Review — Mathematics and Democracy

Brian Wichmann
Brian.Wichmann@bcs.org.uk

1 Introduction

This book [1] is not a conventional one, since the material from each chapter has been previously published by the author, typically with a co-author. This implies a lack of cohesion between the chapters — for instance, the term *Nash equilibrium* is not defined before use. Hence the reader would be well-advised to consult the Glossary. I would have found a list of abbreviations useful. The book does not have a logical flow — I would have expected it would start with a statement of the relevant impossibility theorems [4, 3].

The book reflects the technical interests of Professor Brams rather than being a comprehensive treatment of the topic as given in the title. It is clear that he has no interest in STV since the transfer of a surplus is not mentioned nor is the Droop quota.

2 Approval Voting

Professor Brams is an advocate of Approval Voting which therefore features prominently in the book. He states that the *Mathematical Association of America*, the *American Statistical Association* and the *Institute of Electrical and Electronics Engineers* all use AV. (The abbreviation AV is used in this article alone for obvious reasons.) By way of comparison, the corresponding UK organisations are the *London Mathematical Society*, the *Royal Statistical Society* and the *British Computer Society* all of which use STV by Meek's method. It is claimed that the UN Secretary-General is chosen by AV, but I could find no support for this [2].

Since with approval voting one merely ticks those candidates one 'approves' of, it is reasonable to think of AV as being simpler than preferential voting. Hence if AV satisfies conditions for an election, perhaps it should be used instead of STV? The book does not attempt to answer such a question since there is no logical flow in the information presented to address the issue. For instance, it is not until page 87 that it is mentioned that there is no perfect electoral system.

The first chapter presents a very interesting analysis of AV in the context of a single winner. It is possible to consider that each voter ranks the candidates and then decides where the line between acceptable and unacceptable comes to undertake the approval ballot. Using this model, one can consider the effect of each voter changing the approval line in his/her vote. It is hardly surprising that the change in the approval line can result in many different candidates being elected. The chapter also compares AV with Condorcet which often gives the same result. The material would seem to advocate Condorcet as much as AV. Very many small examples are used to illustrate the logical issues which are helpful in understanding the details of AV. Due to the impossibility results with electoral systems, it seems to me that statistical analysis of the observed behaviour of AV would be useful, but no such analysis is presented in this book. Here, the otherwise very helpful examples can be a disadvantage if they illustrate situations which do not arise in practice. In general, a reasonable case is made for AV when there is just one seat.

It is clear that AV and preferential voting are by no means equivalent, although Brams has shown that they can be usefully compared. However, some other systems certainly are equivalent (in some sense), as with the Supplementary Vote used for the London mayoral elections and the two stage vote used in the French Presidential elections. (Strict equivalence cannot be expected since, for instance, an event between the two votes could influence the result.)

The consideration given to the use of AV for more than one seat is extraordinarily unconvincing. The problem here is that AV is not a proportional system which implies that various additional rules

For this publication, see www.votingmatters.org.uk

need to be introduced in almost all practical elections. All these changes to vanilla AV seem *ad-hoc* and extremely elaborate in some cases. For instance, it is suggested that AV combined with preferences should be analysed using integer programming which is known (in general) to present problems in computational complexity. How could a voter possibly understand what has happened to his/her vote? STV is sometimes criticised from a lack of transparency, but such a version of AV would be totally opaque.

3 Fair Division

The second part of the book is very interesting. The classic application of this theory is to divide a cake into two: one person applies the cut, while the other decides which portion to take. More elaborate versions of this type of approach are considered in detail. In fact, the interest is often in practical applications in which the theory was *not* applied!

There is a presentation of the decision-making process in the US Supreme Court. The nine judges tend to either agree (27 cases of this), or have a very close decision (5 versus 4 in 24 cases) — all the remaining positions are less frequent. This is hardly surprising in view of the political desirability of having a consensus judgement. There is some evidence that the intermediate cases move to the two extremes as a result of discussion and that this bi-modal distribution appears in other contexts.

Another interesting example is that of choosing a cabinet within a coalition government. Here, the examples are outside the USA. Say that there are three coalition parties, A, B and C with six ministerial appointments to make. A reasonable approach would be for the parties to make their choices in the order ABACBA assuming that the relative strengths are A $>$ B $>$ C. Here a mathematical analysis can be made assuming that each party has its own preference list of appointments. Perhaps unsurprisingly, the result of the analysis shows non-monotonic behaviour and also that insincere choices can be advantageous, assuming perfect knowledge. One has to conclude that the conventional practice of a lengthy discussion is not about to disappear in favour of a more formal procedure.

The start of the process of filling cabinet posts is the question of apportionment even before a sequential choice sequence is determined. There is a brief comparison of apportionment methods which shows that the different methods vary in the way the smaller parties would be handled. Again, agreement on such issues would seem problematic unless already codified, such as the apportionment of seats in the US House of Representatives.

Another example of fair division considered in one chapter is that of distributing a number of indivisible items amongst some parties. For instance, it could be the distribution of heirlooms from an estate. (In this case, we ignore the possibility of selling an item which would obtain cash which could then be divided.) Here, no real example is considered but a detailed mathematical analysis is undertaken. The conclusion from this analysis is that significant problems arise unless the number of heirlooms is greater than the number of people to receive them.

One chapter considers a more complex form of division which is called *Adjusted Winner* (AW). The general objectives are proportional and envy-free division. The approach is to start by listing the issues involved, trying to make them as separable as possible. The parties then weight these issues. The higher weight implies that the solution is likely to involve winning on that issue. Fairly simple numerical calculations can then be used to ascertain the optimal solution.

A very detailed analysis is undertaken of applying this method to the Camp David Accord. The author suggests (I think rather optimistically) that AW might well have expedited this agreement, perhaps by two or three years after the Yom Kippur War in 1973. The difficulty here seems to me that people would be reluctant to try a mathematical-based method instead of conventional 'bargaining'. On the other hand, the method clearly obtains a solution very near to the actual agreement and with a high degree of transparence. The example shows that merely specifying preferences would not be adequate here.

4 Conclusions

Readers of *Voting matters* will no doubt be surprised at Professor Brams' dismissal of STV in favour of Approval Voting. If one ignores that issue, then there is much of interest in the book. The relationship between AV and preferential voting given in the first two chapters appears to give support for Condorcet.

Since the book chapters are extracted from previous publications, there is nothing new here. Also, the use of mathematics is sometimes very helpful and on other occasions unlikely to be of interest to the wider public. Surely the use of the Lebesgue

measure (p295) is not a requirement of understanding voting methods.

5 References

[1] S. J. Brams. Mathematics and Democracy — designing better voting and fair-division procedures. Princeton Press. ISBN 978 0 691 13321 8. 2008.

[2] Selecting the Next UN Secretary-General. United Nations Association of the USA, May 2006. See Appendix A. (Available on the Internet.)

[3] D. R. Woodall, An impossibility theorem for electoral systems, *Discrete Mathematics* 66 (1987), 209-211.

[4] K Arrow: Social Choice and Individual Values (2nd edition, Wiley 1962).