On Measuring Transferable Voting Proportionality

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Abstract

Following [16], this paper applies four measures of Party Disproportionality to both categorical and transferable voting (SMP, 2003; and STV, 2007) in Scotland. By the standard of the Sainte–Laguë Index (SLI), the Gallagher Index (GhI) appears less reliable than the simplest Loosemore–Hanby Index (LHI), or the Gini Index (GnI). The proportionality of SMP should not only be compared with that of first preference AV, which proves no less proportional than SMP.

1 Introduction

For comparison between categorical, nontransferable voting (notably Single Member Plurality SMP, — 'First-Past-The-Post') and preferential, transferable voting (especially STV, including Alternative Voting, AV), precision tools are needed to measure proportionality. Despite a generation of sporadic discussion of the best measure of disproportionality, there is still remarkably little discussion of its application to transferable voting.

Substantial references are few and far between. Thus McLean [19, p22] observed:

> The most theoretically defensible definition of proportionality would be one which compared the vector of seat shares with the full matrix of voting preferences ... only STV even attempts to tap voters' preferences below their first. The incompleteness of extant measures of proportionality is therefore less than fair to STV.

More boldly, Lijphart [18, p19] recommended:

Because first-preference and final-count votes can differ substantially, the index

of proportionality calculated on the basis of first-preference votes may present a distorted picture of the actual extent of disproportionality. It is therefore advisable to use the final-count percentages for the calculation of the index of disproportionality.

On the other hand, Gallagher [8, p255] argued that "using later-stage figures overstates the proportionality of STV". Indeed, between STV first and final counts (excluding non-transferable votes), Party Disproportionality may be expected to decrease steeply.

2 Definitions

Around 20 Party Disproportionality indices have been proposed [20]. The simplest measure remains the Loosemore–Hanby Index,

LHI % = 0.5 $\Sigma | V_P \% - S_P \% |$,

where V_P %, S_P % = P-th Party Vote-, Seat-fraction (percent).

Notice that LHI $\% = 0.5 \Sigma | 1 - (S_P\%/V_P\%) | \times V_P\%$: that is, half the total absolute differences, between each Party's *exact* proportionality $(S_P\%/V_P\% = 1)$ and its *actual* $S_P\%/V_P\%$ ratio, weighted by its Vote–fraction $(V_P\% / 100)$. Compare the Gini Index, **GnI**%

$$= 0.005 \Sigma \Sigma | (V_P \% \times S_Q \%) - (S_P \% \times V_Q \%) |$$

= 0.005 $\Sigma \Sigma | (S_P \% / V_P \%) - (S_Q \% / V_Q \%) |$
 $\times V_P \% \times V_Q \%$:

that is, half the total absolute differences, between the $S_P \% / V_P \%$ ratios of each pair of parties, weighted by the product of their Vote–fractions [16, p15].

In a much-cited discussion of various measures of proportionality, Gallagher [9, p40] proposed his own 'Least Squares Index',

GhI% = $[0.5 \Sigma (V_P \% - S_P \%)^2]^{0.5}$.

For this publication, see www.votingmatters.org.uk

At least in academic circles, the Gallagher Index (GhI) has become the most widespread measure of Party Disproportionality [10, p602].

Nonetheless, because Sainte–Laguë (Webster) is the least biased Divisor method of seat *apportionment*, and invulnerable to the paradoxes to which LHI (and GhI) are susceptible, Gallagher [9, p47– 9] recommended a Sainte–Laguë Index "as the standard measure of disproportionality ... probably the soundest of all the measures":

SLI% =
$$\Sigma (V_P \% - S_P \%)^2 / V_P \%$$

= $(\Sigma S_P \%^2 / V_P \%) - 100$.

(All four disproportionality indices are recapitulated in Table 3.3 on page 23.)

3 Previous Findings

Between first and final counts at 13 Irish general elections (multi-member STV, 1961–2002), mean LHI decreased from 7.3 to 3.1 percent overall; remarkably invariant with District Magnitude (M = Seats per Constituency): decreasing from 7.6 to 4.6 percent (M = 3); from 9.3 to 4.3 percent (M = 4); and from 7.6 to 3.6 percent (M = 5). And averaging two Northern Ireland Assembly elections (multi-member STV, 1998–2003: M = 6), LHI decreased from 6.2 to 4.6 percent [16, p21–2].

