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Voting matters

for the technical issues of STV

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Editorial

There are five papers in this issue, the first three being:

- Jonathan Lundell: *Random tie-breaking in STV*.

  Although *Voting matters* has had several papers about tie-breaking, one can see that there is still more to be said on the matter.

- David Hill: *Implementing STV by Meek's method*.

  David Hill has provided an implementation of Meek's method for many years. This implementation has been taken as the 'definition' of the method for the New Zealand elections. In this paper, the details of this implementation are described and contrasted with that of the original *Computer Journal* article.

- Robert Newland: *Computerisation of STV counts*.

  Although Robert Newland died in August 1990, readers may well be surprised at the relevance of this paper for today. Up to his death, he was the leading technical expert on STV within ERS. This paper was located by David Hill and since it was never published, printing it here seemed appropriate. It is hoped that readers will respond to the suggestions made.

The final two papers have a common theme: the form of STV proposed by British Columbia and now being considered for the Scottish local elections to be held next year.

- Jeff O'Neill: *Comments on the STV Rules Proposed by British Columbia*.

  This paper presents the details of an implementation of the British Columbia rules which has been available on the Internet for some time. It is a very simple version of STV in computer terms. Several issues arose from this work which are detailed in the paper.

- James Gilmour: *Developing STV Rules for manual counting to give effect to the Weighted Inclusive Gregory Method of transferring surpluses, with candidates' votes recorded as integer values.*

  The paper is a complete contrast to the previous one. Like the previous paper, the aim is to transfer surpluses by considering all papers, not just the last batch that gave rise to the surplus. The contrast is in its presentation as a manual counting process and the provision of the conventional result sheet. One novelty is (at least within the UK) that the calculations are undertaken with high precision, but the results are presented as integers.

  James Gilmour has produced a proposal and sent it to the Scottish Executive. This proposal, slightly modified, is now on the McDougall web site. Hence the article provides the rationale and background to the proposal.

It is hoped that the contrast between the two methods above will clarify the choices to be made for the Scottish elections. The final choice will be awaited with interest.

Two other items may be of interest to readers. Firstly, the final report on electronic voting in Ireland is due out shortly and will be found at: `http://www.cev.ie/`. Secondly, it has come to my attention that the British Computer Society elect their council by STV but do not provide a result sheet to their electorate — only a list of those elected. Since the transfer of votes will not be visible, this seems to me to be STV in name only. Do readers have other examples of this?

Random tie-breaking in STV

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1 Introduction

The resolution of ties in STV elections is not a settled question. On the contrary, it remains a topic of lively discussion, with several papers published on the subject in these pages; see Earl Kitchener's note, "A new way to break STV ties in a special case" [1] for a summary.

Ties can arise in any STV election during exclusion. With some methods ties can arise at other stages as well; Jeffrey O'Neill [2] lists the cases.

O'Neill also lists four tie-breaking methods. Two methods use the first or last difference in prior rounds to break a tie, and two methods use later preferences—Borda scores or most (fewest) last-place preferences. Brian Wichmann [3] proposes to examine all possible outcomes.

None of these tie-breaking methods is guaranteed to break a tie, since they can themselves result in a tie, or in the case of [3] become so computationally expensive as to be impractical. These cases (strong ties) are typically broken randomly. Some election methods, eg, the Algorithm 123 version of Meek's method [4], rely exclusively on random tie-breaking.

Objections to random tie-breaking fall into two categories. One is a worry that voters and candidates will object to election decisions being made by chance instead of by voter preference. Thus Wichmann [3]: "When a candidate has been subject to a random exclusion in an election, he/she could naturally feel aggrieved." Other objections adduce examples in which it appears intuitively preferable to break a tie based on some measure of voter preference.

All STV election methods rely on random tie-breaking (or at least tie-breaking based on some consideration other than voter preference) to break strong ties. (Ties in first-past-the-post elections are often broken randomly as well, by coin toss, drawing straws, or drawing a high card.)

