches appears critical for democratically governed groups of all sizes to realize their full potential.

Democratic decisions made when choosing between only two candidates or options seem straightforward: the option with the most votes for should win. Unfortunately, in voting situations that have more than two alternatives, democratic decisions rapidly become more problematic. This paper presents a new vote-tallying method, *moderated differential pairwise tallying*, that can improve the quality of single-winner elections. This method is a per-voter hybrid of Condorcet's pairwise comparison with a cardinal-weighted revision of the Borda count which gives all voters control over exactly how their preferences are tallied. We also show that the method maintains the virtues which make pairwise comparisons so appealing while reducing the potential for ambiguous cyclical outcomes. Our hope is that the adoption of this and future advances in social choice theory will help build more responsive democracies and encourage greater civic participation.

For this publication, see www.votingmatters.org.uk

2 The Spoiler Effect and Other Challenges

Before describing the developments in this paper, we would like to provide some motivation and introduce some key ideas. Those readers familiar with the spoiler effect and Condorcet's method of pairwise comparison may choose to go on to the next section.

Consider the following common scenario: two candidates, P and Q, each receive the support of about half of the voters. Selecting between just P and Q is simple: whichever candidate receives more support in a head-to-head comparison can be deemed preferred by the group as a whole. Now consider the effect of adding a third option, M, to the set of candidates. In commonly used *plurality* voting systems, each voter is given one vote to cast for either P, Q, or M. With the addition of alternative M each voter must decide whether or not he would like to change his vote from P or Q to M. If a voter still prefers P or Q to M, then logically he should continue to support his prior top candidate. But the voting system has introduced a risk for those voters who might change their vote from P or Q to M, since removing support from either P or Q might cause the other less desired candidate to win.

The voters who are considering a switch to M are in a quandary. Each voter is forced to weigh the potential benefit of switching his vote to M against the risk of causing his least preferred of the three candidates to win. It is even possible that every voter would actually prefer M to either P or Q but, because of the perceived risk associated with voting for a third alternative, the group will remain polarized and stuck choosing between P and Q.

This M, P, and Q scenario is an example of the third-party *spoiler effect*, where the addition or removal of a non-winning candidate can affect which candidate wins. Almost all voting methods in use today are plagued by variations of the spoiler effect, but it is particularly problematic in plurality-based methods where each voter votes for a single candidate. The prevalence of the spoiler effect in a voting method can directly influence how an election unfolds in several ways.

First, in the face of the spoiler effect, voters are often forced to speculate about which candidates are the top contenders to determine how to avoid a detrimental result. The spoiler effect effectively penalizes voters who do not use a voting strategy which looks to vote only for one of the front-running candidates. This unfortunate reality can cause the outcome of an election to depend more on perceptions of popular opinion than the electorate's true preferences.

In addition, once a voter has decided to support one of the perceived front-runners, there is little incentive for him to consider other alternatives. The perceived risk of supporting a third-party candidate limits which options are fully explored. By focusing attention on the perceived front-runners, the spoiler effect contributes to the polarizing divisiveness surrounding some elections and can influence not only which candidate is elected but the very nature of the preceding democratic debate itself.

These distinctly less-than-democratic outcomes are just a few examples of the potential consequences of using a voting system which is subject to the spoiler effect. The frequent worry about "throwing one's vote away" around election time illustrates the awareness of many voters about the spoiler effect, even if they do not use the term. These issues are so common that they are often accepted as being an inherent part of the election process. However, the potential for electoral results to be influenced by the spoiler effect highlights the very real value of a well-formed, truly democratic voting system. The question then becomes: how can a democratic selection be made between several candidates without inviting the spoiler effect?

As we have mentioned, the spoiler effect is particularly troublesome in plurality methods where each voter has a single vote to cast. In an attempt to reduce the severity of the spoiler effect, many political systems using plurality methods hold a series of smaller contests including primaries, runoffs, or both. Although holding a final runoff between the top two candidates does return to the simple two candidate scenario for the last stage of an election, the voting which determines who will be in the runoff is still subject to the spoiler effect. Recently, Single Transferable Vote (STV) and Alternative Vote or Instant Runoff Voting (IRV) have received a lot of attention for bypassing the logistical need for a separate runoff. However, the implementations of these methods still involve a series of plurality votes and so the spoiler effect still influences election results.

To show how a voting system can be designed to avoid the spoiler effect, we will return to the P, Q, and M scenario. When P and Q were the only two choices, selecting a winner was easy and there was no threat of the spoiler effect. The simplicity of the two candidate scenario holds a clue about how the spoiler effect can be eliminated from democratic decision-making. When candidate M is included in the pool of options and voters are asked to pick one of three candidates, then the spoiler effect appears. Consider instead if we asked voters to pick one candidate out of each possible pair of candidates: {P,Q}, {P,M}, or {Q,M}. With this approach, a voter can still support P over Q, but also express his preference for M over either P or Q, for example. By evaluating the candidates in pairs in this way, the risk of the spoiler effect caused by support for newcomer M is removed. This approach to tallying votes is known as *exhaustive pairwise comparison*.

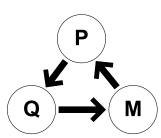
To give voters the freedom to express their preferences over many pairs of candidates, a different kind of ballot is required, something known as a *ranked choice ballot*. Ranked choice ballots, which are also used in STV, IRV, and other methods, allow voters to rank the candidates against each other. Ranked choice ballots allow the concept of pairwise contests to be easily extended to all possible combinations of candidates. For example, if a voter ranked M higher than P on a ranked ballot, then it would be the same as the voter supporting M in the pairwise comparison {P,M}.

The combination of ranked choice ballots and exhaustive pairwise comparison forms the foundation of the voting method first proposed by Condorcet in 1785 [5]. Condorcet's approach effectively holds a simultaneous runoff between every possible pair of candidates. To resolve the P, Q, and M scenario with Condorcet's method, each voter would submit a ballot ranking the three choices. If candidate M won her pairwise comparison against both P and Q, then M would be the *Condorcet winner* and would be selected as the group's most preferred choice.

Pairwise comparisons have the important advantage of *strict candidate pair dependence*: a voter's ranking for candidate M has no effect on the relative standing of P and Q. When exhaustive pairwise comparison produces a Condorcet winner, then the result is *independent from irrelevant alternatives*, meaning that any non-winning candidate can be removed without affecting the outcome. Thus, we have found a method for making a truly democratic decision between multiple candidates without risking the emergence of the spoiler effect.

It is worth noting that plurality-based runoff methods, including STV and IRV, will often eliminate a potential Condorcet winner. The spoiler effect in the early rounds of these plurality-based methods will often lead to a runoff between two less-preferred candidates. A Condorcet winner will always win a runoff election against any of the other candidates and is therefore distinctly the choice of the voters overall.

While exhaustive pairwise comparison has many important virtues when there is a Condorcet winner, unfortunately such a decisive winner does not always occur. When no single candidate wins all of her pairwise sub-contests, a set of more-preferable candidates will often emerge above the rest of the choices. This *top cycle set* wins over all candidates outside the set, but there is no coherent ordering of the candidates within the set [16]. The figure below gives one example of a cycle set, where P wins over M, M wins over Q, but Q wins over P to create a cycle. There exists a class of voting methods known as Condorcet methods which differ only in how they select a winner in the case of a top cycle set. There is little data, however, on how frequently top cycles would occur in real elections, particularly when there are a large number of candidates. Nonetheless, the potential for these ambiguous results with Condorcet's approach means that some further analysis is needed.



In this paper, we pursue an understanding as to why cycles can occur in pairwise tally results and develop a method to reduce their likelihood.

3 Contents of this Paper

The core contribution of this paper is a new pairwise tallying formulation which unifies Condorcet's method with a linear version of the *Borda count*. As we will show, classic pairwise comparison discards critical relative priority information from voters which can resolve cycles. Our new hybrid method, *moderated differential pairwise tallying* (MDPT), is built on the premise that not all voters will choose to strategically maximize their ballot's influence to the Condorcet-style extreme. A new voting parameter called the *moderation span* will be introduced, which allows voters to express slight preferences between candidates they find similarly preferable. We show that when voters use the new moderation parameter, this new tally formulation decreases the chances that a cycle will occur without introducing any dependence on irrelevant alternatives. This concept of individual moderation also suggests intriguing new approaches for resolving top cycles.

This paper is organized as follows. In Section 4 we explore the mathematics of Condorcet's pairwise analysis. The issue of ambiguous cyclical results and their inevitability in the face of Arrow's impossibility theorem is the topic of Sections 5 and 6. We present a fully linear Cardinal utility system in Section 7, but the necessary introduction of a constraint on voter weight in Section 8 leads to a real-valued formulation similar to Borda's method. In Sections 9, 10, and 11 we transform the Condorcet and real-valued Borda tallying methods into a common difference matrix framework. From these transformations we derive our new voter-specified hybrid method, MDPT, in Section 12. We show in Sections 13 and 14 how this new tallying formulation reduces the potential for cyclical results. In Section 15 we discuss some properties and interpretations of moderated tallying. Section 16 focuses on the practical aspects of implementing MDPT, including a suggestion of how voters might cast preference ballots and set the new voting parameter. We conclude in Section 17 and offer some intriguing future directions for this work in Section 18.

For those already versed in social choice theory, we hope that the perspectives on Condorcet, Borda, and strategic voting we present will spark further insights. For those new to some of these concepts or interested in how our proposal might be used, you may want to read the more practical material in Section 16 before diving into the other sections. We have tried to ensure that your effort to grasp these ideas is rewarded with some new and interesting understanding. We would also like to open up these ideas for discussion and look forward to dialog with others interested in improving commonly used democratic group decision methods.

4 Condorcet pairwise tallying

As we described in Section 2, pairwise tallying holds a sub-contest for every pairwise combination of candidates. By considering just the candidate pair in question for each sub-contest, this formulation remains free of the third-party spoiler effect. Based on recently rediscovered manuscript transcriptions, the concept of making group decisions using exhaustive pairwise comparison has a long history. In the 13th century Llull described an iterative procedure for small groups to elect a leader by holding a head-to-head vote for every possible pairwise combination of candidates [8]*. Condorcet proposed the first known pairwise tallying method based on ranked choice ballots in 1785 [5], [11].

We will first present the equations for pairwise tallying and then show an example of the computation. The first step in determining the winner of candidate pair {A,B} with pairwise tallying is to tally how many voters ranked A higher than B,

$$T_{cond}\,[\mathrm{A},\mathrm{B}] = \sum_{v}^{\text{all voters}} \left(\vec{b}_v[A] > \vec{b}_v[B]\right) \quad (1.1)$$

For this paper, $\vec{b}_v$ will always be a voter's real-valued preference ballot vector and the term $\vec{b}_v[A]$ is the preference rating given to candidate A by voter *v*. The relative standing of $\vec{b}_v[A]$ and $\vec{b}_v[B]$ indicates the voter's relative preference for A or B with the expression $(\vec{b}_v[A] > \vec{b}_v[B])$ yielding 1 when $\vec{b}_v[A]$ is greater than $\vec{b}_v[B]$ and zero otherwise. Iterating (1.1) over all pairs of candidates computes a square *pairwise tally matrix*, T_{cond}, using Condorcet's approach. $T_{cond}\,[\mathrm{A},\mathrm{B}]$ contains the number of voters who ranked candidate A higher than B. While not necessary for (1.1), in this paper all preference ratings will be real values. The use of real values will prove significant in later sections.

4.1 Example: Condorcet Tally of One Ballot

For this example we will compute how the single real-valued preference ballot below is added to an aggregate tally.

Voter 1	
1.0	A
0.9	C
0.0	B

* As Llull was well-read in the writings of Arabic scholars, there is some speculation that Llull's ideas may have their roots in prior Arabic thinking, see [12]

This voter has ranked A > C > B. For all the examples in this paper ballots will span the interval [0,1] for simplicity, but any interval is acceptable. When this ballot is tallied using (1.1), the voter's support is added to $T_{cond}\,[\mathrm{A},\mathrm{B}]$, $T_{cond}\,[\mathrm{A},\mathrm{C}]$, and $T_{cond}\,[\mathrm{C},\mathrm{B}]$, as shown in matrix form below.

Voter 1

	A	B	C
A	0	1	1
B	0	0	0
C	0	1	0

Notice that while the ordering of candidates on the voter's ballot can be determined from this matrix, the original spacing between the candidates cannot.

4.2 Picking a Winner from a Condorcet Tally

To determine a choice from a pairwise tally, we first compute a *delta-tally matrix* where each element compares how candidate A fared relative to candidate B. Subtracting the transposed tally matrix T^T_{cond} from T_{cond} yields the difference between row A, column B and row B, column A,

$$D_{cond} = T_{cond} - T^T_{cond} \quad (1.2)$$

In other words, (1.2) tallies the number of voters who ranked candidate A higher than candidate B versus B higher than A across all pairs of candidates in matrix form. The resulting delta-tally matrix, D_{cond}, is anti-symmetric and has zeros on the diagonal (where candidates tie with themselves). From D_{cond} we can easily compute a pairwise win-Boolean matrix W_{cond},

$$W_{cond} = (D_{cond} > 0) \quad (1.3)$$

As before, the $(x > y)$ operator yields 1 (true) when x is greater than y, 0 (false) otherwise. When this operator is applied to a matrix of values it operates on an element-by-element basis. $W_{cond}\,[\mathrm{A},\mathrm{B}]$ will be 1 if $D_{cond}\,[\mathrm{A},\mathrm{B}] > 0$ (ie, when A beats B) and 0 otherwise. Overall contest results are then determined by examining the full contents of this pairwise win matrix.

Together (1.1), (1.2), and (1.3) perform Condorcet's pairwise analysis. If all the win-Booleans in a candidate's row in W_{cond} are 1 (except the 0 diagonal where candidates tie with themselves), this candidate has won a direct comparison with every other candidate in the election and is distinctly the best choice. As mentioned in Section 2, a candidate who wins all of her pairwise comparisons is termed a *Condorcet winner.* Since the delta-tally matrix is anti-symmetric, a Condorcet winner will also have her corresponding column in W_{cond} all 0 since she will not have lost to anyone.