4 New Findings

Between first and final counts, the 2007 Irish General Election STV LHI decreased steeply, from 12.0 to 4.3 percent overall: and from 16.0 to 6.4 percent (M = 3); from 13.0 to 8.9 percent (M = 4); and from 10.1 to 4.8 percent (M = 5)[3]. And the 2007 Northern Ireland Assembly (NIA) STV LHI decreased from 7.0 to 3.3 percent (M = 6: Table 3.1).

Taagepera and Grofman [20, p671] sustained five Disproportionality indices, including: LHI; GhI; GnI; and SLI ('chi-square'). Between STV first and final counts, all four indices decreased at each of the last 14 Irish general elections (1961–2007) overall.

However, between STV first and final counts at the 2003 NIA Election, LHI, GnI and SLI decreased; but GhI actually *increased* ($2.9 \rightarrow 3.4$ percent). And between the 1998 and 2007 NIA elections, *first* count LHI, GnI and SLI increased, but GhI decreased; while *final* count LHI, GnI and SLI decreased, but GhI increased (Table 3.1).

Further scrutiny reveals that, at the 1965 (M = 4) and 1981 (M = 5) Irish general elections, LHI, GnI

and SLI decreased, but GhI increased, between STV first and final counts. Again, in two out of 32 Councils at the 2007 Scottish local elections, LHI, GnI and SLI decreased; but GhI increased. Table 3.2 summarises these findings.

Nonetheless — regardless of measure — Table 3.3 shows that the 2007 Scottish Council Elections (STV: M = 3–4) proved significantly more proportional than their 2003 predecessors (SMP). Indeed, Party Disproportionality proved substantially higher in 2003 (SMP mean LHI = 24.9 percent: non–PR) than in 2007 (STV First \rightarrow Final Count mean LHI = 14.9 \rightarrow 8.7 percent: semi–PR \rightarrow full PR).

Calculating 'exact' GhI proves exceptionally tedious: necessitating the disaggregation not only of the votes for each elected independent candidate (also needed in calculating SLI); but also of every single *unelected* independent. Both GhI and LHI are often miscalculated; mainly by aggregating minor parties and/or independent candidates (as if they represented a single party).

Another complication was the protraction of Scottish STV counts [11, p227]:

"The requirement to continue transferring votes when there [are] two continuing candidates and only one place remains to be filled is unnecessary and its effects are undesirable".

All STV final count Disproportionality measures presented here exclude those superfluous transfers; following the long-established Irish convention.

5 Controversy

The Independent Commission on the Voting System [13, p26] contended that AV "is capable of substantially adding to [SMP] disproportionality"; while the Independent Commission on PR [12, p118] also maintained that "AV can produce a hugely disproportionate result". And according to the Electoral Reform Society (www.electoral-reform.org.uk), AV "can be less proportional than" SMP; a view echoed in the long-awaited desk review by the Ministry of Justice [14, 155] — disclosing neither evidence nor reference.

Moreover, the Electoral Reform Society [7, p42] argued that AV would *actually* have proved less proportional than SMP at all of the last three UK general elections:

"In the 1997 election feeling was running so strongly against the Conservatives that AV would simply have helped several more Lib Dem and Labour voters swap preferences and defeat Conservatives in seats where the Tories were ahead under FPTP. In 2001 it would also have swollen the Labour majority. In 2005 the evidence from opinion polling suggests that Labour would have once again had a larger majority under AV than FPTP" [SMP].

So it is of some interest to look more closely at recent UK general elections; mainly supposing that SMP Party Votes = AV first preferences; and that, from third-placed LibDem candidates, two-thirds of votes transfer to Labour candidates, and one third to Conservative candidates.

Table 3.4 attempts such a crude estimation for the three main parties in 2005 in England (where Conservative exceeded Labour votes, but Labour much exceeded Conservative Seats). Between AV first (SMP) and final counts, Vote–fractions increase for both the Conservatives (by 1.7 percent) and Labour (by 2.6 percent); and decrease for the LibDems (by 4.3 percent); while Seat–fractions increase for both Labour (by 2.1 percent) and the LibDems (by 2.3 percent); and decrease for the Conservatives (by 4.4 percent).