2 Prior-round tie-breaking

The rationale for forwards tie-breaking (using O'Neill's terminology) appears to be that it gives greatest weight to first preferences. O'Neill [2] argues for backwards tie-breaking:

> A more important problem, is that forwards tie-breaking does not use the most relevant information to break the tie. The most relevant information to break a tie is the previous stage and not all the way back to the very first stage. By immediately looking to the first stage to break the tie, the ERS97 rules allow the tie-breaking to be influenced by candidates eliminated very early in the process and also by surpluses yet to be transferred. Instead, if we look to the previous stage to break a tie, candidates eliminated early on in the process will have no influence in breaking the tie. In addition, it allows for surpluses to be transferred which gives a more accurate picture of candidate strength.

Carrying O'Neill's argument to its logical conclusion, however, the "most relevant information" is not in any prior round, but rather in the current round—and the current round declares a tie.

Prior-round tie-breaking encourages insincere voting. Consider this election fragment, with two candidates to exclude:

5	A
4	B
1	CB

Excluding C, we have:

5 A
5 B

and must now break the tie. Prior-round tie-breaking requires that we exclude B, since A led B 5-4 in the previous round. So voter CB, believing that the first choice (C) is likely to be excluded, is encouraged to insincerely vote B (or BC) so as not to jeopardize B's chances in the event of an A–B tie.

Prior-round tie-breaking is especially troublesome in the context of Meek rules, since it violates Meek's Principle 1: If a candidate is eliminated, all ballots are treated as if that candidate had never stood. But if C had never stood, A and B would have been tied.

3 Later-preference tie-breaking

Kitchener [5] points out a problem case for random tie-breaking:

> An extreme case can arise where there is one seat and the electors are the same as the candidates; for example, if a partnership is electing a senior partner. Each candidate may put himself first, and all, except candidate A, put A second. Under most present rules, one candidate then has to be excluded at random, and it may be A. There is no way of getting over this unreasonable result without looking at later preferences. . . .

The smallest such election:

1 A
1 B A
1 C A

Prior-round tie-breaking methods are of no help in the first round, and a random choice excludes A, the consensus choice, one third of the time. Kitchener proposes to use Borda scores to break the tie; we must still randomly break a strong B-C tie, but A survives and is elected.

This case is related to a problem with STV in general, pointed out by Meek [6]. "A related point, and probably the strongest decision-theoretic argument against STV, is the fact that a candidate may be everyone's second choice but not be elected."
. . . and also related to the general problem of premature exclusion.

Kitchener concedes that there is a problem with Borda tie-breaking, as there is with any tie-breaking method that relies on later preferences.

> It is a fundamental principle of STV that later preferences should not affect the fate of earlier ones; this encourages sincere voting, but means that some arbitrary or random choice must be made to break ties, which can give unreasonable results.

Responding to the Borda tie-breaking suggestion, David Hill [7] objects: "What matters is that tactical considerations have been allowed in, where STV (in its AV version in this case) is supposed to be free of them."

This point is crucial. In any election system, the rules, including the method of breaking ties, must of course be specified in advance. When we look at the partnership election example above, we interpret the ballots as the sincere expression of the voters, and so read the ballots as favoring A. But as both Hill and Kitchener observe, once later-preference tie-breaking is introduced, we must expect insincere voting. In the face of later-preference tie-breaking, B and C, to maximize their chances of winning (after all, each is their own first choice) must resort to bullet voting (American English—one might say characteristically AmE—for plumping). The ballots would then read,

1 A
1 B
1 C

. . . and we're forced to resort to a random choice. This seems a shame, since it does appear from the presumably sincere ballots in the initial profile that both B and C prefer A to the other. The partners might be well advised to adopt a special rule forbidding each to vote for herself. In that case, we would have:

1 abstain
2 A

. . . and A wins outright.

4 Random tie-breaking

An advantage claimed by Meek [6] for STV is that "There is no incentive for a voter to vote in any way other than according to his actual preference." One of Meek's motivations for proposing a new STV method is to come closer to that ideal. Likewise Warren [8], "It is one of the precepts of preferential voting systems that a later preference should neither help nor harm an earlier preference."

Any election method relies for its properties on the implicit assumption that voters will vote sincerely, that is, that their ballots will reflect, within the limitations of the specific method, their true preferences. Without sincere votes, any election method fails to reflect the will of the electorate, on the principle of garbage in, garbage out. It is perverse to use tie-breaking methods that reintroduce incentives for voters to vote insincerely. Hill and Gazeley [9]:

> In considering this, we need to take into account, among other things, that the true aim of an election should not be solely to match seats as well as possible to votes, but to match seats to the voters' wishes. Since we do not know the wishes we must use the votes as a substitute, but that makes it essential that the votes should match the wishes as far as possible. That, in turn, makes it desirable that the voters should not be tempted to vote tactically.