4.3 Example: Condorcet Winner

For this example we will examine a three voter, three candidate election which produces a Condorcet winner.

Voter 1		Voter 2		Voter 3	
1.0	A	1.0	B	1.0	C
0.9	C	0.2	C	0.8	A
0.0	B	0.0	A	0.0	B

As in Example in section 4.1, we can compute what each voter will contribute to the tally matrix.

Voter 1

	A	B	C
A	0	1	1
B	0	0	0
C	0	1	0

Voter 2

	A	B	C
A	0	0	0
B	1	0	1
C	1	0	0

Voter 3

	A	B	C
A	0	1	0
B	0	0	0
C	1	1	0

Summing all of these contributions produces the following pairwise tally matrix.

T_{cond}

	A	B	C
A	0	2	1
B	1	0	1
C	2	2	0

To compute the corresponding delta-tally matrix, we subtract T_{cond}^T from T_{cond}, so $D_{cond}\,[\mathrm{A},\mathrm{B}] = (T_{cond}[\mathrm{A},\mathrm{B}] - T_{cond}\,[\mathrm{B},\mathrm{A}])$. From the delta-tally we then compute the pairwise win-Boolean matrix.

D_{cond}

	A	B	C
A	0	1	-1
B	-1	0	-1
C	1	1	0

W_{cond}

	A	B	C
A	0	1	0
B	0	0	0
C	1	1	0

The winner of this election is candidate C who wins over both A and B. This can also be seen directly from D_{cond} since C's row is all positive except the 0 diagonal.

4.4 Properties of Condorcet Tallying

Since each matrix element $T_{cond}\,[\mathrm{A},\mathrm{B}]$ depends only on relative ballot positioning of the associated pair of candidates, each element of T_{cond} possesses *strict candidate-pair dependence*. This desirable property is a direct result of Condorcet's method of tallying votes. Since each voter's full support is given to his preferred candidate in every pairwise sub-contest, all pairwise results are independent of the presence of any candidate not in the pair. Therefore, when there is a Condorcet winner, pairwise tallying exhibits the very desirable property of *independence from irrelevant alternatives*: any non-winning candidate can be added or removed from the contest without changing the result. As a consequence of this independence, pairwise tallying is free of the spoiler effect. The catch, however, is that a Condorcet winner does not exist for all possible collections of ballots.

5 Coinciding Cyclical Majorities

As Condorcet discovered in 1785, even though each voter submits a strictly ordered list of candidates, the set of pairwise contest results can form an ambiguous cycle [3, pg 193]. One collection of ballots which produces a cyclic outcome is shown in Example below.

5.1 Example: Coinciding Cyclical Majorities

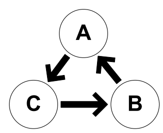
In this example we will modify one of the ballots from Example in 4.2 such that the pairwise tally results in a cycle. We will switch Voter 1's rating of candidates B and C so that the previous winner, C, is now at the bottom of his ballot.

Voter 1		Voter 2		Voter 3	
1.0	A	1.0	B	1.0	C
0.9	B	0.2	C	0.8	A
0.0	C	0.0	A	0.0	B

Notice that each voter is using a different rotation of the same preference order. Tallying these ballots produces the following win-Boolean matrix.

W_{cond}

	A	B	C
A	0	1	0
B	0	0	1
C	1	0	0



Each candidate wins one of their pairwise comparisons and loses the other one, forming a cycle: A wins over B, B wins over C, but C also wins over A. A cycle can also be shown graphically using a win edge-graph as above, where directed edges point from the loser to the winner of each pairwise sub-contest.

5.2 Cause of Cycles

There are several names for the cycle phenomenon, including "majority rule cycle" and "cyclical majority." In his writings on the topic, Condorcet often used the word "contradictoire" when referring to cycles [5]. The potential for cyclical results is also often referred to as Condorcet's "paradox": although each voter submits a ranked ballot, it is possible for the group tally to have no coherent ranking. We prefer the term *coinciding cyclical majorities* which emphasizes that the pairwise majority-rule victories which produce the cyclical result all occur from the same collection of ballots. Regardless of the name, the potential for ambiguity in the ranking produced by pairwise tallying can seem paradoxical, contradictory, or at least disconcerting. Under some situations a result with no decisive winner may be acceptable, but in general we require a method which can resolve any possible set of input ballots. This paper shows why coinciding cyclical majorities are actually an expected result of the mathematics of pairwise tallying and what can be done to reduce their likelihood.

Contrary to potential misconception, the potential for coinciding cyclical majorities is not so much due to underlying irrational preferences of the voters as it is a product of how votes are tallied. As noted earlier, Condorcet's method of maximizing a voter's influence over each pairwise sub-contest independently means that pairwise tallying is immune to the spoiler effect. However, since each pairwise sub-contest is tallied based only on the relative rank order of the two candidates involved, information regarding relative preference magnitudes is entirely lost. The sign of the preference difference between the candidates is conserved, but the strictly pairwise perspective in Condorcet's evaluation cannot distinguish between a voter's significant, modest, or trivial preference differentials.

As noted by Saari [14], the emergence of cycles can be seen as a result of this information loss: coinciding cyclical majorities occur because of the distortion of all voter priorities to the same weight by Condorcet's style of pairwise voter influence maximization. On occasion, the distortion caused by the non-linearity of Condorcet's binary pairwise comparison can overwhelm a weaker consensus and this incoherence of pairwise victories may manifest as a cycle. As we will show in Section 10, relative voter priority information loss and the resulting potential for cycles become even more problematic as the number of candidates increases.

Condorcet discovered cycles in the late 18th century, but it would be another 150 years before a now famous theorem would more clearly explain the obstacles to designing an optimal social choice method.

6 Arrow's Impossibility Theorem

In his 1951 book *Social Choice and Individual Values* [1], economist Dr. Kenneth Arrow proposed a list of properties that an ideal social choice method or voting system would possess:

1. unrestricted domain (or universality);
2. positive association of values (monotonicity);
3. independence of irrelevant alternatives (binary independence);
4. non-imposition (or citizen sovereignty);

5. non-dictatorship.* †

Arrow's famous *impossibility theorem* concludes that a democratic voting system with three or more options cannot achieve all of these desirable properties for all possible collections of ballots. In his 1972 Nobel Prize lecture, Arrow concluded: "*Certainly, there is no simple way out. I hope that others will take this paradox as a challenge rather than as a discouraging barrier*" [2].

While Arrow's theorem may seem discouraging at first, Condorcet's pairwise analysis provides some hope. When there is a Condorcet winner, pairwise analysis achieves all of the desirable properties Arrow listed. For many collections of ballots a winner can thus be found without relaxing any of these properties. The challenge then becomes two-fold: reducing the occurrence of cycles while maintaining Arrow's properties and resolving cycles when they occur with minimal relaxation. There have been several proposals for how to resolve cycles since Condorcet discovered them, including ones by Condorcet [19], Tideman [17], Schulze [15], and Green-Armytage [7]. We will instead directly address reducing the prevalence of cyclical majorities by pursuing a deeper understanding of information loss in pairwise analysis which causes cycles.

* There exist many variations of this famous impossibility theorem, including a 1963 version which replaces the monotonicity and non-imposition criteria with the Pareto efficiency. Monotonicity, however, is in its own right a frequently discussed property of social choice functions so we have chosen to use the original version of the theorem.

† Expanded description of Arrow's five properties:

a) Unrestricted domain means that (1) each voter must have the freedom to rank all of the choices available in any order, (2) the voting mechanism must be able to process all possible sets of voter preferences, and (3) it must consistently give the same result for the same profile of votes — no randomness is allowed.

b) Monotonicity, also termed "positive association of social and individual values", means that a change in a candidate's placement on a ballot (either higher or lower), if it causes a change in the candidate's ranking, can only result in a change in final ranking in the same direction.

c) Indepedence of/from irrelevant alternatives means that if A is preferred to B out of the choice set {A,B} by the electorate as a whole, then introducing a third alternative X, thus expanding the choice set to {A,B,X}, must not make B preferred to A. This property is also referred to in the literature as *binary independence*.

d) Non-imposition means citizens must be free to vote for the candidate(s) of their choice.

e) Non-dictatorship means no single voter determines the entire contest outcome.

7 An idealized, real-valued linear system perspective

Since the potential for cycles results from the distortion of differential preference magnitudes, an idealized choice function that will avoid this pitfall seems straightforward. We can instead simply sum every voter's real-valued preference for each candidate,

$$\vec{\tau}_{benefit-cost} = \sum_{v}^{\text{all voters}} \vec{b}_v \qquad (1.4)$$

With this approach, all relative preference information is aggregated into the tally vector $\vec{\tau}_{benefit-cost}$ and the winner is then the candidate with the highest component. Since it is a completely linear system, (1.4) does not distort preferences at all.

In this idealized framework, each voter's preference ballot can be interpreted as a vector of *expected cardinal utilities* or *von Neumann-Morgenstern utilities* [20]. The preference value assigned to each possible outcome would be the voter's expected benefit from that outcome minus any associated cost. Under such a system it would be considered the social responsibility of every voter to understand how to map his personal utility into the group's summation. The optimal solution for the group as a whole is then the alternative with the highest tally in the benefit-cost sum. This choice function is also known as the Bentham-Edgeworth sum of individual utilities. It can also be thought of as a Range Voting variant where each voter can pick his own range.

If all voters submit appropriately scaled ballots, then (1.4) represents a social choice function that effectively achieves all of Arrow's five stated properties. Unconstrained ballots mapped linearly by a sum into a global tally vector represent a truly unrestricted domain. Full linearity implies positive association of values, as all changes to a voter's preference ballot are positively conducted directly into the tally. Irrelevant alternatives never affect the ranking of other candidates because ballots are unconstrained and each candidate can be given any value completely independent from the others. With this function, voters are also free to vote as they wish for each candidate and no voter determines the entire election results (unless all other voters agree that they should).

However, even though each voter should constrain the envelope of his preference schedule to only exert his appropriate level of influence, it is in the voter's interest to maximize the influence of his preference schedule. This reality causes this function to be unusable. Contentious decisions and differing concepts of individual social responsibility could easily cause an endless escalation of ballot vector magnitudes as competing factions wrestle for control of a decision outcome.

8 Real-world constraints

Rescaling ballots to standardize the minimum and maximum preference values that a voter can express appears to be an easy solution to the issue of voter self-interest which renders (1.4) effectively unusable. Subtracting the minimum preference value and then dividing by the ballot span $\Delta_v = \max(\vec{b}_v) - \min(\vec{b}_v)$ normalizes each voter's ballot to a standard interval between 0 and 1,

$$\tau_{\text{rvborda}} = \sum_{v}^{\text{all voters}} \frac{\vec{b}_v - \min(\vec{b}_v)}{\Delta_v} \qquad (1.5)$$

Similar to (1.4), (1.5) retains all information regarding the relative priorities of a voter but also limits the maximum magnitude of support a voter can express for a candidate. We will refer to this approach as a real-valued Borda method but note that it is also equivalent to Range Voting when all voters mention every candidate and rescale their ballot.*

8.1 Example: Real-valued Borda

To show how real-valued Borda differs from pairwise tallying, we will tally the same set of ballots as in Example in 5.1. With Condorcet-style tallying these ballots produce a cyclic result because of the loss of relative preference magnitude information.

Voter 1		Voter 2		Voter 3	
1.0	A	1.0	B	1.0	C
0.9	B	0.2	C	0.8	A
0.0	C	0.0	A	0.0	B

Since all voters' ballot spans are 1.0 and all ballots start at 0.0, computing the real-valued Borda tally vector for this example is a simple sum of the candidate components. Note, however, that the results would be equivalent if any ballot was scaled to span [0, 100], shifted to span [13], [14], or some combination of the two.

$\vec{\tau}_{\text{rvborda}}$	
1.9	B
1.8	A
1.2	C

For these ballots B gets the highest total and is thus the winner.

8.2 Properties of Real-valued Borda

A real-valued Borda tally is distinguished from the classic ordinal Borda tally, where preference metrics are constrained to an evenly spaced, strictly ordered ranking. By forcing preferences onto a constrained ordinal number line, classic ordinal Borda has no means to express either that one candidate stands apart from the others or that two candidates are equally preferable. For example, with ordinal tallying the ballots in Example in 8.1 would produce a tie since all middle candidates would be effectively forced to 0.5. We use unconstrained cardinal-weighted ballots because we believe ballots should be instruments for representing all of a voter's relative preference information for any viable alternatives (an extension of Arrow's unrestricted domain).

In addition, when candidates are added to or removed from the middle of an ordinally constrained ballot, the voter's expressed preference value will change for all other candidates except the top and bottom choices. In contrast, inserting or removing a candidate from the middle of a real-valued ballot causes no change in the preference expression for the other candidates. The use of ordinally constrained ballots causes unnecessary and detrimental dependence on all the other alternatives under consideration to be introduced into a Borda-style tally. The use of real-valued preference ratings will also prove crucial in the development of our moderated differential pairwise tallying method.

The introduction of the normalization by ballot span for the real-valued Borda method in (5) has addressed the primary issue with (4). However, this introduction of a limit on the strength of a voter's authority renders Arrow's five properties mutually unachievable. Although (5) achieves Arrow's four other properties, it sacrifices independence from irrelevant alternatives. This loss of independence from irrelevant alternatives opens the door for strategic voting: as Borda himself observed, choice functions like (5) which are sensitive to less relevant alternatives work "only for honest men" [3, p. 215].