Consequently, between AV first and final counts, absolute deviations $|S_P \% - V_P \%|$ increase for the Conservatives (by 1.7 percent); but decrease for both Labour (by 2.6 percent) and the LibDems (by 4.3 percent). Under AV, Labour become less over-represented; the Conservatives become more under-represented; and the LibDems — far more under-represented than the Conservatives under SMP — become less under-represented. As a result, SMP Disproportionality (LHI = 16.6 percent) — much as expected — lies somewhere between AV first and final counts (LHI = 18.7 \rightarrow 16.1 percent). Thus in 2005, AV would have been more-or-less as disproportional as SMP — *despite increasing the Labour majority!*

Estimated AV results for the 2001 General Election were similar. Even in 1997, when SMP Disproportionality, as measured by LHI and GhI, fell below that of both AV first and final preferences, SMP approximated AV first count GnI and final count SLI (Table 3.4).

Despite the crudity of these estimates (and their assumptions), only trifling differences in Party Disproportionality separate SMP from AV. It remains possible to devise artificial examples of AV exceeding SMP Disproportionality; and real countries differ in their political arrangements, perhaps weakening international comparisons.

Nonetheless, it is of interest to compare the last 10 general elections in the UK (SMP, 1970–2005) and Australia (AV, 1983–2007); with similar numbers of parties (in terms of voters). SMP seats have proved less proportional to party votes (mean LHI = 19.0 percent) than AV Seats to *first* preferences (mean LHI = 16.0 percent); and significantly less proportional than AV Seats to *final* preferences (mean LHI = 11.9 percent). Table 3.5 gives the details.

6 Discussion

What can be made of the new findings? By Gallagher's "standard measure of disproportionality" [9, p49], SLI (like both LHI and GnI) decreased — as expected — in five cases, between aggregate STV first and final counts; whereas GhI increased (Table 3.1 and Table 3.2). Thus GhI appears a less reliable Disproportionality index, at least for transferable voting.

Lijphart [17, p59–60] recommended GhI as steering "a middle course between the Rae and Loosemore-Hanby indices. Its key feature is that it registers a few large deviations much more strongly than a lot of small ones". (The Rae and Loosemore–Hanby indices measure party *average* and *total* Disproportionality, respectively; but the merit of any hybrid measure remains unclear). Comparing two hypothetical election results, with the same LHI (and GnI = 5.0 percent), his intuitively "much more proportional situation" returned a "much lower" GhI (2.2 < 5.0 percent); but a slightly higher SLI (1.3 > 1.0 percent) [15, p9] — prefiguring the real GhI anomalies reported here.

Calculating GhI is complicated by the problem of 'lumped residuals' [10, p603–5]. Table 3.3 compares 'exact' LHI, GhI, GnI and SLI, between 2003 SMP and 2007 STV first and final counts, in Scottish Council elections.

LHI detects proportionality changes between STV first and final counts more reliably; and simply quantifies overall party over– (or under–) representation. LHI also proves highly correlated with the theoretically preferable GnI (satisfying Dalton's Transfer Principle), analogous to the widely used *Gini Coefficient* of income or wealth inequality.

The definitive Sainte–Laguë Index is easily calculated, but may exceed 100 percent [16, p8]. That Philip Kestelman: Transferable Proportionality

problem may be solved by transforming SLI into a [8] Gallagher, M (1986): 'The Political Borooah Index, Consequences of the Electoral Syste

BrI% = 100 - 1 / [
$$\Sigma$$
 (S_P % / 100)² / V_P %]
= SLI% / (SLI% + 100).

BrI is analogous to another measure of income or wealth inequality [16, p15].

Preferential, transferable voting (including AV) liberates voters from the tactical constraints of non-transferable, categorical voting (like SMP). Accordingly, comparing categorical voting party Disproportionality with transferable *first* preference Disproportionality is both artificial and unfair. Transferability allows voters to designate sincere *first* preferences for a much wider spectrum of less popular parties (and independents); reassured that lower preferences are *transferable* to more popular parties.