5 Voter psychology

One might counter that, except in small elections, the chances of a tie are sufficiently small that a voter ought to ignore the possibility of a tie altogether and vote sincerely. This argument is problematic on two fronts. First, our methods should work with small elections as well as large ones (and the line between small and large elections is not well defined). Second, especially in a high-stakes election, the voter's estimation of the risk associated with voting sincerely is likely to be wrong.

Computer security authority Bruce Schneier, interviewed in *CSO Magazine* [10], comments:

> Why are people so lousy at estimating, evaluating and accepting risk?... Evaluating risk is one of the most basic functions of a brain and something hard-wired into every species possessing one. Our own notions of risk are based on experience, but also on emotion and intuition. The problem is that the risk analysis ability that has served our species so well over the millennia is being overtaxed by modern society. Modern science and technology create things that cannot be explained to the average person; hence, the average person cannot evaluate the risks associated with them. Modern mass communication perturbs the natural experiential process, magnifying spectacular but rare risks and minimizing common but uninteresting risks. This kind of thing isn't new—government agencies like the [US] FDA were established precisely because the average person cannot intelligently evaluate the risks of food additives and drugs—but it does have profound effects on people's security decisions. They make bad ones.

For our purposes, read *tactical voting decisions* for *security decisions*. Rational insincere voting is bad enough; insincere voting based on faulty information or poor tactics is even worse.

6 A note on weighting votes in later-preference tie-breaking

Consider this election profile (BC rules, two to be elected, quota 10):

12	AB
7	BC
9	C
2	D

A is elected, and D is excluded, leaving B and C tied with nine votes each in the third round. If we break the tie with Borda scores:

A	36	(elected)
B	24+21 =	45
C	14+27 =	41
D	6	(excluded)

C is excluded, and B is elected as the last candidate standing for the second seat.

Notice in particular that while B receives only the two transferable votes from the AB voters (a quota of 10 being retained by A, who is elected), B gets full credit for all 12 AB votes in the Borda tiebreaker.

I suggest that the AB voters, having elected A, must carry only the transferable weight of their votes in calculating the tie-breaking Borda score. Otherwise these voters *double dip*, not only electing A, but also participating disproportionately and decisively in the tie-breaking elimination of C and subsequent election of B.

If we calculate the Borda scores using the weight of transferable votes (that is, votes currently allocated to hopeful candidates), we have:

A		(elected)
B	4+21 =	23
C	14+27 =	41
D		(excluded)

Calculated with the vote weights that give rise to the tie itself, the Borda score now breaks the tie to eliminate B, and C is elected.

The same argument applies to any method that breaks ties with later preferences. Votes committed to already-elected candidates should not be counted again in breaking subsequent ties.

7 A better later-preference tie-breaking method

The chief problem with STV tie-breaking with Borda scores is that it violates the principle of later-no-harm, and it does so in an especially egregious way. Suppose that six candidates are in the running, that I have voted ABC, and that B and C are tied for elimination. The Borda scores for B and C pick up four and three points, respectively, from my ballot. If the three points that my ballot contributes to C's Borda score is the margin for C's victory over B in the Borda tiebreaker, then my later mention of C has led directly to the defeat of B, even though I prefer B to C.

Consider an alternative later-preference tiebreaker. For the sake of simplicity, I will describe it for two-way ties, and then extend it to *n*-way ties. To break a tie, compare the ballots that prefer B to C to the number of ballots that prefer C to B, weighted as described in the note above. Exclude the less-preferred candidate. Break strong ties randomly.

This method, like all later-preference methods, violates later-no-harm, but it preserves a property that I will call *later-no-direct-harm*. My ranking of ABC will not harm B's chances in a BC tie. In the case of a BC tie, my ballot will either have no effect (the margin of B over C or vice versa without my ballot is sufficient that my ballot makes no difference), or it will cause the BC tie to be broken in favor of B, my preferred candidate in the tie (B and C are strongly tied without my ballot), or my ballot will convert a one-vote C advantage (without my ballot) to a strong tie (with my ballot), giving B an even chance in a random tiebreak.