Ballot span normalization limits the ability of an individual voter to set her own weight of influence but, as we will now illustrate, encourages the adoption of a voting strategy. When there are larger numbers of candidates under consideration, a voter's ballot may be significantly stretched by irrelevant alternatives. This ballot stretching decreases a voter's authority over the true contenders, increasing the po-

* More information about Range Voting, visit the Center for Range Voting at `http://www.rangevoting.org/`

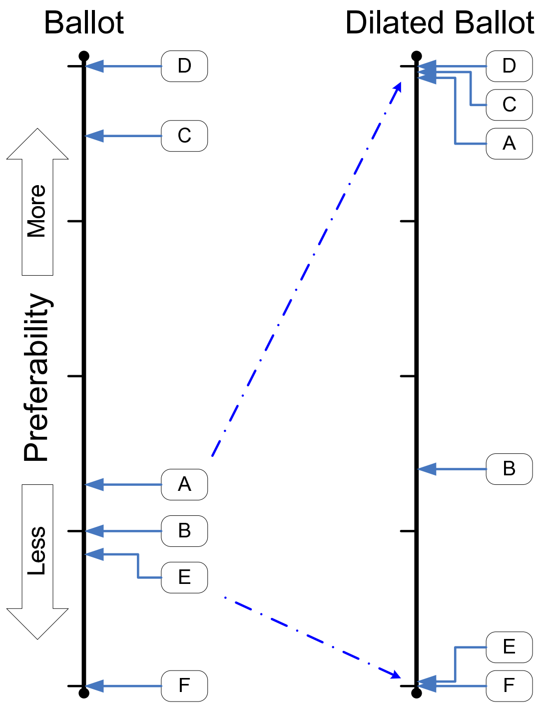


Figure 1.1: In this hypothetical example, a voter's true preferences are shown at left, while a strategically dilated version is shown at right. If the voter believes D, C and F have no chance in the election, it is in her best interest to strategically dilate and clip her ballot so that the contending candidates A, B and E define the span. While this distorts preference information for the candidates the voter believes are non-contenders, it maximizes the weight of influence between those considered to be of real importance.

tential gains from ballot manipulation. If a voter can predict the top contending candidates, she can increase her influence in the end decision by dilating and clipping her ballot to span just these top contenders; an example of this strategy is shown in Fig. 1.1. In the face of large-scale speculative gaming of this kind, the decision outcome becomes predominantly dependent on perceptions of popular opinion instead of true voter preference. We term this adverse situation of outcome dependence on perceived top contenders a form of *speculative indeterminacy*.

It is worth noting that while ordinal preference ballots preclude dilating and clipping, they encourage other strategic possibilities which are even worse. Instead of dilating their ballots, voters are encouraged to stuff the middle of their ballot with irrelevant candidates to increase their authority between the top contenders. This strategic reordering of candidates on the ballot further obscures the true wishes of the voters, yielding close to meaningless results. When there are large numbers of candidates, the potential gain from strategic voting with ordinal Borda increases as there are more irrelevant alternatives that can be stuffed in the middle of an ordinal ballot.

The only reasonable goal of any voting strategy is to elect the highest possible candidate from the voter's sincere preference schedule. As we have just described, with Borda's method voters can manipulate the placement of perceived non-contenders to increase the influence of their ballot over the front-runners. All vote tallying systems which do not exhibit strict candidate pair dependence will effectively encourage some kind of similar speculative voting strategy. When voters no longer express their true preferences to a social choice function, the election result cannot reflect the true desires of the electorate.

One way of interpreting Condorcet's tallying method is that it automatically maximizes each voter's influence since the voter's full influence is expressed between each candidate pair. Therefore, a strategic voter cannot do anything to change a pairwise sub-contest where her sincerely preferred candidate falls on the losing side. The property of strict candidate pair dependence limits a voter to trying much riskier indirect strategies. A voter can only attempt to flip a pairwise contest to a candidate she finds less preferable in the hope of creating a cycle which might end up resolving in her favor.

Unfortunately, as shown in Example in 5.1, when there are more than two candidates Condorcet's influence maximization also invites cyclical outcomes. In contrast, Borda-style methods with ballot span normalization always yield a distinct outcome but encourage several forms of strategic voting. In the following sections, we will transform these two classic tallying methods into a common delta-preference framework. This new framework will clarify their similarities and differences, and suggest a hybrid method which can exhibit the desirable properties of both methods.

9 Pairwise difference matrix of a vector

In preparation for the use of concise matrix notation throughout the rest this paper, we introduce an operator that computes the pairwise *difference matrix* of a vector. This operator will be used to express both

Condorcet's and Borda's methods into a unified matrix formulation. First, we build a square matrix M from a vector $\vec{v}$ by replicating the vector in each matrix column,

$$M = [\,\vec{v}\,\ldots\vec{v}] \tag{1.6}$$

The difference matrix of $\vec{v}$ is then defined as the column-replicated matrix M minus its transpose,

$$\mathrm{DiffM}\,(\vec{v}) = M - M^T \tag{1.7}$$

The subtraction of the transpose yields an anti-symmetric matrix which contains the pairwise difference between every combination of components in the incoming vector. The element [A,B] of the resulting matrix is equal to the difference in value between components [A] and [B] of the vector: $\mathrm{DiffM}\,(\vec{v})\,[\mathrm{A},\mathrm{B}] = \vec{v}\,[A] - \vec{v}\,[B] = -\mathrm{DiffM}\,(\vec{v})\,[\mathrm{B},\mathrm{A}]$ All the diagonal elements of $\mathrm{DiffM}\,(\vec{v})$ are 0. Note that this subtraction of the transpose is the same operation used in (1.2) to find the Condorcet delta-tally. We also note that the DiffM operator is linear, a property we will use to reorder operations in the development of the re-factored formulations that follow. When this operator is applied to a voter's preference ballot vector, we will refer to the resulting matrix as the voter's *delta-preference matrix* $\mathrm{DiffM}(\vec{b}_v)$.

9.1 Example: A Delta-Preference Matrix

For this example we will consider the same ballot as in Example in 4.1.

Voter 1	
1.0	A
0.9	C
0.0	B

As described, the [A,B] element of $\mathrm{DiffM}(\vec{b}_v)$ is given by $\vec{b}_v[A] - \vec{b}_v[B]$.

Voter 1

	A	B	C
A	0	1.0	0.1
B	-1.0	0	-0.9
C	-0.1	0.9	0

10 Condorcet pairwise tallying in difference matrix form

In preparation for forming a hybrid method that unifies the underlying approaches of Borda and Condorcet, we will reformulate Condorcet pairwise tallying employing the above difference matrix operator. The voter's delta-preference matrix $\mathrm{DiffM}(b_v)$ is used to compute the pairwise tally matrix across all pairs of candidates in a single element-by-element matrix operation,

$$T_{\mathrm{Cond}} = \sum_{v}^{\text{all voters}} \left(\mathrm{DiffM}\left(\vec{b}_v\right) > 0\right) \tag{1.8}$$

This equation is the equivalent matrix form of the classic pairwise comparison in (1.1).

In (1.2) we computed the Condorcet delta-tally D_{Cond} from pairwise tally T_{Cond}. We will now derive an equation for determining the Condorcet delta-tally directly from ballots by combining (1.2) and (1.8). We can reorder the differencing of T_{Cond} and its transpose from (1.2) to be inside the ballot tallying summation by again employing the delta-preference matrix $\mathrm{DiffM}(\vec{b}_v)$, which then gives D_{Cond} as:

$$\sum_{v}^{\text{all voters}} \left(\left(\mathrm{DiffM}(\vec{b}_v) > 0\right) - \left(\mathrm{DiffM}(\vec{b}_v) > 0\right)^T\right) \tag{1.9}$$

This delta-tally formulation is equivalent to performing both (1.1) and (1.2) but is tabulated directly from the voters' ballots without the need of the intermediate pairwise tally.

To condense (1.9) we will use the signum function which is defined as

$$\mathrm{sgn}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases} \tag{1.10}$$

Signum can also be written as the difference of two inequalities $\mathrm{sgn}(x) = (x > 0) - (x < 0)$ or equivalently $\mathrm{sgn}(x) = (x > 0) - (-x > 0)$. Since the delta-preference matrix $\mathrm{DiffM}\,(b_v)$ is always anti-symmetric, its transpose is equal to its negation: $(\mathrm{DiffM}\,(b_v) > 0)^T = (-\mathrm{DiffM}\,(b_v) > 0)$ $= (\mathrm{DiffM}\,(b_v) < 0)$. Using this, (1.9) simplifies to

$$D_{\mathrm{Cond}} = \sum_{v}^{\text{all voters}} \mathrm{sgn}\left(\mathrm{DiffM}(\vec{b}_v)\right) \tag{1.11}$$

This equation is the direct difference matrix expression of the Condorcet delta-tally in (1.2).

10.1 Example: Direct Delta-tally of Condorcet Winner

For this example we will use the set of ballots from Example in 4.2 which produced a Condorcet winner.

Voter 1		Voter 2		Voter 3	
1.0	A	1.0	B	1.0	C
0.9	C	0.2	C	0.8	A
0.0	B	0.0	A	0.0	B

Using (1.11), we can directly compute what each voter will add to the delta-tally.

Voter 1

	A	B	C
A	0	1	1
B	-1	0	-1
C	-1	1	0

Voter 2

	A	B	C
A	0	-1	-1
B	1	0	1
C	1	-1	0

Voter 3

	A	B	C
A	0	1	-1
B	-1	0	-1
C	1	1	0

As in Example in 4.1, the voter's original spacing between candidates cannot be recovered from his contribution to the Condorcet delta-tally. As expected, summing all voter contributions produces the same aggregate delta-tally as we found in Example in 4.1.

$D_{\rm Cond}$

	A	B	C
A	0	1	-1
B	-1	0	-1
C	1	1	0

As before, C wins both of its pairwise comparisons and is therefore the winner.

It is also worth noting that since (1.11) is equivalent to (1.2), (1.11) will yield an ambiguous cyclical outcome for the same ballot collections as (1.2).

11 Real-valued Borda Tallying in Difference Matrix Form

We will now transform the vector Borda tallying method from Section 8 into an equivalent difference matrix form. This new form will have a similar structure to the Condorcet delta-tally presented in (1.11). To start, we can compute the delta-Borda tally matrix for a Borda tally vector produced by (1.5),

$$D_{\rm rvborda} = {\rm DiffM}(\vec{\tau}_{\rm rvborda}) \qquad (1.12)$$

The $D_{\rm rvborda}$ matrix contains the same information on the relative standing of candidates as the $\tau_{\rm rvborda}$ vector, but it possesses a similar anti-symmetric structure to the Condorcet delta-tally in (1.2) and (1.11). We can commute the difference matrix operator in (1.12) to inside the summation across voters, creating an equivalent delta-Borda matrix formulation that implements the preference differencing operation on a per ballot basis similar to (1.11),

$$D_{\rm rvborda} = \sum_{v}^{\text{all voters}} \frac{{\rm DiffM}(\vec{b}_v)}{\Delta_v} \qquad (1.13)$$

As in (1.5), Δ_v is the span of the voter's ballot, i.e. $\Delta_v = \max(\vec{b}_v) - \min(\vec{b}_v)$. ${\rm DiffM}(\vec{b}_v)$ is the delta-preference matrix from the voter's ballot as described in Section 5. Normalizing ${\rm DiffM}(\vec{b}_v)$ by ballot span limits the voter's contribution to a given delta-tally component to ± 1 , since $\max\Big({\rm DiffM}(\vec{b}_v)\Big) = \Delta_v$.

In the absence of exact ties, the tally vector from (1.5) always yields a distinctly ordered candidate ranking. The delta-Borda matrix derived from this tally vector contains the same ranking information and, therefore, the win matrix produced by this delta-Borda tally will also yield the same unique, cycle-free candidate ranking. In other words, since Borda tallying is a linear process that does not distort relative preference magnitude information, it will never yield an ambiguous cyclical result. This statement is true for the vector-based tabulation from (1.5) as well as the equivalent difference matrix form in (1.13).

11.1 Example: Real-valued Borda Delta-tally

To demonstrate that (1.13) is equivalent to (1.5), we will perform a real-valued Borda delta-tally using the ballots from Example in 8.1.

Voter 1		Voter 2		Voter 3	
1.0	A	1.0	B	1.0	C
0.9	B	0.2	C	0.8	A
0.0	C	0.0	A	0.0	B

Each voter's contribution to the delta-tally is the voter's delta-preference matrix divided by the span of his ballot. All ballot spans are 1.0 for this example.

Voter 1

	A	B	C
A	0	0.1	1.0
B	-0.1	0	0.9
C	-1.0	-0.9	0

Voter 2

	A	B	C
A	0	-1.0	-0.2
B	1.0	0	0.8
C	0.2	-0.8	0

Voter 3

	A	B	C
A	0	0.8	-0.2
B	-0.8	0	-1.0
C	0.2	-1.0	0

For the Condorcet case in Example in 10.1, only a voter's ordering of the candidates could be determined from his contribution matrix. For the real-valued Borda contribution matrices above, the relative magnitudes of delta-preference are conserved. The aggregate delta-tally is the sum of all voter contributions. As in the Condorcet case, we determine the winner by computing a win-Boolean matrix.

D_{rvborda}

	A	B	C
A	0	-0.1	0.6
B	0.1	0	0.7
C	-0.6	-0.7	0

W_{rvborda}

	A	B	C
A	0	0	1
B	1	0	1
C	0	0	0

As in Example in 8.1, candidate B is the winner. This difference-matrix form for real-valued Borda also demonstrates that determining a winner from a Borda tally is very similar to finding a Condorcet winner. In both circumstances, the winning candidate will have her row in the win-Boolean matrix all 1 (except to diagonal). The key difference is that the linearity in Borda tallying guarantees that it will never yield a cycle.

In the next section, we will combine the transformed Condorcet and Borda tallying formulations in (1.11) and (1.13) to reduce the prevalence of cyclical outcomes.