Consequently, seats may well prove less proportional to AV *first* preferences than to SMP Votes; and SMP should also be compared with AV *final* preference Disproportionality. Any fear that AV for British MPs may prove less proportional than SMP may be reasonably disputed on the basis of crude estimates for England, 1997–2005 (Table 3.4); and of comparing the UK (SMP, 1970–2005) with Australia (AV, 1983–2007: Table 3.5).

7 References

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Election	STV First \rightarrow Final Count Disproportionality Index						
	LHI%	GhI%	GnI%	SLI%			
1998	$6.0 \rightarrow 3.8$	$3.6 \rightarrow 2.4$	$9.3 \rightarrow 5.7$	$4.7 \rightarrow 1.8$			
2003	$6.4 \rightarrow 5.4$	$2.9 \rightarrow 3.4$	9.4 ightarrow 7.0	6.1 ightarrow 2.0			
2007	7.0 ightarrow 3.3	3.1 ightarrow 2.6	$10.3 \rightarrow 5.2$	7.3 ightarrow 1.2			

Table 3.1: Party Disproportionality Index: Northern Ireland Assembly Elections, 1998–2007.

Electoral Office for Northern Ireland (2004); and

Electoral Office for Northern Ireland (2007).

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http://www.eoni.org.uk/index/elections/assembly-election-2007
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Table 3.2: Party Disproportionality Index: Country, Year and District Magnitude (selected), 1965–2007.

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Country, Year :	STV First \rightarrow Final Count					
District Magnitude (M)	Disproportionality Index					
[Aggregate Seats (S)]	LHI %	GhI %	GnI %	SLI %		
Irish Republic, $1965 : M = 4 [S = 44]$	$6.0 \rightarrow 5.8$	$4.4 \rightarrow 5.4$	$8.1 \rightarrow 7.6$	$3.6 \rightarrow 2.2$		
Irish Republic, 1981 : M = 5 [S = 75]	$5.3 \rightarrow 4.1$	$3.4 \rightarrow 3.5$	$7.9 \rightarrow 4.3$	5.7 ightarrow 0.7		
Northern Ireland, $2003 : M = 6 [S = 108]$	$6.4 \rightarrow 5.4$	$2.9 \rightarrow 3.4$	$9.4 \rightarrow 7.0$	$6.1 \rightarrow 2.0$		
Scotland (Moray), 2007 : M = 3–4 [S = 26]	$21.3 \rightarrow 14.4$	$6.2 \rightarrow 6.5$	$32.5 \rightarrow 17.5$	$37.3 \rightarrow 12.7$		
Scotland (Stirling), 2007 : M = 3–4 [S = 22]	$12.9 \rightarrow 11.1$	$8.3 \rightarrow 9.1$	$16.2 \rightarrow 13.1$	10.7 ightarrow 6.6		

Data Sources: Dáil Éireann (1962–2007); Electoral Office for Northern Ireland (2004);

Bochel & Denver [1]; and

32 Scottish Council Websites or Personal Communications.

Data Sources: Chief Electoral Officer for Northern Ireland (1998);