That is, my ABC ballot either has no effect on breaking a BC tie, or it benefits B.

By *later-no-direct-harm*, I mean that the fact that I have ranked the later preferences BC will not harm my favorite in the potential tie between B and C. Later-no-harm is not avoided; my ABC preference could break a tie in favor of B, and B could subsequently defeat my first preference, A, whereas A might have prevailed had C won the BC tiebreaker. Any harm to A, however, will come indirectly, in a later round—and it would be rude for me to complain that the BC tie was broken on the basis of my preference for B over C.

Generalizing to breaking an *n*-way tie for exclusion:

1. Find the first mention of any member of the tied set of candidates on each ballot, and calculate the total such mentions for each of the candidates, using the transferable weight of each ballot. Ignore ballots that do not mention at least one tied candidate.
2. If all *n* candidates are still tied, exclude one tied candidate at random; *finis*.
3. Otherwise, remove from consideration for exclusion the candidate (or a random choice from the tied set of candidates) with the highest score from step 1.
4. If only one candidate remains, exclude that candidate; *finis*.
5. Otherwise, *n* is now the remaining number of tied candidates (that is, less the reprieved candidates from step 3); continue at step 1.

If the tie is for a winner rather than an exclusion, then remove from consideration the candidate with the lowest rather than the highest score. This is simply single-winner STV (AV or IRV) with weighted ballots, and suggests an alternative to the proposed algorithm for breaking a tie for exclusion: break an n-way tie for exclusion by counting an STV election (again with weighted ballots) with n candidates and $n-1$ winners; exclude the single loser.

It's worth noting that a similar procedure based on lowest preferences (along the lines of Coombs tie-breaking) does not satisfy the principle of later-no-direct-harm. For example, if candidates X, Y and Z are tied for exclusion and I have ranked those candidates XYZ, it's possible that my preference for Y over Z is decisive in favor of Y, and that Y but not Z beats X in a head-to-head tiebreaker; thus my preference for Y over Z decides the tiebreak in favor of Y over X, contrary to my preferences.

Likewise, Condorcet ranking is equivalent to the proposed method for two-way ties, but violates later-no-direct-harm in the general *n*-way-tie case.

The proposed tie-breaking method—let's call it *weighted first preference*—differs from prior-round tie-breaking methods in that it considers the preferences of all voters (suitably weighted), and not only voters who have ranked the tied candidates first (after elections and exclusions) in a prior round.

Hill and Gazeley [9] observe, in the context of Sequential STV:

> With this new version, should it be recommended for practical use? That depends upon whether the user is willing to abandon the principle that it should be impossible for a voter to upset earlier preferences by using later preferences. Many people regard that principle as very important, but reducing the frequency of premature exclusions is important too. We know that it is impossible to devise a perfect scheme, and it is all a question of which faults are the most important to avoid.
>
> In considering this, we need to take into account, among other things, that the true aim of an election should not be solely to match seats as well as possible to votes, but to match seats to the voters' wishes. Since we do not know the wishes we must use the votes as a substitute, but that makes it essential that the votes should match the wishes as far as possible. That, in turn, makes it desirable that the voters should not be tempted to vote tactically.
>
> They would not be so tempted if they felt confident that later preferences were as likely to help earlier ones as to harm them, and if they could not predict the effect one way or the other. At present, we see no reason to doubt that these requirements are met.

The proposed method for breaking ties satisfies the same criteria: later preferences are as likely to help earlier ones as to harm them, and voters cannot predict the effect one way or the other. This is not the case for other preference-based tie-breaking methods discussed in these pages.

Whether this slight opening of the door to a violation of later-no-harm is justified by the benefit of breaking ties non-randomly (in most cases) is, in David Hill's words [7], a matter of judgment.

8 Summary

Arguments for various nonrandom tie-breaking implicitly assume sincere voters. But the introduction of those very methods undermines that crucial precondition, and without sincere voters the arguments fail.

When O'Neill argues [2] that "forwards tie-breaking does not use the most relevant information to break the tie," and that later rounds reflect better information, the logical conclusion of his argument is that the most relevant information is not in a prior round at all, but rather in the current round that gives rise to the tie. That information is, simply, that the candidates have equal support, by the means we've chosen to measure that support.