12 Moderated Differential Pairwise Tallying: A Hybrid

We will now bring together the desirable properties of Condorcet and real-valued Borda tallying, balancing Condorcet's strict candidate pair dependence with real-valued Borda's undistorted transmission of relative priority information. A significant form of information loss in Condorcet's pairwise comparison is the removal of degree for each voter's smaller differential preferences. Cycles are more prevalent due to information loss when no distinction is made between candidates far apart versus close together on a voter's ballot. Making use of the similar structure of (1.11) and (1.13), we can address this shortcoming in Condorcet's method by forming a parameterized hybrid of Condorcet and Borda tallying using a linear sigmoid. A linear sigmoid introduces a proportional, sloped linear region around near equal preference to the classic signum from (1.11). This sloped region will address issues caused by the hypersensitive step transition in a signum function. We define this linear sigmoid function as

$$\text{linsgn}(x,h) = \begin{cases} \dfrac{x}{h} & \text{if } \mid x \mid < h \\ sgn(x) & \text{if } \mid x \mid \geq h \end{cases} \tag{1.14}$$

The parameter h is the half-width of the linear region of the sigmoid, with the equation's conditional written in terms of the magnitude (absolute value) of *x*. As $h \to 0$, the linear region vanishes towards the signum's step discontinuity.

Using this linear sigmoid we can insert a parameterized Borda-like proportional region into the saturated, binary comparison of Condorcet's method. To control the width of this linear region we introduce the *moderation span*, m_v. This voter specified parameter allows each voter to choose where on the tallying continuum between the Condorcet and Borda methods her ballot will be tallied. Moderated differential pairwise tallying (MDPT) can be written in matrix form as

$$D_{\text{Mod}} = \sum_{v}^{\text{all voters}} \text{linsgn}\left(\text{DiffM}(\vec{b}_v), m_v\right) \tag{1.15}$$

We can also compute the same moderated delta-tally element-by-element and with less abstraction,

$$D_{\text{Mod}}[\text{A},\text{B}] = \sum_{v}^{\text{all voters}} \begin{cases} \dfrac{\vec{b}_v[A]-\vec{b}_v[B]}{m_v} & \text{if } |\,\vec{b}_v[A]-\vec{b}_v[B]\,| < m_v \\ \operatorname{sgn}\left(\vec{b}_v[A]-\vec{b}_v[B]\right) & \text{if } |\,\vec{b}_v[A]-\vec{b}_v[B]\,| \geq m_v \end{cases} \tag{1.16}$$

As they are equivalent, both (1.15) and (1.16) divide the difference in candidate preference values by the voter's moderation span, m_v, reverting to the previous Condorcet formulation when the difference between the candidates is greater than m_v. Regardless of any voter's moderation span or candidate placement, each matrix element in D_{mod} depends only on the pair of candidates in question. Because of this property, MDPT possesses the same property of strict candidate pair dependence as Condorcet's classic pairwise tallying.

12.1 Example: Moderated Differential Pairwise Tallying

For our example of MDPT we will use the collection of ballots from Examples 5.1 and 8.1. With pairwise tallying these ballots produced an ambiguous cycle, while a winner was found using real-valued Borda. For this example we will set each voter's moderation span to half of their ballot span, 0.5.

Voter 1		Voter 2		Voter 3	
1.0	A	1.0	B	1.0	C
0.9	B	0.2	C	0.8	A
0.0	C	0.0	A	0.0	B

We next compute the moderated delta-preference matrix for each voter. All preference differentials smaller than 0.5 are moderated.

Voter 1

	A	B	C
A	0	0.2	1.0
B	-0.2	0	1.0
C	-1.0	-1.0	0

Voter 2

	A	B	C
A	0	-1.0	-0.4
B	1.0	0	1.0
C	0.4	-1.0	0

Voter 3

	A	B	C
A	0	-1.0	-0.4
B	1.0	0	-1.0
C	0.4	1.0	0

The sum of these voter contributions produces the moderated delta-tally from which we can also determine a win-Boolean matrix.

D_{mod}

	A	B	C
A	0	0.2	0.2
B	-0.2	0	1.0
C	-0.2	-1.0	0

W_{mod}

	A	B	C
A	0	1	1
B	0	0	1
C	0	0	0

The winner for this moderated example is candidate A. This hybrid method produced a different result than the Borda and Condorcet methods. With full ballot span normalization, the winner for these ballots is B as shown in Example in 11.1. With Condorcet's pairwise tallying in Example in 5.1, these ballots produced a cycle. To resolve the cycle, the two voters on the winning side of one pairwise contest in the Condorcet tally need to moderate sufficiently to change the pairwise result. In this example, Voter 2 and Voter 3 both indicated that they found A and C relatively similar, allowing A to win. If the voters' moderation spans were expanded to 1.0, then the result of the sub-contest between A and B would also flip and B would be the winner. This result would be the same as the Borda case since moderated tallying with all $m_v = 1.0$ is equivalent to real-valued Borda.

We will discuss some details of how this new vote tallying mechanism works and why a voter might choose to moderate in the following sections.

13 Moderation as a transfer function

Moderation in MDPT can be understood as follows. If option A is much more preferable than B, the signum function limits the support of A with respect to B to $+1$. Conversely, if B is much more preferable than A, then the differential opposition of A to B is similarly limited to -1. If the difference in preference between A and B is less than the voter's moderation span, then the expression of relative support or opposition between the candidates will be

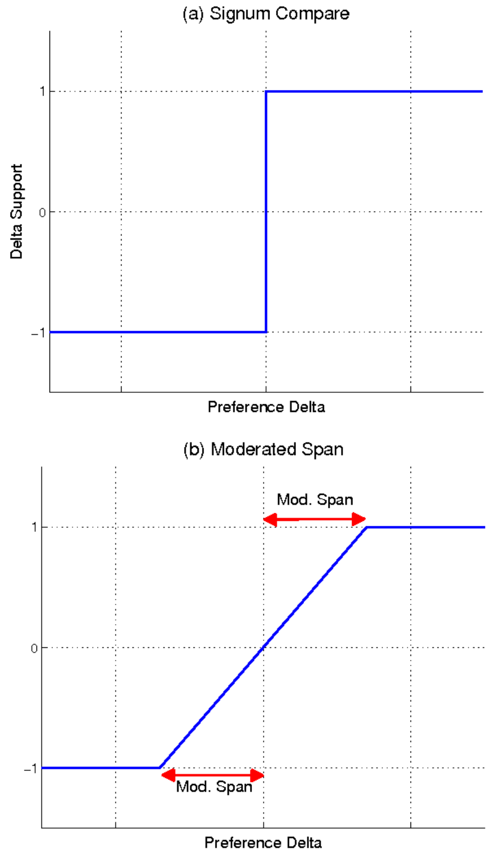


Figure 1.2: Voter support of one candidate over another based on the preference delta of those candidates on the voter's ballot. The left graph shows this transfer function for Condorcet pairwise comparison, the right for moderated pairwise comparison.

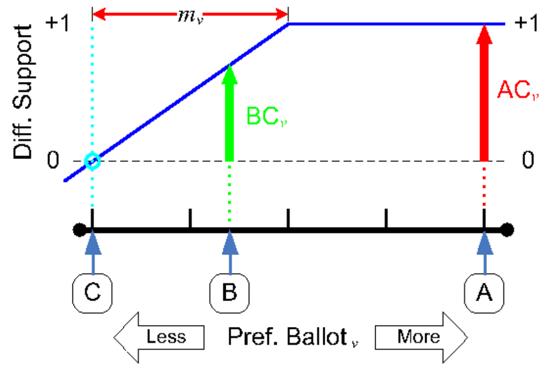


Figure 1.3: Assessing candidates A and B with respect to C, where the voter's ballot is shown below the sigmoid. The voter's contribution into the tally matrix for each sub-contest is represented by the height of the BC and AC arrows. Note that B receives moderated support while the voter exerts full support for A.

less than the voter's full weight. The moderation span introduces a linear region into a voter's contribution to the delta-tally matrix. With this addition to pairwise tallying, the voter can choose to moderate his expression of relative support/opposition for alternatives he finds nearly equally preferable. Fig. 1.2 shows a voter's differential support versus delta-preference for pairwise tallying using both Condorcet's classic quantization and MDPT.

When m_v is shorter than the smallest distance between candidates on a voter's ballot, the voter's tally contribution is equivalent to Condorcet's comparison in (1.1). If a voter's moderation span is equal to the whole span of his ballot, the voter's delta-preferences will all fall within the moderated linear region as the magnitudes in the delta-preference matrix are bounded by ballot span. In this circumstance, the voter's tally contribution is equivalent to that from the linear delta-Borda tally in (1.5). When a voter's span of moderation is set between these two extremes, this partial linearity allows for a hybrid of strong and moderate opinion. The effect of the moderation span on tally contributions is demonstrated in Figs. 1.3 and 1.4.

We note that the transfer function of the linear sigmoid bears some resemblance to the dilating and clipping voter strategy shown in Fig. 1.1. With this strategy a voter effectively created a linear region over the perceived front-runners and compressed the ends of his ballot. In (1.15), however, the $\mathrm{linsgn}(x, h)$ function is applied to a voter's delta-preference matrix $\mathrm{DiffM}(\vec{b}_v)$ instead of directly to the expressed preference ballot vector $\vec{b}_v$. This procedure mitigates the need for one of the two degrees of freedom in such a speculative strategic interval voting strategy: the position of a strategic interval. In the same way that Condorcet's pairwise tallying assesses the relative ranking of all possible candidate combinations independently, MDPT evaluates all transpositions of the voter's desired moderation span centered around each candidate on the ballot. Essentially, m_v defines the extent of a moderation interval around each candidate. The width of the voter's moderation span is the remaining degree of freedom in voting, allowing the voter to choose the level of influence for his smaller differential preferences.

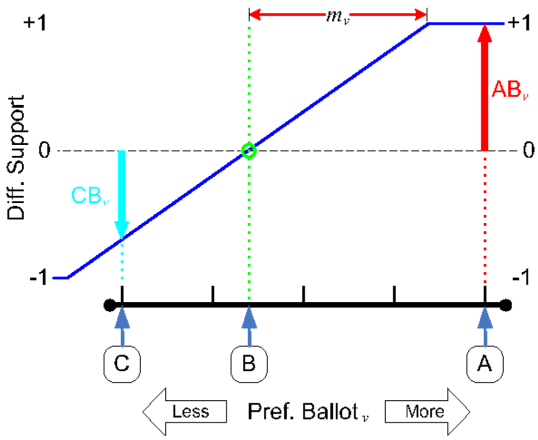


Figure 1.4: To assess candidates with respect to B, the linear sigmoid is transposed to center around candidate B. In sub-contest CB, the voter's opposition to C is moderated to the same degree as in BC from Fig. 1.4 since the delta-tally matrix is anti-symmetric (BC = −CB). In sub-contest AB, A is considered significantly more preferable to B and receives full support.

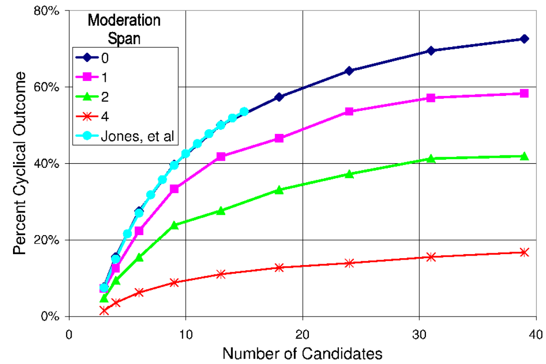


Figure 1.5: This graph shows how the percentage of random elections producing a cyclical result grows with the number of candidates considered for different moderation levels. Each of our data points represents 7,500 elections using seven random ballots. For each election, seven random ballots with uniform candidate distribution were created and then rescaled to span [0,10]. When all moderation spans are 0 (blue diamonds), the results are found to be equivalent to data gathered by Jones et al [9] for classic Condorcet pairwise analysis (light blue circles). When the moderation span for these random ballots is set 40% of ballot span, the data shows a dramatic decrease in the probability of cycles.

14 The cycle reducing effect of moderation

To demonstrate the cycle reducing effect of moderation, we simulated elections using uniform random ballots. Many other voting models for simulating perhaps more realistic elections exist, including issue-space methods [4]. We have chosen instead to use uniform randomness because it readily produces cycles and allows us to show the effect of moderation in the most general way.

For each data point in Fig. 1.5 and 1.6, we computed 7,250 elections. Each simulated election used seven random ballots possessing a uniform candidate distribution. After randomly selecting a preference value for each candidate, we rescaled every ballot to span 0 to 10 so that a particular moderation span had the same meaning for each ballot. Fig. 1.5 shows how the moderated span extension to classic Condorcet pairwise tallying can reduce the occurrence of cyclical results. When voters choose to moderate the expression of their differential preference over candidates they find similarly preferable, cycles are less likely to occur.

For the case when no voters choose to moderate (all $m_v = 0$), Fig. 1.5 shows a sharp growth in the percentage of elections resulting in cycles as the number of candidates increases. One conclusion from this result is that, in classic pairwise analysis, elections with a large number of candidates are much more likely to produce cycles. This result agrees with the data for classic pairwise tallying of random ordinal ballots in Jones et al [9], which is also shown in Fig. 5. For electorates which choose to moderate, however, cycles are significantly less likely to occur even with more than 30 candidates.

Fig. 1.6 presents another view of this same cycle probability simulation. This view shows that as moderation increases the probability of a cycle goes to 0 even for large candidate fields.

15 Discussion

The addition of the moderation span addresses an important shortcoming of Condorcet's pairwise tallying. Although classic pairwise tallying is hardened against manipulation, it does not allow voters to express any difference between their various priorities. All delta-preference magnitudes are treated as the same. We would like to suggest three ways of interpreting this new concept of moderation.