Council	LHI%		GhI%		GnI%		SLI%	
Council	SMP	STV	SMP	STV	SMP	STV	SMP	STV
Aberdeen City	19.5	$8.8 \rightarrow 4.9$	13.2	$6.2 \rightarrow 3.2$	20.8	$12.1 \rightarrow 5.8$	16.7	$7.0 \rightarrow 2.4$
Aberdeenshire	14.6	$14.6 \rightarrow 8.0$	7.1	$8.2 \rightarrow 3.9$	21.5	$22.1 \rightarrow 10.7$	20.2	$20.0 \rightarrow 7.5$
Angus	28.7	$12.2 \rightarrow 7.4$	16.2	6.4 ightarrow 4.1	35.1	17.1 ightarrow 9.3	42.7	$11.6 \rightarrow 3.0$
Argyll + Bute	32.7	$19.4 \rightarrow 7.9$	9.5	7.2 ightarrow 3.2	41.5	$27.7 \rightarrow 12.0$	55.6	$26.5 \rightarrow 6.7$
Clackmannanshire	13.7	$7.8 \rightarrow 5.1$	10.2	6.1 ightarrow 3.8	16.9	$10.9 \rightarrow 6.0$	11.7	$6.1 \rightarrow 1.3$
Dumfries+Galloway	17.6	$11.4 \rightarrow 6.7$	8.2	$5.3 \rightarrow 3.7$	23.3	$14.7 \rightarrow 8.2$	20.8	$12.1 \rightarrow 6.0$
Dundee	7.6	$10.6 \rightarrow 6.3$	4.7	$6.3 \rightarrow 4.3$	10.6	$12.3 \rightarrow 7.0$	6.6	$7.7 \rightarrow 2.8$
East Ayrshire	22.7	$9.1 \rightarrow 4.3$	18.8	$5.3 \rightarrow 3.2$	25.5	$12.4 \rightarrow 5.0$	23.5	8.9 ightarrow 1.7
E Dunbartonshire	23.7	$15.1 \rightarrow 10.9$	15.8	$11.7 \rightarrow 8.2$	27.7	$22.1 \rightarrow 14.1$	28.5	$20.1 \rightarrow 6.5$
East Lothian	33.3	$16.3 \rightarrow 10.0$	27.2	10.5 ightarrow 6.4	37.9	$23.7 \rightarrow 13.8$	49.8	$19.6 \rightarrow 7.8$
East Renfrewshire	17.1	$10.4 \rightarrow 8.1$	9.8	$6.6 \rightarrow 4.9$	22.2	$15.5 \rightarrow 10.2$	20.6	$10.0 \rightarrow 4.8$
Edinburgh	24.3	$10.6 \rightarrow 5.1$	20.7	$6.5 \rightarrow 4.3$	34.9	15.1 ightarrow 6.7	43.0	$8.8 \rightarrow 1.7$
Falkirk	14.0	$15.9 \rightarrow 12.2$	6.0	8.7 ightarrow 8.0	20.9	$18.3 \rightarrow 13.0$	21.1	$16.6 \rightarrow 10.6$
Fife	19.7	9.7 ightarrow 3.6	12.1	5.0 ightarrow 2.1	24.5	$12.8 \rightarrow 5.6$	22.1	8.7 ightarrow 2.6
Glasgow	42.3	$17.1 \rightarrow 8.0$	34.1	$11.5 \rightarrow 5.1$	44.1	$21.2 \rightarrow 8.8$	72.7	$18.4 \rightarrow 4.2$
Highland	29.4	$23.2 \rightarrow 10.9$	4.6	7.9 ightarrow 4.0	41.5	$33.4 \rightarrow 16.0$	56.4	$39.4 \rightarrow 12.9$
Inverclyde	25.3	$11.7 \rightarrow 7.3$	19.8	$6.6 \rightarrow 4.9$	33.3	$14.8 \rightarrow 9.7$	39.8	$11.6 \rightarrow 6.3$