Meek [6] drives this point further home with his Principle 1: "If a candidate is eliminated, all ballots are treated *as if that candidate had never stood*." Prior-round tie-breaking typically, though not exclusively, depends on preferences for candidates who have been excluded in the tie-breaking round. To consider those preferences violates Meek's Principle 1.

Later-preference tie-breaking (eg. Borda or Coombs) encourages insincere voting by violating the later-no-harm principle.

The encouragement of insincere voting is too high a price to pay for partially excluding chance from STV election methods. We should prefer random tie-breaking in all cases.

If preferences must be considered in breaking ties, then ties should be broken on the basis of overall earliest preferences, using transferable ballot weights.

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Implementing STV by Meek's method

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1 Introduction

At the time of the original implementation of STV by Meek's method [1] we were feeling our way. Later thought has shown that, in some respects, the details can be improved while keeping the overall plan. Thus my own later implementation, as part of a suite of programs to deal with the whole election process rather than just the vote counting, and to include other versions of STV as well as the Meek version, made some changes from that original implementation. The aim of this paper is to describe those changes and the reasons for them.

My program is written in the Pascal computer language. While designed to be used under the MS-DOS operating system, it can also be easily accessed from Windows XP.

In [1] Woodall gave mathematical proof that the Meek formulation has a unique solution for any given voting pattern, and that the method necessarily converges upon that solution. Strictly speaking that proof assumes infinite mathematical precision. In this paper I refer to that proof even though my implementation has only finite precision. Provided that the degree of precision is adequate, the approximation to Woodall's proof will be close enough for practical purposes.

2 Terminology

In [1] we used the term 'weight' for the fraction, of each vote or part of a vote received, that a candidate retains. This has now become known as the candidate's 'keep value', to be in accordance with the traditional term 'transfer value'.

We also used 'excess' for the amount of vote remaining after all candidates mentioned in the voter's preferences have received their shares. The more traditional, but longer, term 'non-transferable' is now used for this.

3 Arithmetic

In [1] the numbers of votes and the keep values were declared as 'real' variables in the computer sense. These would be represented in the computer in floating-point form, which is necessarily only approximate and there is no guarantee that exactly the same approximations will be used on different computer systems. Given the robustness of the Meek method, it is highly improbable that a different candidate would ever be elected because of this, except perhaps in the case of a tie, but it is thought wise to avoid even the possibility.

It is therefore better to make sure that the numbers are so represented that, although still approximate because only a finite number of decimal places is used, the results are necessarily identical on all computers. To achieve this, floating-point methods are avoided altogether, each 'real' number being represented by a pair of integers, integer arithmetic on computers being exact.

Assuming 32-bit integers to be available, the maximum allowable integer is 2147483647 so to allow 9 decimal places for the fractional part is safe and convenient. Thus a number such as 123.456, for example, is represented as a pair of integers with 123 as the value of its integral part and 456000000 as its fractional part. Adding or subtracting such numbers is simple enough, the integral parts are added or subtracted, and the fractional parts are added or subtracted. If the resulting fractional part exceeds 999999999, then 1000000000 is subtracted from it and 1 is added to the integral part. Similarly, if the resulting fractional part is negative, then 1000000000 is added to it and 1 is subtracted from the integral part. There is no need to worry about the whole number, rather than just its fractional part, ever being negative; that never happens within the Meek method.

Multiplication and division are not so simple, and special routines are necessary to enable them to be performed with no risk of overflow.

In principle, a fixed number of significant figures might be preferable to a fixed number of decimal places, but all that really matters is that the precision should be great enough as to ensure that the use of more precision would be virtually certain not to change the outcome. The fixed 9 decimal places undoubtedly satisfies this and is convenient.

4 Quota definition

Meek's formulation [2] used the integral part of $1+T/(s+1)$, where T is the total number of active votes and s is the number of seats to be filled. He obviously intended that the initial 1 of this formula should be replaced by 1 in the last decimal place used, when not working solely in integers. An alternative approach is that of the second edition of Newland and Britton [3] in ignoring the initial 1 altogether if the calculation comes out exactly, while adding extra rules to ensure that no more than s candidates can be elected even in exceptional cases. In [1] we adopted the Newland and Britton approach (with the necessary extra rules) because the number of decimal places that would be used by a floating-point implementation was unknown.