First, voters may simply wish to express slight

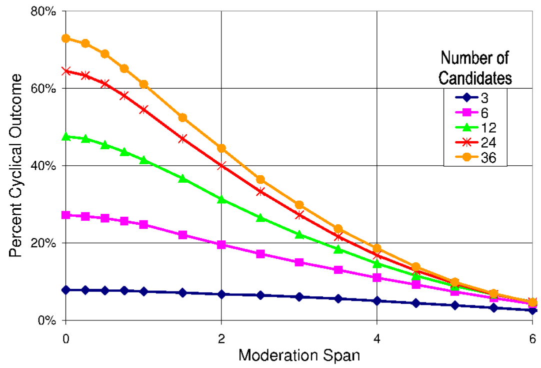


Figure 1.6: This graph shows how the probability of a cyclical outcome decreases with increasing moderation span for several different numbers of candidates. The results were produced using the same uniform random ballot method as Fig. 1.5 with seven ballots. As the moderation spans grow, the prevalence of cycles drops off towards 0. When all moderation spans cover the voters full ballot ($m_v = 10$in this case), the computation is equivalent to a real-valued Borda tally and will never result in a cycle.

preferences. Some Condorcet methods allow voters to express that they consider a pair of candidates equally preferable. Moderation extends this idea to create a smooth continuum between considering two candidates equally preferable and expressing full support for one over the other. For example, a voter might strongly prefer A over C, but have only a small preference for A over B. The moderation span permits the voter to express these differences in priority. Such slight preferences could represent some form of uncertainty a voter has for whether A or B is actually better.

A second perspective on the moderation span is that it gives voters the freedom to choose not to strategically maximize their voting influence over candidate pairs they find similarly preferable. At $m_v = 0$, MDPT tallies a voter's ballot using Condorcet's pairwise analysis, meaning all delta-preference magnitudes are maximized. When m_v is equal to ballot span, only the single largest delta-preference on the voter's ballot is maximized. Moderation gives voters control over the level of strategic maximization for their ballot. In fact, a voter could choose to expand m_v beyond the size of their ballot, which brings us to our final comment.

The third interpretation of moderation we suggest is that it allows a form of voluntary *interpersonal comparison of utilities*. In Section 7 we described an idealized, cardinal utility method for making social choices. The issue which makes this method unusable is well known in economic theory: while individuals can determine the relative costs and benefits of different potential outcomes for themselves, there is no general, well-defined way of comparing the utilities of individuals. However, as discussed by Sen [17], the common background and experiences of members of a society do allow at least a limited form of interpersonal comparison of utilities. While pairwise analysis operates on the premise that all voters will try to maximize their own influence over a decision, the voter-specified moderation span leaves open the possibility that whole communities will be able to see their individual preferences in a broader perspective. When many voters choose to vote moderately, the group can make decisions with more of a Borda-like, shared benefit-cost perspective. Some voters may recognize they do not have as much at stake in a particular decision as others and perhaps then set their moderation spans greater than the span of their ballot. Although the reality of contentious and consequential elections requires that any well-formed vote tallying method be hardened against manipulation, voluntary moderation enables more moderate groups of people to also use pairwise analysis.

The concept of moderation also seems applicable even in the case where there are only two candidates on the ballot. In Sections 2 and 4 we discussed how picking between just two candidates avoided complications from the spoiler effect and was therefore fairly straightforward. However, as we have just described, there are circumstances where a voter may wish to express a slight preference for one candidate over another. The moderation enhancement to classic pairwise tallying in MDPT gives voters this flexibility, even in the two-candidate scenario.

When voters are provided the freedom to express moderate opinion, we assert that a candidate who emerges on top of all head-to-head comparisons with every other candidate under consideration is distinctly the best choice. We term such a candidate a *moderate Condorcet winner*. If no voters choose to moderate, then the moderate Condorcet winner is equivalent to the classic Condorcet winner. When voters do choose to moderate, then the moderate and classic Condorcet winners will sometimes differ. In particular, as we showed in Section 14, a moderate Condorcet winner occurs more often than a classic Condorcet winner.

We refer to a voting method as *moderate Condorcet winner definite* if it always selects the moderate Condorcet winner when one exists. MDPT is intrinsically moderate Condorcet winner definite. Since the moderation span gives voters control over the strategic expression of their ballot, we claim that this proposed criterion is an improvement over the classic Condorcet winner criterion and thus should be a requisite property of any well-formed social choice function.

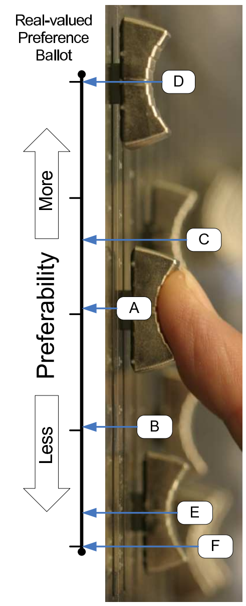
That said, MDPT does not on its own constitute a complete social choice function. Cyclical majorities can still occur, particularly if voters choose not to moderate in contentious decisions. It would certainly be possible to use a cycle breaking scheme like those proposed by Llull [8], Condorcet [21], Tideman [18], or Schulze [15] on top of MDPT. Since our approach uses real-valued preference ballots, the cardinal-weighted approach proposed by Green-Armytage [7] could also easily be employed. However, as we alluded to in Section 6, we believe that the emphasis in designing a cycle resolution scheme should be on minimally relaxing independence from irrelevant alternatives. Most existing cycle resolution methods are defined in terms of properties of the aggregate tally. We suggest instead that an approach could be built using the concept of moderation. Such a method would resolve cycles by removing edges from the win-edge graph based on which candidate pairs individual voters find most similarly preferable. A voter priority driven method of this type could more directly minimize the amount of compromise individual voters would need to make for the group to reach a coherent decision.

16 Practical Aspects

In the thick of the mathematical detail of our method, it can be difficult to keep track of the bigger picture question: how would it work in actual elections? In this section, we take a step back to address this more practical question. To begin, we will describe how a moderated preference ballot would be cast. We will then describe how elections could be set up to work with our system. Finally, we will discuss some properties of MDPT which should make a transition to this new system easier.

16.1 Casting a moderated ballot

We will now describe how a voter might cast a real-valued preference ballot with a moderation span. This new form of voting will require a new user interface, but with a good design we believe it will be quite intuitive. Our vision for this new interface involves the use of sliders to move candidates up or down on the ballot. These sliders could either be mechanical sliders like those in the figure below, or graphical sliders on a computer screen controlled with a mouse or a touchscreen. Forming a ballot would then be a simple matter of pushing the sliders up or down until the voter is happy with the positions of the various candidates.



As an example of how a voter could shape her real-valued preference ballot, consider the following scenario:

16.2 Example: Casting a moderated ballot

After careful consideration, a voter has determined her preferences for six candidates in an election. She might start creating her ballot by simply placing the six candidates in order, with her most preferred at the top of her ballot and an equal spacing between the rest, as shown in Fig. 1.7 (a). Suppose she has a strong preference for D, C, or A over any of the other candidates, but does not have strong preferences between those three. She would then separate D, C, and A from the rest and shrink the space between them, as shown in Fig. 1.7(b). Next, if she particularly dislikes candidate F, then she would move F further down (Fig. 1.7(c)).

After placing the six candidates, the voter has two further decisions to make: (1) how to set her moderation span to indicate which candidate pairs she finds similarly preferable, and (2) where to place a default value which would be given to any unrated candidates.

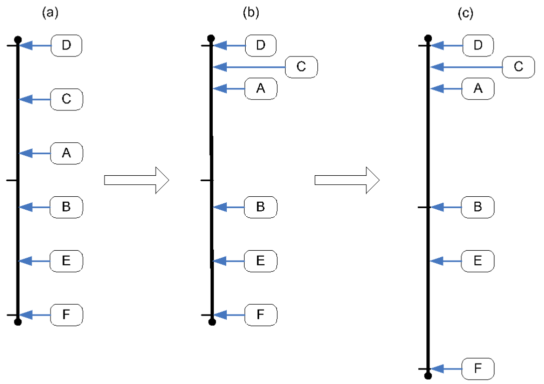
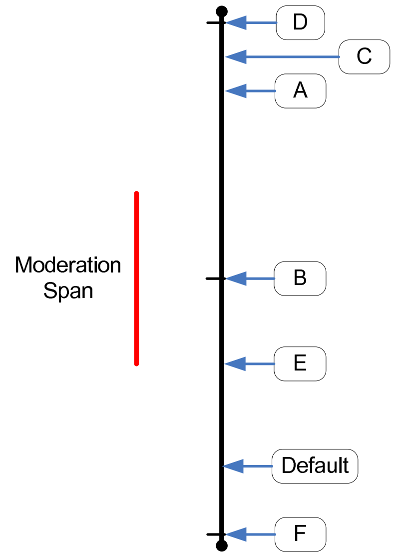


Figure 1.7: Shows the incremental construction of the ballot for Example in 16.2.



	A	B	C	D	E	F	Def
A	-	1.0	-0.2	-0.4	1.0	1.0	1.0
B	-1.0	-	-1.0	-1.0	0.5	1.0	1.0
C	0.2	1.0	-	-0.2	1.0	1.0	1.0
D	0.4	1.0	0.2	-	1.0	1.0	1.0
E	-1.0	-0.5	-1.0	-1.0	-	1.0	0.6
F	-1.0	-1.0	-1.0	-1.0	-1.0	-	-0.4
Def	-1.0	-1.0	-1.0	-1.0	-0.6	0.4	-

Figure 1.8: Shows the final ballot and moderation span for Example in 16.2, including the contribution matrix for the ballot. The moderation span scales all shorter preference differences on the voter's ballot.

Based on her final ballot rankings, Fig. 1.7(c), the voter would set her moderation span to a distance approximately equal to the space between candidates A and B. This span would indicate her strong preference for the higher of two candidates on her ballot separated by that distance, like A compared to B, or E compared to F. It would also indicate her smaller preference for candidates placed more closely on her ballot, like D compared to C or A.

The final step is that the voter should also specify a default value which will be given to any candidates she has not rated. This default value will usually be near the bottom of the ballot, with only candidates that the voter strongly dislikes below the default. Since this voter finds F particularly unpreferable, she would place her default value between E and F. If there are other candidates in the election, G and H, for example, the system would register that this voter prefers them both over F, even though she has not explicitly ranked them on her ballot. Our voter's ballot is now complete. Fig. 1.8 shows the final ballot at left, as well as how the ballot would be tallied, at right.

16.3 Setup of an election

One of the main goals of this method is to enable voting over a wide range of options. As discussed in Section 2, methods built on pairwise analysis eliminate the need for primaries or other methods of limiting available options. By reducing the potential for cyclical majorities, MDPT allows voters the widest possible range of alternatives. However, when many options are available, voters have to put in greater effort to determine their preferences. It is therefore fundamentally important that voters have easy access to clear information about all alternatives. In circumstances where many voters do not have the time or expertise to form considered opinions, we suggest the use of a *delegable proxy representation system*, similar to that proposed by Green-Armytage [6]. Such a system would allow voters to proxy their voting weight to the representative of their choosing for a given issue, achieving a free-form proportional representation structure and a more responsive democratic process.

In addition, when there are many candidates on the ballot, there must be a way to handle the candidates that a voter does not wish to rate. As mentioned in the description of how to cast a moderated ballot, our suggestion is to allow voters to specify a default value. This default value would be assigned to any unrated candidate. Again, we anticipate that this default value would typically be placed at or near the bottom of the voter's ballot.

16.4 Transition to MDPT

A transition to this new system would require an adjustment for voters who are used to picking a single candidate. Significant voter and poll-worker education campaigns would be necessary for all voters to feel comfortable with this or any new voting system. However, MDPT has a couple features which should make the transition easier.

The first attractive feature of the new system is the removal of the need to vote strategically based on which candidates are top contenders. MDPT would allow a voter to support any candidate of his choice - regardless of that candidate's popularity - without feeling that his vote might be "thrown away." This innovation means that voting can be a simple expression of preference. With our method, voters can feel they have greater choice in an election and that their vote is, therefore, more meaningful.

Flexibility is the second feature of this voting system which should ease any transition. In addition to the moderated preference ballot described above, voters may use our system to cast other styles of ballots with which they are more familiar. If a voter wants to vote only for his favorite candidate, then he can simply put that candidate at the top of his ballot and leave all others at the bottom. A voter who prefers to do a basic ranking of candidates could list them in his preferred order and set the moderation span to 0. We believe voters will appreciate this freedom to express their preferences, and that this appreciation will translate into a better experience while voting and a greater value being placed on the democratic process.

17 Conclusion

In this paper we have presented moderated differential pairwise tallying (MDPT), a per-voter hybrid of the methods of Condorcet and Borda. The foundation for this method is based on Condorcet's pairwise tallying, which has the important property of strict candidate pair dependence. As we described, however, the classic formulation of pairwise tallying discards all voter priority information. This information loss can cause ambiguous, cyclical results for some collections of ballots. At the other end of the spectrum, we examined a real-valued Borda method which always returns a coherent result. The necessary division by ballot span in Borda methods introduces dependence on irrelevant alternatives and encourages speculative voting strategies. To add some Borda-style linearity to pairwise tallying, we developed the voter-specified moderation span. As we have shown, for electorates that choose to widen their moderation spans, cycles will occur less frequently and group consensus will be easier to find.

The introduction of the voter-specified moderation span, in conjunction with real-valued preference ballots, is an important enhancement to Condorcet's pairwise tallying method. Providing voters the freedom to express moderate differential preferences partially addresses the critical information loss issue with classic pairwise tallying. We also proposed to replace the classic Condorcet winner definite criterion for voting methods with a new moderate Condorcet winner definite criterion. Often, cycle resolution will not even be necessary when voters choose to moderate over their diverse opinions. It is only when voters choose not to moderate in contentious decisions that the remaining potential for cycles requires some additional resolution. In the next section we will discuss how the tools we have presented in the paper could be extended to resolve cycles or provide a framework for more directly comparing social choice functions.

18 Future Research

We believe there are some intriguing directions for further research based on the material in this paper. Through the use of real-valued preference ballots and pairwise delta-tallying, we have expressed Condorcet and Borda's tallying methods in a unified framework. This perspective highlighted the similarities and differences of these methods. It appears that other common social choice methods can also be expressed using real-valued preference ballots and pairwise delta-tallying. This delta-preference framework is a potential foundation for a generalized approach for comparing social choice methods.

The concept of individual moderation introduced in this paper provides a new foundation for constructing a complete democratic group decision system. As discussed in Section 15, the additional needed component is a cycle resolution method which minimizes dependence on less-relevant alternatives. With such a system, the spoiler effect would be a thing of the past. Primaries and other methods of artificially pruning the scope of alternatives under consideration would also no longer be necessary.