Midlothian	43.4	$17.2 \rightarrow 14.7$	34.3	$12.3 \rightarrow 9.8$	48.4	$23.2 \rightarrow 17.1$	82.5	$23.2 \rightarrow 15.6$
Moray	29.7	$21.3 \rightarrow 14.4$	12.2	6.2 ightarrow 6.5	37.1	$32.5 \rightarrow 17.5$	44.1	$37.3 \rightarrow 12.7$
North Ayrshire	29.0	$14.6 \rightarrow 5.8$	22.9	$6.9 \rightarrow 3.0$	32.9	$19.9 \rightarrow 8.6$	37.6	$15.4 \rightarrow 3.8$
North Lanarkshire	23.3	$11.2 \rightarrow 8.1$	18.5	$7.2 \rightarrow 5.3$	25.6	$15.6 \rightarrow 9.6$	25.1	$14.6 \rightarrow 7.9$
Orkneys	41.5	$29.4 \rightarrow 17.3$	11.9	$7.4 \rightarrow 5.8$	48.0	$41.7 \rightarrow 21.6$	77.5	$58.9 \rightarrow 22.7$
Perth + Kinross	5.9	7.9 ightarrow 6.7	3.8	$5.2 \rightarrow 4.5$	9.6	$11.3 \rightarrow 8.6$	7.9	$8.0 \rightarrow 4.5$
Renfrewshire	15.7	$13.1 \rightarrow 4.7$	12.6	8.1 ightarrow 2.9	21.7	$15.0 \rightarrow 7.2$	17.2	$12.5 \rightarrow 3.9$
Scottish Borders	22.9	$14.3 \rightarrow 10.3$	8.2	6.0 ightarrow 4.4	35.3	$18.8 \rightarrow 14.1$	46.3	$17.2 \rightarrow 11.0$
Shetlands	34.9	$32.6 \rightarrow 19.2$	14.7	7.8 ightarrow 6.1	42.7	$45.6 \rightarrow 23.6$	60.2	$68.8 \rightarrow 24.8$
South Ayrshire	21.8	$6.2 \rightarrow NA$	16.6	$3.2 \rightarrow NA$	24.1	$7.0 \rightarrow NA$	28.3	$6.7 \rightarrow NA$
South Lanarkshire	30.2	$13.1 \rightarrow 6.2$	23.9	7.0 ightarrow 4.0	32.7	$17.0 \rightarrow 9.3$	38.6	$14.1 \rightarrow 5.0$
Stirling	36.1	$12.9 \rightarrow 11.1$	25.2	8.3 ightarrow 9.1	37.5	$16.2 \rightarrow 13.1$	56.7	10.7 ightarrow 6.6
W Dunbartonshire	32.2	$20.1 \rightarrow 6.9$	26.2	$10.0 \rightarrow 4.4$	33.6	$24.4 \rightarrow 10.2$	42.4	30.1 ightarrow 7.6
Western Isles	28.0	$23.3 \rightarrow 14.5$	8.5	6.0 ightarrow 4.5	38.7	$34.6 \rightarrow 20.0$	47.8	$39.1 \rightarrow 18.0$
West Lothian	16.0	$15.6 \rightarrow 3.9$	11.4	8.7 ightarrow 2.7	21.8	19.1 ightarrow 5.8	21.3	$17.6 \rightarrow 2.5$
Mean	24.9	$1\overline{4.9} \rightarrow 8.7$	15.3	$7.4 \rightarrow 4.8$	30.4	$20.3 \rightarrow 11.2$	37.0	$19.6 \rightarrow 7.6$
95%CLmean Upper	28.3	$17.1 \rightarrow 10.2$	18.2	$8.1 \rightarrow 5.5$	34.0	$23.5 \rightarrow 13.0$	44.2	$25.0 \rightarrow 9.8$
Lower	21.5	$12.7 \rightarrow 7.3$	12.3	$6.6 \rightarrow 4.2$	26.7	$17.0 \rightarrow 9.4$	29.8	$14.2 \rightarrow 5.4$