When working solely in integers, or to only 2 decimal places as in Newland and Britton rules, there are advantages in their formulation, but those advantages are minimal where greater precision is used. For my implementation, therefore, I have included the addition of 0.000000001 to the quota, so that no extra rules are needed, while it is very hard to believe that such a tiny increment will ever cause any disadvantage.

5 Output

In [1], mainly because we were still feeling our way at that time, more output was given than now seems sensible, producing two tables at each stage of the iteration, one to say, in effect, "Where are we now?", the other to say "What are we going to do about it?" There is really no need for any output for those iterations that do not elect or exclude any candidate, so immediate output has been cut down to just showing the names of candidates elected or excluded as those events occur, with storage in computer files of enough information to allow various forms of table to be easily produced when wanted.

There is also provision for an animated form of output, showing coloured lines on the screen performing the transfers of votes. This is deliberately slowed down to make it easy to watch.

6 Ties

In the event of a tie, where a candidate must be excluded and two or more are exactly equal in last place, [1] gave only a pseudo-random choice as the solution. In my implementation, I was persuaded by ERS Technical Committee to include the traditional 'ahead at first difference' criterion as a first tie-breaker, with a pseudo-random choice only if that did not solve it.

Strictly speaking this is contrary to Meek's stated principles on which his method is based, and was somewhat against my will, but it is unreasonable to expect to win every argument, and it does no real harm, particularly as ties hardly ever occur in real elections.

The pseudo-random method used is similar except that [1] calculated random numbers only if and when required. I have found it more convenient to assign such numbers to the candidates in the first instance and thus to have them already available if wanted. However I change the assigned numbers at each stage so that, if A is randomly preferred to B on the odd stages, then B is preferred to A on the even stages.

7 Election

In [1] candidates were not deemed elected until the end of an iteration. The keep values having converged, it was then considered whether any additional candidate had achieved the quota. Further thought has shown that it is absolutely safe to elect as soon as a candidate reaches the quota during the iterations and at once to start adjusting that candidate's keep value, along with those of any others already elected. This follows from Woodall's proof, given as part of [1], that if there is a feasible vector, then there is a unique solution vector — see that proof for the definitions of those terms.

8 Convergence

Both in [1] and my present implementation, the overall plan consists of iterations within iterations, the outer iterations being the operations up to and including the exclusion of a candidate, the inner iterations being the successive adjustments of keep values.

In [1] the inner iterations were taken as having converged when each elected candidate's votes were individually close enough to the current quota. This has been simplified to saying that the sum of the current surpluses of all the elected candidates must be no greater than 0.0001. It is almost certain in any case that, if such a small sum of all surpluses is ever reached, the lowest candidates are tied and further iterations would not separate them. Because of the short-cut exclusion rule mentioned below, however, it hardly ever happens that iterations need to proceed so far.

9 Short-cut exclusion rule

During the iterations, if it is found that the lowest candidate's current votes plus the total surplus of the elected candidates is less than the current votes of the next lowest candidate, it is certain that, if the iterations were continued all the way to convergence, that lowest candidate would necessarily still be the lowest and would have to be excluded. It is therefore safe to exclude the candidate at once. The next iterations will then start from a different point than would otherwise have been the case, but it follows from Woodall's proof that the next solution vector will still be the same, so the eventual result must be unchanged.

To see that, in these circumstances, the lowest candidate cannot catch up, it should be noted that the total number of votes remains unchanged and the effect of reducing the keep values of elected candidates is to pass their surplus votes to other candidates or, possibly, to non-transferable. If all the surpluses are passed to the lowest candidate, that candidate would necessarily, given the conditions, remain the lowest. If some are passed to other candidates that is even worse for the lowest, even if some of those candidates become elected.

The only point that needs more thought is to consider what happens if some surplus becomes non-transferable, resulting in a reduction of the quota. If n votes become non-transferable, the extra surplus created thereby is $mn/(s+1)$ where m is the number of elected candidates so far, and s is the number of seats. We know that m is less than s, because otherwise all seats are filled and the whole election is over. Therefore $mn/(s+1)$ is less than n, which shows that the amount that could have gone to the lowest candidate has been reduced.