While there remains work to be done, the potential for significant positive social impact from advances in social choice theory cannot be overstated. We are grateful for the encouraging style at the close of Arrow's Nobel Prize lecture and we would likewise encourage others in this quest to further understanding in this vital and challenging field.

19 Acknowledgements

The authors would like to thank the many people who reviewed drafts of the paper for their ideas and helpful feedback, including Tim Dirks, Jay Hanks, Bruce Carol, Karl Obermeyer, Per Danzl, Jeff Moehlis, and William Durham. The authors would like to especially thank Roger Sewell and James Green-Armytage for their insightful theoretical perspectives on these developments, as well as the anonymous reviewer for his/her insightful comments and focus on the practical side. Finally, both authors would like to thank Dave Daly for his writing expertise, help in forming the framework for this paper, and bringing focus to the relation to Arrow's properties.

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About the Authors

Peter Lindener

Peter is an independent researcher who has chosen to dedicate his life to the advancement of truly democratic social decision systems. After the tenuous nature of the 2000 U.S. presidential election, Peter started thinking on the deeper theoretical issues of larger scale group decision processes. His reflections on the virtues and flaws of Condorcet and Borda's tallying methods provided the initial foundation for the rest of this meaningful collaboration. In addition, Peter's style of always debating the other side of an issue really aided the volleyball nature of this joint work.

Joseph "Joey" Durham

Joey is a PhD student in Control, Dynamical Systems, and Robotics at the University of California, Santa Barbara. While always interested in politics and government, Joey's interest in Social Choice Theory was sparked by conversations with Peter in 2004. Joey's continued insights about the nature of voting's contextual interdependency ultimately led to our current focus on the strategic game aspects of Social Choice. Joey's effort in writing up these shared developments has helped bring a needed clarity to this work.

Casual vacancies and the Meek method

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1 Introduction

If a casual vacancy occurs in a body that has been elected by STV, caused, for example, by an elected member resigning, there is a difficulty because to hold a by-election for just the one vacant seat would, usually, result in the dominant party (or other interest group) gaining the seat, whereas the vacancy may have arisen by the resignation of a candidate from a minority group. The ideal solution, in many ways, would be that of Thomas Wright Hill's 1819 version of STV [1, 2] in which a substitute would be elected only by those electors who had, in the first place, elected the resigning candidate – but that solution is not possible in these days of secret voting.

A possible solution is for the remaining members to co-opt a suitable replacement and that may be perfectly satisfactory in some cases, but in most cases it would not be thought a good plan.

A properly representative result would be attained if all seats were declared vacant and a full new STV election held, but it would not be at all satisfactory to put other people's seats at risk because one had resigned. Those correctly elected in the first place, for a given term, must be allowed to continue and to complete their term.

A solution that is sometimes advocated is not to have either co-option or a new election, but to recount the original votes, treating the resigned candidate (and any other candidate who no longer wishes to be considered) as withdrawn, and the remaining elected members as "guarded", meaning that they cannot be excluded. Thus the exclusion rule changes, from excluding the candidate who currently has fewest votes, to excluding the non-guarded candidate who currently has fewest votes.

It should be noted that any such recounting is likely to break the rule that later preferences should not upset a voter's earlier preferences because adherence to that rule requires that later preferences are not looked at until the fates of earlier preferences have been definitely determined. When recounting, later preferences will have been looked at, and acted upon, in making the initial count and that cannot be undone. Provided that voters can be assured that it cannot happen on the initial count, the thought that a casual vacancy could occur later and need to be dealt with, is rather unlikely to worry anyone much.

There remain some problems: (1) if the voting pattern has been published, as I believe it should be, it is possible to determine with certainty who a replacement will be and, in a party situation, that could lead to pressure on someone to resign; (2) in a party situation, there may be no spare candidate of the same party. This could be an advantage, though, in that it might persuade parties to offer more candidates in the first place in case of such an eventuality, thus improving the choice for voters; (3) if the count were made in the ordinary way, except for observing the guarding criterion, it could result in too many candidates exceeding the quota simultaneously, typically two candidates doing so where there is only one vacant seat.

If the first two of those problems are not regarded as too serious, and such a method is to be adopted, how should the third problem be dealt with? Whatever is done must be compatible with the particular STV rules in use. Here I am concerned with the situation under the Meek rules.

2 Artificial examples of the problem

Example 1. Like many artificial examples this is intended merely to illustrate a point, and so the fact that something so extreme is unlikely in practice need not disturb us. Suppose three seats are occupied by A, B and Z, and Z resigns. After redistributing Z's votes appropriately, the votes are

10	A	
10	B	
100	CA	(See Appendix 1 for
60	DB	a detailed explanation)
30	EDA	
20	EDB	

The normal quota is 230/4 = 57.5 and C and D have both passed it, while A and B are guarded. Is it right to take a "first-past-the-post" type of solution and say C has more votes than D and should take the seat, or is it right to take an STV type of solution and say that E's votes must be redistributed first giving D 110 to C's 100? I strongly believe that the second of those approaches is preferable.

Example 2. If that is accepted, we need to note that a similar situation can arise even though too many candidates have not passed the quota. Consider the following: again three seats are occupied by A, B and Z, and Z resigns. After redistributing Z's votes appropriately, the votes are

10	A	
10	B	
100	CA	(See Appendix 1 for
50	DB	a detailed explanation)
19	EDA	
20	EDB	
20	FDA	
21	FDB	

The normal quota is 250/4 = 62.5; C has passed it, while A and B are guarded. Is it right to elect C, even though D, E and F between them have 130 votes to C's 100? I do not think that it is. The trouble arises because the normal quota is really irrelevant – the logic of its calculation depends upon no candidate being guarded.

These examples are highly artificial, and it might be thought that such a problem would hardly ever happen in practice, but experience suggests that it happens more frequently than would be guessed as likely. The possibility must be allowed for.

3 A suggested solution

A solution that seems to meet the requirements admirably has been suggested to me by Douglas Woodall. It works by treating any non-guarded candidate who exceeds the quota as "checked". In ordinary English, "checked" can have more than one meaning, and it is used here in two senses. First it means that the candidate's name has been marked for special treatment; secondly it means that the candidate's progress has been held up. A checked candidate is not yet elected, but is otherwise treated exactly as if elected, in having a reduced keep value to redistribute surplus votes.

The count proceeds exactly as normal (except that exclusions are of the lowest non-guarded candidate in each case) until no candidate remains who is not either guarded or checked. After that, each counting of the votes must be taken to convergence, not using any short cut of excluding a candidate before convergence. In my own implementation, convergence is taken as having been reached when the total surplus is no more than 1/10000 of a vote.

When convergence is reached, to the degree of accuracy defined in the rules, if there are too many guarded and checked candidates to fill all seats, a candidate must be excluded. All checked candidates will then have a quota of votes and the one with the highest keep value is excluded.

The counting continues until the number of remaining candidates equals the number of seats to be filled, when all those remaining are elected.

Trying this on Example 1 above, C is not elected but checked. When an exclusion becomes necessary, E is excluded as having the fewest votes of C, D and E. D now has more than a quota and is checked. When an exclusion next becomes necessary the keep values of C and D are 0.521 and 0.474 respectively. C is therefore excluded leaving A, B and D to be elected.

Trying it on Example 2 above in a similar way, E and F are the first to be excluded as having fewest votes. When an exclusion next becomes necessary the keep values of C and D are 0.594 and 0.457 respectively. C is therefore excluded leaving A, B and D to be elected.

So in both these cases, the correct result, in my opinion, is attained.

It should be noted that such a solution is not available for those versions of STV that do not redistribute votes (when appropriate) to already-elected candidates. In those versions there is no equivalent of the keep value of a candidate nor, so far as I can see, anything else that could usefully be employed to give a similar effect.

4 Example of a real non-party election

The test, though, must be how it behaves with real elections. It has been tried on 17 elections where political parties were not involved, each election being used several times as each sitting candidate in turn was taken as having resigned. The results seem to me to be satisfactory. As an example, an election with 11 candidates (A, B, ..., K) for 3 seats, and 58 votes, has been chosen. The votes are set out in Appendix 2.

Those elected were GHJ. If G were to resign and the votes were recounted without any guarding, those elected would be AHK, showing that J had been thrown out because somebody else had resigned, which would not be a sensible outcome. Using the proposed system, those elected would be AHJ, bringing in A to replace G, but not throwing anyone out.

Satisfactory results have also been found if two or more sitting candidates resign simultaneously.

5 A party-based election

Where an election is conducted on political party lines, and there are some non-elected candidates of the various parties, it might be expected that, if someone resigns, the vacancy would probably be someone else of the same party. The complete voting patterns of the Glasgow City Council 2007 elections have been published and these are a valuable resource of real party-based STV elections. The actual counting was not by the Meek method, but a Meek count can be carried out on them nevertheless.

It is a pity that, in general, the Scottish parties did not make the best use of STV in that, except for Labour, they usually put up only 1 candidate per ward. However the Hillhead Ward is an exception. Here those elected, both in fact and by Meek counting, were one each of the Labour, Liberal Democrat, Scottish National and Green parties, while there was also an unelected Labour candidate, and an unelected Liberal Democrat candidate.

Using the proposed method, if the Labour councillor were to resign, the other Labour candidate would be the replacement, but if the Liberal Democrat councillor were to do so, the other Liberal Democrat candidate would be the replacement.

It is not suggested that, if a councillor resigns, someone of the same party ought necessarily to be the replacement. The correct replacement is what the voters want, even if of a different party. However, in a party-based election, it would be a little odd if the correct replacement were not of the same party, where such a person is available. The observed result, using the proposed system, does follow the expected pattern.

As it happens, the other Labour candidate would be the replacement if the Scottish National or Green candidate were to resign, but it is not claimed that this indicates anything special.

6 Comparison with a plain recount

In the party-based election discussed above, it is found that if it were merely rerun normally, without guarding, the sitting members would be elected anyway, and the same pattern of filling the vacancies occurs. That is good – the aim is to get the right solution in difficult cases, not to change the solution in easier cases. It is one of the virtues of the proposed system that if a plain recount would elect all the sitting candidates, then the result always agrees with that of such a plain recount.

7 Acknowledgements

I thank Brian Wichmann and Nicolaus Tideman for much helpful discussion and comments on an earlier proposal of mine, and Jonathan Lundell for helpful discussion on this proposal. However the main acknowledgement must go to Douglas Woodall for suggesting the current proposal and for agreeing so selflessly to let me put it forward in this paper. My own earlier proposal was both more complicated and less effective and I have abandoned it.

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Appendix 1: Examples 1 and 2

Artificial examples can be very useful as illustrations of a problem, but they should not be so unrealistic as to be impossible in practice. It might be thought impossible for A and B to have had enough support to have been elected originally, yet have so little at the recount, yet it is possible.

For Example 1, let there be 22 candidates for 3 seats and votes

10	A
10	B
2	FZDB
18	GZDB
3	HZDB
17	IZDB
4	JZDB
16	KZDB
5	LZCA
15	MZCA
6	NZCA
14	OZCA
7	PZCA
13	QZCA
8	RZCA
12	SZCA
9	TZCA
11	UZCA
30	ZEDA
20	ZEDB

On an initial count ABZ are elected. If Z resigns, and all candidates except ABCDE are then unwilling to stand, we get

10	A
10	B
100	CA
60	DB
30	EDA
20	EDB

with A and B as sitting members, as in Example 1.

For Example 2, similarly, let there be 22 candidates for 3 seats and votes

10	A
10	B
2	GZDB
26	HZDB
3	IZDB
15	JZDB
4	KZDB
19	LZCA
1	MZCA
14	NZCA
6	OZCA
13	PZCA
7	QZCA
12	RZCA
8	SZCA
11	TZCA
9	UZCA
19	ZEDA
20	ZEDB
20	ZFDA
21	ZFDB

On an initial count ABZ are elected. If Z resigns, and all candidates except ABCDEF are then unwilling to stand, we get

10	A
10	B
100	CA
50	DB
19	EDA
20	EDB
20	FDA
21	FDB

with A and B as sitting members, as in Example 2.

9 Appendix 2: A real example

These are the votes in the real non-party election discussed above.

ACBHJIKFDE
ACH
ACIFHJEK
AHDCGFEBJI
AHICDFEBGJ
BCK
BFGHKAECDJ
CBEJIHKFGAD
CGEHAJBDIK
CKABHGDJ
EGKH
EGKHF
GAHE
GEBFA
GEHFCBJIDA
GEKCB
GFECBADHKJ
GFEHACBKJI
GHFCBAEI
GKEHCJDBAF
GKIH
HABCEFGIKD
HABCJ
HACFGBKIJE
HAGKCJIBEFD
HAJFEBCGKID
HBACFK
HBCEFJAIKG
HCA
HCA
HCBGFEKAJI
HDFCEGABJI
HGCKBFEAJD
HGKCBFAJDE
HIBCDG
HIJEGFBACKD
HIKGCABJEFD
HJA
HJACBEFGIKD
HJAIEGCDBKF
HJCABIGEKFD
HJCFGEDBAI
HJIGAB
HJIGKABCFE
HJKABCDEFG
HKB
HKBCEGJAFI
HKCBGAJEFI
HKJ
HKJABC
J
JCBK
JHICBGEFDA
JHICGFBAED
JHKACGIBFE
JICHKBAFGE
KABCFG
KHACJIGBED

Alternative Voting in Proportion

Philip Kestelman

Abstract

Measurably fairer than either Single Member Plurality (SMP), Supplementary Voting (SV) or Two–Round Voting (TRV), even the simplest, fully transferable electoral system — Alternative Voting (AV) — is poorly comprehended. In particular, considering both first and final preferences, AV is rarely if ever less proportional than SMP; and would constitute a reasonable, provisional reform of the UK House of Commons.