Table 3.3: Party Disproportionality Index: Scotland: SMP (2003); and STV (2007).

Notes:

NA = Not Available. First \rightarrow Final Count

Loosemore-Hanby Index,	LHI%	$= 0.5 \Sigma S_P \% - V_P \% ;$
Gallagher Index,	GhI%	$= [0.5 \Sigma (S_P\% - V_P\%)^2]^{0.5};$
Gini Index,	GnI%	= 0.005 $\Sigma \Sigma \mid (S_P \% \times V_Q \%) - (S_Q \% \times V_P \%) \mid$; and
Sainte-Laguë Index,	SLI%	$= \Sigma (S_P \% - V_P \%)^2 / V_P \%:$

where

 $V_P\%, S_P\%$

= P-th Party Vote-, Seat-fraction (*percent*).

Data Sources: Rallings C & Thrasher T (2003): Local Elections Handbook 2003. LGCEC, Plymouth; Bochel & Denver [1]; and

32 Scottish Council Websites or Personal Communications.

Table 3.4: Party Disproportionality Index: Three General Elections, England, 2005–1997.

Year: System Vote / Seat Party: Fraction (percent) Count (Basis) Fraction Conservative Labour Lib Dem 2005: SMP V% S% 38.0 36.8 37.7 54.3 24.3 8.9 (Actual) S% - V%-1.2+16.6-15.42005: AV First V% S% 32.4 56.4 24.3 11.2 38.0 37.7 (Estimate*) S% - V%-5.6+18.7-13.140.3 56.4 AV Final V% S% 39.7 32.4 20.0 11.2 (Estimate*) S% - V%-7.3 +16.1- 8.8 Disproportionality LHI% GhI% GnI% SLI% 2005: SMP 16.6 16.0 22.2 17.1 2005: AV First 18.7 16.6 22.3 17.2 $\rightarrow 16.1$ \rightarrow Final* $\rightarrow 14.0$ $\rightarrow 18.2$ $\rightarrow 11.7$ 2001: SMP 18.1 16.1 21.6 16.3 2001: AV First 18.8 16.3 20.3 14.9 \rightarrow 18.0 \rightarrow Final* $\rightarrow 15.7$ $\rightarrow 18.6$ $\rightarrow 13.2$ 1997: SMP 16.5 14.9 20.1 14.6 1997: AV First 19.5 17.0 20.0 15.4 \rightarrow Final* $\rightarrow 19.0$ $\rightarrow 17.0$ $\rightarrow 19.3$ $\rightarrow 14.5$

Actual SMP and Estimated AV for the three main parties.

* Estimate, based on following Main Assumptions:

Constituency Party SMP (actual) Votes = AV first preference votes ;

Third–placed LibDem:	$\frac{2}{3}$ Votes \rightarrow Labour;	$\frac{1}{3}$ Votes \rightarrow Conservative ;
Third–placed Labour:	$\frac{2}{3}$ Votes \rightarrow LibDem;	$\frac{1}{3}$ Votes \rightarrow Non–transferable ;
Third-placed Conservative:	$\frac{2}{3}$ Votes \rightarrow LibDem;	$\frac{1}{3}$ Votes \rightarrow Non–transferable .

Data Sources: Rallings, C and Thrasher, M *eds* (1998): Britain Votes 6: British Parliamentary Election Results 1997. Ashgate, Aldershot; and Electoral Commission (2001, 2005).

United	Kingdom (S	SMP)	Australia (AV)				
Election	Parties	LHI% †	Election	Parties	Count: LHI% †		
(Year)	$(N_{P})*$		(Year)	(N _P)*	First	Final	TCP ‡
1970	2.46	8.8	1983	2.68	15.2	14.3	11.2
1974 (Feb)	3.13	19.9	1984	2.81	11.8	10.9	7.9
1974 (Oct)	3.15	19.0	1987	2.90	13.6	12.7	9.8
1979	2.87	15.3	1990	3.37	17.1	9.9	5.1
1983	3.46	24.2	1993	2.91	14.1	11.4	7.4
1987	3.33	20.9	1996	3.23	18.8	16.2	12.6
1992	3.06	18.0	1998	3.46	20.5	10.4	6.4
1997	3.22	21.2	2001	3.43	18.2	10.6	4.9
2001	3.33	22.1	2004	3.26	15.8	11.3	6.6
2005	3.59	20.7	2007	3.10	15.0	10.9	5.2
Mean	3.16	19.0	Mean	3.12	16.0	11.9	7.7
(95%CI)	(2.9–3.4)	(16–22)	(95%CI)	(2.9–3.3)	(14–18)	(10–13)	(6–10)

Table 3.5: Single Member Plurality: UK, 1970–2005 and Alternative Voting: Australia, 1983–2007.

Notes

*	Parties (Number), N _P where $V_P \%$	= $1 / \Sigma (V_P \% / 100)^2$, = $P-th$ (SMP / AV First Count) Party Vote–fraction (<i>percent</i>).
†	Loosemore-Hanby Index, LH where $S_P \%$, $V_P \%$	$I\% = 0.5 \Sigma S_P \% - V_P \% ,$ = <i>P</i> -th Party Seat-, Vote-fractions (percent).
‡	TCP = Two-Candidate Prefer	ed (Exhaustive Vote Distribution).

Data Sources: Rallings, C and Thrasher, M *eds* (2000): British Electoral Facts 1832–1999. Ashgate, Aldershot; Electoral Commission (2001, 2005); Australian Electoral Commission (www.aec.gov.au); and Psephos Website (http://psephos.adam-carr.net/countries/a/australia).