Similar arguments show that, if two or more lowest candidates have a total number of votes that, together with the current surplus, is less than the votes of the candidate next above, it is safe to exclude them all at once, provided that enough would remain to fill all seats. I have not implemented this (except in the special case where several lowest candidates have zero votes) believing it to be simpler to explain what is going on if only one at a time is excluded.

With traditional style STV it is important that rules are firmly laid down as to whether or not multiple exclusions are to be made, because it can change the result. Thus, for example, Newland and Britton rules [3] insist that multiple exclusions must be made when possible, whereas Church of England rules [4] insist on only one at a time. With Meek rules, however, it is optional, as the result is necessarily the same either way. The fact that I exclude only one at a time is not intended to suggest that there is anything wrong, within a Meek system, with multiple exclusions if others wish to use them.

10 Equality of preference

Meek [2] suggested allowing voters to express equality of preference where desired. In [1] this option was not included. My program does include the option but there are some difficulties involved, as explained in detail in [5]. I continue to hold the conclusion expressed there that "the complications may be too many to be worth it ... [but] the facility is strongly valued by a significant number of electors".

11 Constraints

Not proposed by Meek, the program also allows constraints, whereby a maximum number, or a minimum number, may be laid down for certain categories among those elected. I dislike such constraints in principle [6], but they are necessary in certain circumstances in the Church of England [4] and, if the Church ever wished to update its procedures to use Meek-style STV, it would be necessary to demonstrate that it could cope with this additional complication.

At present the main thing for which constraints may be wanted is the filling of casual vacancies, where this is done by recounting the original votes with the late occupier of the vacant seat withdrawn. The constraint that is then necessary is to disallow exclusion of any existing seat holder.

12 STV in New Zealand

Those working on the introduction of STV for certain elections in New Zealand, having decided that the Meek rules were what they wanted, had my implementation available to them, and most of its details given above, such as the 9-decimal place working, and the figure of 0.0001 for the total surplus to indicate convergence, have been incorporated into their Act of Parliament [7].

There is, of course, no objection to these details having been used, but I hope that it will not become 'folklore' that they must be used and that Meek has not been properly implemented otherwise.

13 Acknowledgements

I thank both the editor and the referee for suggestions that led to substantial improvements in this paper.

14 References

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[5] I.D. Hill. Difficulties with equality of preference. *Voting matters*, issue 13, 8-9, 2001.

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[7] New Zealand Government. Local Electoral Regulations 2001. Schedule 1A.

Editorial Postscript

I (Brian Wichmann) also have an implementation of the Meek algorithm, written in Ada 95. I have not made this generally available for several reasons: firstly, it lacks any system for preparing the data and even any adequate diagnostics on incorrect data (and hence is just a counting program); secondly the program has a number of extensions written to aid some investigations (typically reported in *Voting matters*); thirdly the program does not perform the arithmetic exactly correctly. There are a number of small differences between my Ada 95 version and the version in this paper; ties are broken differently and I will exclude several candidates together having the same number of votes provided it is safe to do so.

In 2000, I did perform a check of the Meek implementation described in this paper against the original version published in 1987 [1]. The report of this validation can now be found on the McDougall website. One interesting finding was that a test (M135) was actually a tie between two candidates for exclusion. However, both programs performed slightly different calculations in approximating the solution in such a way that neither reported a tie and the differences in the rounding resulted in a different exclusion. This is not considered a fault as, where there is really a tie, either result is acceptable.

Computerisation of STV counts

Robert Newland
(deceased)

This note, located by David Hill, appears not to have been published. It is dated February 1983. It is unclear why it was not published. Since it raises many interesting issues, it is reproduced here. Readers may wish to comment on the proposals. We hope to include their comments in a subsequent issue of *Voting matters* — Editor.

(1) It has often been suggested that STV counts should be computerised to save time/money. I have always regarded that view as unrealistic. Much of the time of any election count is taken up with preliminaries, such as envelope-slitting in postal ballots, unfolding voting papers, checking their authenticity, and, in public elections, reconciliation of numbers of papers issued.