1 Introduction

Following the 1950 UK General Election, a London *Times* editorial (27 February 1950) reproached the Liberal Party for over–nominating Parliamentary candidates: "nothing can excuse the irresponsible spattering of the electoral map with hundreds of candidatures for which there was never the remotest chance of substantial support, but which might just deprive the members elected of certainty that they represented the majority of their constituents ... in the great majority of constituencies, the Liberal Party can best serve Liberalism by leaving, or helping, its supporters to judge for themselves which of the two larger parties can do most to put the Liberal spirit into practice".

Over half a century later, British MPs are still elected by *Single Member Plurality* (SMP: so-called 'First-Past-the-Post'); and hardly less forlornly, the Liberal Democratic Party (LDP) nominates candidates in most constituencies. *Alternative Voting*(AV: single member majority, instant run-off) enables voters to express their *sincere* first preferences; ensuring that MPs represent absolute majorities of constituency voters.

However (like SMP), AV scarcely mediates proportional representation (PR). Indeed, the Electoral Reform Society [3, p42], claimed that AV "could produce even more disproportional results than FPTP"; and that AV would have so proved at the last three UK general elections (1997, 2001 and 2005). And following the 2005 General Election, John Curtice (*Independent*, 10 May 2005) contended that AV "would have produced an even more disproportional result – a Labour majority of 98" (instead of 66 Seats).

The Independent Commission on the Voting System (the Jenkins Commission) likewise claimed that AV "is capable of substantially adding to [SMP] disproportionality" [10, p26]. The Independent Commission on PR [9, p118] also maintained that "AV can produce a hugely disproportionate result". The same view was echoed in the long-awaited desk review by the Department of Justice [12, p155], — with neither evidence nor reference.

This article seeks to contest that formidable consensus; and to highlight widespread misunderstandings of AV — both parliamentary and public. It is argued that, bearing in mind not only first preferences, AV has rarely if ever proved less proportional than SMP. In practice, AV has proved more proportional than SMP — and *measurably* so.

2 Quantifying Proportionality

Proportionality mainly concerns the relationship between party vote-fractions and seat-fractions: but *which* vote-fractions? The answer may seem obvious in categorical voting systems (SMP and Party Lists) — overlooking tactical voting (and personal votes for party candidates). The problem arises acutely in preferential, transferable voting systems, notably: Supplementary Voting (SV: contingency voting); Two-Round Voting (TRV: double balloting); Alternative Voting (AV); and multi-member Single Transferable Voting (STV).

How should the proportionality of *transferable* voting be measured? And in particular, how should we compare proportionality between categorical and transferable voting?

For the 1997 General Election in Britain, Dunleavy *et al* [2, p5] calculated a *Deviation from pro-*

portionality (DV) of 21 percent for SMP (actual); and 23.5 percent for AV (estimated): implying that AV was less proportional than SMP. The Jenkins Commission [10, p47] adopted that "statistical measuring rod known as a DV score".

DV is simply the overall difference between over-represented (or under-represented) party vote- and seat-fractions; the *Loosemore-Hanby Index:*

LHI *(%)* $= 0.5\Sigma\mathbf{abs}(V_P\% - S_P\%)$,

where $V_P\%$ = *P-th* Party vote-fraction*(%)*
and $S_P\%$ = *P-th* Party seat-fraction *(%)* [13, p13]

Table 1 illustrates the calculation of LHI for the 2009 European Parliamentary Election in Britain (d'Hondt regional closed party lists). Notice the substantial over-representation of the top four parties; and the large contribution of unrepresented party voters (8.5 percent = 48 percent of LHI = 17.6 percent): implying some need for transferable voting, and the limitation of an exclusive focus on *first* preference disproportionality.

Jenkins [10, p47] equated "full proportionality" with DV = LHI = 4–8 percent; and *full PR* elections may be characterised, a trifle more generously, as yielding LHIs under 10 percent. *Semi-PR* — 'broad PR' — elections may then be defined by LHIs ranging 10–20 percent (say); leaving *non-PR* elections with LHIs over 20 percent.

Thus the last three UK general elections (1997–2005: SMP), with LHIs of 21–22 percent, typified non-PR. Nominally PR, the first three European Elections in Britain (1999–2009), with LHIs ranging 14–18 percent, have proved consistently semi-PR. The fairest *Sainte-Laguë* regional party apportionments would have yielded LHIs ranging 6–10 percent (full PR) [13, pp 12, 21: updated].

An Additional Member System (SMP-plus) elected the Scottish Parliament (1999–2007), the National Assembly for Wales (1999–2007) and the London Assembly (2000–2008). In their first three elections, LHIs (between party list votes and total seats) ranged 11–13 percent, 11–19 percent and 14–15 percent, respectively: all semi-PR [13, p21: updated].

At the 2007 Scottish Council Elections (multi-member STV), first preference LHI averaged 15 percent (ranging 6–33 percent): largely semi-PR [14, p23]. The last three Irish general elections (multi-member STV) have also proved semi-proportional to first preferences (1997–2007: LHI = 12–13 percent); though fully proportional over the previous 10 elections (1965–1992: LHI = 3–10 percent) [13, p22: updated].

All three Northern Ireland Assembly elections (1998–2007: multi-member STV) mediated full PR, first count LHIs ranging 6–7 percent [13, p21: updated]. And at the last 10 Maltese general elections (1966–2008: multi-member STV), first preference LHIs ranged 1–9 percent: also fully proportional — despite the wrong party winning four STV elections — before compensation with additional members in 1987, 1996 and 2008 (Table 2).

3 Other Disproportionality Measures

The simplest — and perhaps most intuitive — measure of party total disproportionality, LHI is only one of several different indices [14, pp18-19]. Notice that LHI *(percent)*:

$$= 0.5\Sigma\mathbf{abs}(1 - S_P\%/V_P\%) \times V_P\%$$

Compare another promising disproportionality measure, the *Gini Index* GnI *(percent)*:

$$= 0.005\Sigma\Sigma\mathbf{abs}(V_P\% \times S_Q\% - S_P\% \times V_Q\%)$$

GnI is analogous to the widespread *Gini Coefficient*, measuring inequality of income or wealth.

In academic circles, the most widespread measure [7, p602] is the *Gallagher Index* GLI *(percent)*:

$$= \sqrt{0.5\Sigma(V_P\% - S_P\%)^2}$$

Nonetheless, in the much-cited article proposing this 'Least Squares Index', Gallagher [6, p49], recommended, "as the standard measure of disproportionality", the *Sainte-Laguë Index* SLI *(percent)*:

$$= \Sigma(V_P\% - S_P\%)^2/V_P\%$$

The Sainte-Laguë (Webster) method is the least biased divisor method of seat *apportionment*, and invulnerable to the paradoxes to which LHI and GhI are susceptible [13, p12].

4 Transferable Voting Proportionality

When calculating the party total disproportionality mediated by transferable voting, Lijphart recommended [15, p19], that: "Because first-preference and final-count votes can differ substantially, the index of proportionality calculated on the basis of first-preference votes may present a distorted picture of the actual extent of disproportionality. It is therefore advisable to use the final-count percentages for the calculation of the index of disproportionality".

On the other hand, Sinnott [17, p117], maintained that "we have no option but to compare parties' first preference votes with their shares of the seats"; while Gallagher [5, p255] argued that "using later-stage figures overstates the proportionality of STV".

Indeed, between STV first and final counts (excluding non-transferable votes), party total disproportionality may be expected to decrease steeply. At the Northern Ireland Assembly elections (1998–2007), LHI decreased from 6–7 to 3–5 percent; and at the Scottish Council Elections (2007), mean LHI decreased from 15 to 9 percent [14, pp22–23]. At the last 10 general elections in Ireland (1977–2007), mean LHI decreased from 8 to 3 percent [13, p22: updated]; and in Malta (1966–2008), from 3 to 2 percent (Table 2).

At the 2003 Northern Ireland Assembly Election, LHI, GnI and SLI decreased, but GhI actually *increased*, between STV first and final counts. And at all 32 Scottish Council Elections in 2007, LHI, GnI and SLI decreased consistently; but GhI increased in two councils [14, p22].

Between first and final counts at the last 10 Maltese general elections, LHI, GnI and SLI usually decreased, unlike GhI in 1996 and 2003 (Table 2). Judged by the standard of the Sainte-Laguë Index [6, p49], LHI appears more reliable than GhI, at least for measuring *transferable* voting disproportionality.

5 Single Member Transferable Proportionality

Table 3 illustrates the calculation of LHI in the most rudimentary form of transferable voting — *Supplementary Voting* (SV) — at the 2008 London Mayoral Election. Between SV first and second counts, party total disproportionality (LHI) — the *wasted* vote-fraction — decreased from 57 to 52 percent (47 percent, excluding non-transferable votes).

However, the SV second count non-transferable vote-fraction (nine percent) exceeded the winner's margin of victory (six percent). Voting for neither first count front-runner — and ignorant of both — those non-transferable voters might have preferred another candidate. SV effectively disfranchises such voters; obliging them to contemplate tactical *(insincere)* second 'preferences', in order to avoid complete vote-wastage (just like SMP).

That problem is partially solved by *Two-Round Voting* (TRV), as in the notorious 2002 French Presidential Election (Table 4a). Between TRV first and second rounds, LHI decreased steeply, from 80 to 18 percent. In TRV (unlike SV), voters enjoy the advantage of expressing their second preferences in the full knowledge of both first preference front-runners.

The 2002 French Presidential Election exposed another flaw in *truncated* preferential voting: the possibility that even sincere first preferences may prove recklessly fissiparous. Between TRV first and second rounds, the *Effective Number of Parties* decreased extremely, from 8.7 to 1.4 parties; and — with voters doubtless chastened tactically by that earlier experience — rather more narrowly in 2007, from 4.7 to 2.0 parties (Table 4b).

Between SV first and second counts (Table 3), and between TRV first and second rounds (Table 4a), party total disproportionality never increases in each constituency. What about national *aggregate* disproportionality? With only a few English mayors elected by SV, there is no real example of such aggregation.

However, a similar form of TRV elects Parliamentary Deputies in France (Table 4b). Averaging 2002 and 2007 French general elections, LHI halved, from 29 to 15 percent, between TRV first and second rounds; leaving TRV *non-PR* overall (like SMP).

The main remedy for the tactical constraints of SMP, SV and TRV is fully transferable voting — and arguably, full preference completion — *Alternative Voting* (AV), in the case of single member constituencies. The classic example is the 1998 Australian General Election in Blair, Queensland (Table 5). Both SMP and SV would probably have elected the racist Hanson (One Nation); while TRV might well have elected Clarke (Labor); whereas AV actually elected Thompson (Liberal). Between AV first and final counts, party total disproportionality (LHI) decreased steadily, from 78 to 47 percent.

The Australian House of Representatives furnishes the only real example of AV national aggregation. At the last 10 general elections, the UK (1970–2005: SMP) and Australia (1983–2007: AV) exhibited comparable numbers of parties (in terms of voters). AV seats have proved more proportional even to *first* preferences than have SMP seats to votes (mean LHIs = 16 and 19 percent, respectively); and significantly more proportional to *final* preferences (mean LHI = 12 percent) [14, p25].

6 Comparing AV with SMP

Closer to home, is there something peculiar about the UK which altogether invalidates such international comparison? What of the claim that, at the last three UK general elections, AV would have proved even less proportional than SMP?

At the 2005 UK General Election, as the Electoral Reform Society [3, p42] explained: "The reason AV would swell the Labour majority is that the second preferences of Lib Dem supporters still tend to favour Labour over Conservative".

Which implied that SMP (actual) party voters were expressing AV *first* preferences; and that seats would be less proportional to AV first preferences than to SMP votes. The former implication seems rather implausible. After all, unlike SMP (partially tactical) voting, AV allows voters to express (wholly sincere) *first* preferences for a wider spectrum of less popular parties; secure in the knowledge that their lower preferences are *transferable* to more popular parties.

Accordingly, the latter implication is plausible, but irrelevant. AV *first* preference disproportionality may well exceed SMP disproportionality; but we are hardly comparing like with like. That comparison between AV and SMP is both artificial and unfair.

Yet just suppose that we equate SMP (actual) party votes with AV first preferences; and assume that, from LDP third-placed candidates, one third of votes are transferred to Conservative candidates, and two-thirds to Labour candidates; and that, from both Conservative and Labour third-placed candidates, two-thirds of votes are transferred to LDP candidates. At the last three general elections in England (1997–2005), for the three main parties, such crude estimates yield the following results [14, p24].

At the 2005 General Election, between AV first and final counts, Labour become slightly less over-represented; and the Conservatives become more under-represented; but the LDP becomes less under-represented. Then SMP (actual) disproportionality lies between AV first and final count (estimated) disproportionality — however measured. Thus AV would have been more-or-less as disproportional as SMP — *despite increasing Labour's overall majority!*

Likewise in 2001, SMP (actual) LHI and GhI lie between AV first and final count (estimated) LHI and GhI. Moreover, SMP (actual) GnI and SLI exceed even AV first count (estimated) GnI and SLI.

At the 1997 General Election — with tactical voting (LDP-supporters voting Labour, and Labour-supporters voting LDP) at its height — AV first and final count (estimated) LHI and GhI exceed SMP (actual) LHI and GhI. Nonetheless, SMP (actual) GnI approximates AV first count (estimated) GnI; while SMP (actual) SLI approximates AV final count (estimated) SLI — theoretically more satisfactory — again leaving AV and SMP comparably disproportional.

At the 1997 General Election in Britain, Dunleavy *et al* [2, p15], simulated SV and AV outcomes by applying regional voters' second preferences (sample-surveyed) to each constituency. LDP transfers were divided between 18 percent Conservative, 49 percent Labour and 33 percent other parties (or non-transferable); making AV (simulated) more disproportional than SMP (actual). However, SMP (actual) party votes were equated with AV *first* preferences; and AV *final* preference disproportionality was not considered.

Thus comparing SMP with AV disproportionality is not straightforward. It depends on what is being compared with SMP party votes (AV first and/or final preferences); and on how the overall vote-seat relationship is measured (the choice of index).

7 Comprehending Transferability

The Jenkins Commission [10, p39], considered that "the decisive objection to AV on its own ... was its potential short-term unfairness to Conservative party supporters ... parties in adversity should not be treated unfairly".

Yet the Conservatives were *not* the sole party in adversity! SMP seats have long under-represented LDP voters — both absolutely and relatively — far more than Conservative Party voters.

Even at the 1997 UK General Election, party-specific disproportionalities $(S_P\% - V_P\%)$ measured: Conservative, -5.6 percent; and LDP, -9.8 percent $(S_P\% / V_P\%$ = 0.82 and 0.42, respectively), [10, p24]

To be sure, Jenkins was not advocating AV alone; but topped-up AV (AV-plus). However, the 1998 Commission made no attempt to dispute other arguments against AV, raised in the *Note of Reservation by Lord Alexander* [10, pp53–55], and summarised without comment in the 2008 Department of Justice desk review [12, pp35–36].

Lord Alexander disclosed fundamental problems in understanding even the simplest method of fully transferable voting; notably wondering: "Why should the second preferences of those voters who favoured the two stronger candidates on the first vote be totally ignored ... ? Why, too, should the second preferences of ... those who support the lower placed and less popular candidates ... be given equal weight with the first preferences of supporters of the stronger candidates?".

Somewhat lamely, the Commission [10, p25], acknowledged, without discussion, that "the second or subsequent preferences of a losing candidate, if they are decisive, are seen by some as carrying less value ... and so contributing less to the legitimacy of the result, than first preference votes".

Nonetheless, the Commission argued that AV "would increase voter choice ... and thus free ... voters ... from a bifurcating choice between realistic and ideological commitment or ... voting tactically". Yet Lord Alexander contended that "AV could further heighten ... tactical voting"!

Such misapprehensions are not confined to these shores. Even the Australian House of Representatives Joint Standing Committee on Electoral Matters [11, p113] — quoted without comment in Farrell and McAllister [4, p56] — defended obligatory full preference completion: "there is a strong chance that an optional preferential system will eventually lead to voters casting only one preference as the realisation sinks in to voters that, to indicate second and subsequent preferences, will decrease the possibility that their most preferred candidate will win".

On the contrary: transferable voters need reassuring that expressing their lower preferences cannot prejudice the chance of electing their higher preference candidates. However (unlike Australians), British voters need not be obliged to express lower preferences: after all, some LDP-supporters may view both Conservative and Labour parties with equal distaste!

8 Conclusions

It is arguable that SMP and AV disproportionalities are strictly incommensurable. Nonetheless, the proportionality of categorical voting should not only be compared with that of first preference, transferable voting. As Reilly [16, p176], put the matter: "assessments of preferential voting systems on the basis of their proportionality which do not consider the impact of lower-order preferences ... offer a misconceived and sometimes misleading interpretation of the true relationship between seats and votes".

In each single member constituency, where no candidate enjoys an absolute majority of votes, even SV (or TRV) would be more representative, less wasteful and more equitable than SMP. Likewise considering both first and final preferences, AV is never less proportional than SMP in each constituency.

It may be possible to devise artificial examples where every constituency is more proportional to AV final preferences than to SMP party votes; but less proportional overall [I.D. Hill, personal communication, 2006]. However, there appears to be no such published argument against AV, which needs some plausibility — and evidence — to be fully persuasive.

Taking account of both first and final counts overall, AV has mediated semi-PR in Australia; while multi-member STV has mediated semi-PR in Scotland, and full PR in the Irish Republic. And with LHIs well under 10 percent, both Maltese MPs and Northern Irish MLAs have proved fully proportional even to STV first preferences.

The measurement of electoral proportionality remains debatable. Not much has changed since Gallagher [6, p33], lamented "surprisingly little discussion of what exactly we mean by proportionality and how we should measure it". The Gallagher Index (GhI) has largely displaced the Loosemore-Hanby Index (LHI), which appears more reliable for evaluating transferable voting disproportionality. (And unlike LHI, GnI and SLI, calculating 'exact' GhI necessitates disaggregating unrepresented party voters [7, pp603–5]).

LHI retains the advantage of simplicity; being the fraction of total seats, transferred from over– to under-represented parties, for exact PR. LHI is analogous to the little-known *Robin Hood Index* of inequality: the fraction of total income, transferred from rich to poor people, for complete equality [18, p41].

Hart [8, p276] rendered the sentiment, expressed in that 1950 *Times* editorial (quoted above), rather more forcefully: "Liberals should be forced to choose between the two other parties". At the 1951 General Election, 77 percent fewer Liberal candidates were nominated; Labour won a plurality of votes (48.8 percent); but Conservatives won an absolute majority of MPs (51.4 percent); and SMP mediated full PR (LHI = 4.1 percent) [12, p92]!

AV would resolve that dilemma; allowing minor party-supporters, in hopeless constituencies, to express their *sincere* first preferences, ultimately *transferable* to major party candidates. Guaranteeing voters absolute majority representation in every constituency, AV could inaugurate less confrontational politics between parties competing for second preferences; and even a choice of candidate within parties.

Long infantilised by SMP, British voters and MPs may need a decade to appreciate even the simplest, fully transferable electoral system. Thanks to tactical considerations, categorical voting (SMP or Party Lists) is no simpler than preferential voting (STV, including AV), which would introduce novel challenges. AV could provide a valuable learning experience; and need not be inhibited by fears that AV might prove less proportional than SMP.

The Electoral Reform Society find AV "only a very minor ... step in the right direction" (towards multi-member STV); while Baston [1, pp6, 25, 50], considers AV a worthwhile reform: "AV could be introduced quickly and simply – it would not require complex legislation, new boundaries or a referendum . . . It does not justify the hopes (or fears) of those who regard it as a piece of pro-Labour manipulation".

AV for MPs in the UK would disclose voters' genuine preferences; while frustrating anything like their proportional representation (much like SMP). Once the principle of fully transferable voting is established, more radical electoral reforms (like AV-plus or multi-member STV) may well follow; needing more complex legislation, boundary revision and a referendum.

At the 1997 UK General Election, Labour promised a referendum on what would have been AV-plus. Over a decade later, Labour could still redeem that pledge — not wholly, but in substantial part — by offering AV in time for the 2010 General Election.

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Table 1: Party Votes, Seats and Disproportionality
(*d'Hondt regional closed party lists*):
European Election (MEPs): Great Britain*, June 2009.

Party	Number		Fraction		Seat-to-vote Fraction	
	Votes (V)	Seats (S)	Votes (V%)	Seats (S%)	Ratio (S%/V%)	Deviation (S% − V%)
Total	15,076,935	69	100.0	100.0	1.00	0.0
Conservative	4,138,394	25	27.4	36.2	1.32	+ 8.8
UK Independence	2,498,226	13	16.6	18.8	1.14	+ 2.3
Labour	2,381,760	13	15.8	18.8	1.19	+ 3.0
Liberal Democrat	2,080,613	11	13.8	15.9	1.16	+ 2.1
Green	1,303,748	2	8.6	2.9	0.34	− 5.7
British National	943,598	2	6.3	2.9	0.46	− 3.4
Scottish National	321,007	2	2.1	2.9	1.36	+ 0.8
Plaid Cymru	126,702	1	0.8	1.4	1.72	+ 0.6
English Democrat	279,801	0	1.9	0.0	0.00	− 1.9
Christian	249,493	0	1.7	0.0	0.00	− 1.7
Socialist Labour	173,115	0	1.1	0.0	0.00	− 1.1
No2EU	153,236	0	1.0	0.0	0.00	− 1.0
Others (V% < 1%)	427,242	0	2.8	0.0	0.00	− 2.8
Over-represented	11,546,702	65	76.6	94.2	1.23	+17.6†
Under-represented	3,530,233	4	23.4	5.8	0.25	−17.6

***** Great Britain (England, Scotland, Wales) and Gibraltar (South West Region).

† LHI = 17.6 percent (*semi-PR*). Compare Sainte-Laguë LHI = 9.6 percent (*full-PR*)‡.

‡ The Sainte-Laguë method was used to *apportion* 72 MEPs between 12 UK Regions: South East (10); London and North West (8); Eastern (7); South West, West Midlands, Yorkshire & Humberside and Scotland (6); East Midlands (5); Wales (4); and North East and Northern Ireland (3): most fairly *proportionating* regional seats to electorates.

Data source : Guardian (9 June 2009).

Table 2: Number of parties, and party total disproportionality, by selected index (multi-member STV first → final count: before top-up compensation), and General Election: Malta, 1966-2008.

Election (Year)	Parties (N_v)	LHI%	GhI%	GnI%	SLI%
1966	2.4	9.0 → 1.6	7.3 → 1.5	12.1 → 2.2	10.4 → 1.5
1971	2.0	1.1 → 0.5	1.0 → 0.5	1.6 → 0.5	1.1 → 0.0
1976	2.0	0.8 → 1.1	0.8 → 1.1	0.8 → 1.1	0.0 → 0.0
1981	2.0	3.2 → 2.8	3.2 → 2.8	3.2 → 2.8	0.4 → 0.3
1987	2.0	3.4 → 3.8	3.3 → 3.8	3.5 → 3.8	0.7 → 0.6
1992	2.1	1.7 → 1.4	1.5 → 1.4	2.1 → 1.4	1.8 → 0.1
1996	2.1	4.5 → 4.0	4.0 → 4.0	5.2 → 4.0	2.1 → 0.6
1998	2.0	2.0 → 0.3	1.8 → 0.3	2.6 → 0.3	1.3 → 0.0
2003	2.0	2.1 → 1.9	1.8 → 1.9	2.4 → 1.9	0.8 → 0.1
2008	2.1	3.5 → 2.5	2.9 → 2.5	4.4 → 2.5	2.2 → 0.2
1966-2008	Mean	3.1 → 2.0	2.8 → 2.0	3.8 → 2.0	2.1 → 0.4

Effective number of parties, $N_v = 1/\Sigma(V_P\%/100)^2$
where $V_P\% = P$-th Party (first count) vote-fraction (percent).

Parties : Despite the opportunities afforded by multi–member STV, Malta has become a two–party polity. For Irish and Scottish Council (2007) voters, the median number of parties (N_v) is four; while Australia (AV) and the UK (SMP) are three–party polities.

Parliament : Despite very low disproportionality, the party with an *absolute majority* of total STV final preferences (50.1–51.6 percent, excluding NT votes) won a *minority* of STV seats (31/65 = 48 percent) in 1981, 1987, 1996 and 2008; and (in 1987, 1996 and 2008) was compensated with four top–up seats (final count best losers), thereby securing a bare overall majority (35/69 = 51 percent).

Main data source : www.maltadata.com/alltrans.xls

Table 3: Mayoral Election (Supplementary Voting): London, June 2008.

Candidate (Party)	First → Second count Vote−fraction (percent)	Seat−fraction (percent)
Boris Johnson (Conservative)	43.2 → 48.4	100.0
Ken Livingstone (Labour)	37.0 → 42.6	0.0
Brian Paddick (Liberal Democrat)	9.8 → 0.0	0.0
Seven Others (V% < 5 percent)	10.0 → 0.0	0.0
Non-transferable (NT)	0.0 → 9.0	0.0
Total Disproportionality (LHI%) = Unrepresented Voters (percent)	56.8 → 46.8 (excluding NT)	

Data sources : Guardian, 3 May 2008; and London Elects (www.londonelects.org.uk).

Table 4: Presidential and General Elections (two-round voting): France, 2002-2007.

Table 4a: Presidential Election: France, April-May 2002.

Candidate (Party)	First → Second round Vote-fraction (percent)	Seat-fraction (percent)
Jacques Chirac (Rally for the Republic)	19.9 → 82.2	100.0
Jean-Marie Le Pen (National Front)	16.9 → 17.8	0.0
Lionel Jospin (Socialist Party)	16.2 → 0.0	0.0
François Bayrou (Union for French Democracy)	6.8 → 0.0	0.0
Arlette Laguiller (Workers' Struggle)	5.7 → 0.0	0.0
Jean-Pierre Chevènement (Citizens' Movement)	5.3 → 0.0	0.0
Noël Mamère (Greens)	5.2 → 0.0	0.0
Nine Others (V% $<$ 5 percent)	24.0 → 0.0	0.0
Total Disproportionality (LHI%) = Unrepresented Voters (percent)	80.1 → 17.8	

Table 4b: Presidential and General Elections: France, 2002-2007.

Election	Year	TRV first → second round	
		Parties (N_v)	LHI%
Presidential	2002	8.7 → 1.4	80.1 → 17.8
	2007	4.7 → 2.0	68.8 → 46.9
General	2002	5.3 → 3.0	31.1 → 18.0
	2007	4.4 → 2.8	26.9 → 11.2

Data source : French Interior Ministry website (www.interieur.gouv.fr/sections/a_votre_service/elections/resultats).

Table 5: Blair, Queensland General Election (Alternative Voting): Australia, 1998.

Candidate (Party)	Count: Vote-fraction (percent)				
	1	2-5	6	7	8
Pauline Hanson (One Nation)	36.0	+ 0.6 = 36.6	+ 0.6 = 37.2	+ 1.7 = 38.9	+ 7.7 = 46.6
Virginia Clarke (Labor)	25.3	+ 0.7 = 26.0	+ 1.7 = 27.7	+ 1.6 = 29.3	– 29.3 = 0.0
Cameron Thompson (Liberal)	21.7	+ 0.4 = 22.1	+ 1.1 = 23.1	+ 8.6 = 31.8	+ 21.6 = 53.4
Brett White (National)	10.3	+ 0.3 = 10.6	+ 1.4 = 11.9	– 11.9 = 0.0	0.0
Neal McKenzie (Democrats)	3.6	+ 1.1 = 4.8	– 4.8 = 0.0	0.0	0.0
Four others (V% $<$ 2 percent)	3.2	– 3.2 = 0.0	0.0	0.0	0.0
Total Disproportionality (LHI%) = Unrepresented Voters (percent)	78.3	77.9	76.9	68.2	46.6

Data source : Australian Electoral Commission website (www.aec.gov.au).