With computerised counts, input would be time-consuming, whether by operators working in pairs to ensure accuracy, or whether by special equipment reading special voting papers presented in succession. Voting machines capable of accepting preferences seem an unlikely investment for infrequent public elections.

The time required for manual STV counts can be exaggerated, while any saving in time/money in computerised counts is doubtful or marginal. Unless there are other positive advantages to be gained from the computerisation of STV counts, it seems wrong to deprive candidates and others of the opportunity of witnessing manual counts.

(2) As Stephen Freeland said in his recent paper, COUNTING STV BY COMPUTER, "the existing 1976 procedures for counting STV elections represent a balance between technical refinement and speed of counting". Indeed, the 1976 procedures included improvements over earlier procedures both in technical refinement *and* in speed of counting. The current (1976) procedures are probably the best that can be achieved in manual counts.

Although little can be said in favour of computerisation of STV counts if the objective is merely the supposed saving of time/money, nevertheless, if computerisation is intended, the opportunity can be taken of incorporating improved counting procedures into STV which are not practicable in manual counts.

One minor improvement is obvious. It would be absurd to write a computer program restricting the calculation of quota, $V/(N+1)$, and of transfer values, to two decimal places. Using more decimal places would, on occasion, lead to a different, better, result. Since the results of manual and computer counts would then no longer be comparable, it would be sensible to make other improvements to achieve even better, different, results.

(3) In my COMPARATIVE ELECTORAL SYSTEMS where I was concerned primarily with the comparison of systems employing manual counts, I indicated briefly in section 7.8(c), Further Refinements, two areas of improvement not practicable in manual counts, viz., (i) the recommencement of counts from the beginning after exclusions, and (ii) the transfer of voting papers to next preferences even though already elected.

Stephen Freeland discusses the first of these in his paper. Following exclusion, often some voting papers are non-transferable. In consequence, towards the end of the count, candidates are elected without the quota: votes are of unequal effect.

The remedy is to re-commence the count ab initio after each exclusion. (A)

Non-transferable papers showing preferences only for excluded candidates would be discarded, and a new, lower, quota would be calculated. Eventually all candidates would be elected on attaining the same (lowest) quota: votes would be of equal effect.

Non-transferable papers showing preferences for already elected candidates would now be used to help elect those candidates: there would be fewer non-transferable papers.

Moreover, a well-known tactical voting ploy would be pre-empted. Suppose that in an election with quota 9, candidate A has 10 voting papers: 9 AB, 1 AC. The count proceeds thus:

A	10	−1	9
B	-	+0.9	0.9
C	-	+0.1	0.1

Under current rules, the elector who voted AC can maintain his support for A, but increase his support ten-fold for C by voting ZAC, where Z is not the elector's genuine first choice, but is believed to have little or no support. The count proceeds:

A	9		9
B	-		-
C	-	+1	1
Z	1	−1	-

There is an inherent danger that many such tactical voters might elect Z unintentionally.

Such tactical voting is pre-empted if the count is re-commenced after the exclusion of Z:

A	9	
B	-	
C	-	
Z	1	excluded.

New start:

A	10	−1	9
B	-	+0.9	0.9
C	-	+0.1	0.1

(4) In manual counts, it is standard practice, in transferring a consequential surplus, only to examine, and where appropriate transfer, those papers, all of one value, last received, which gave rise to the surplus. It is sometimes suggested that *all* the papers of an elected candidate should be examined and where appropriate transferred, since they all contributed to the existence of the surplus. This is an apparently attractive argument, but such a procedure, by itself, is unsound.

Suppose that in an election with quota 8, candidate A has 10 papers marked ABCD, B has 8 papers, and C has 7 papers. The count proceeds:

A	10	−2	8	
B	8		8	
C	7	+2	9	−1

It would clearly be unsound to examine and transfer any of the original 7 papers for C while the larger number of 8 papers for B have no further effect on the count. The 8 papers for B remain unexamined because B had already attained the quota, and the surplus of A was transferred, passing over B, direct to C.

The remedy is to transfer voting papers to next preferences even if already elected, thereby enabling all voting papers of an elected candidate to be examined when a consequential surplus is transferred. (B)

Electors would then be more equally represented.

Suppose in an election with quota 10, preferences for candidates A, B, C are shown on 30 voting papers: 20 AB, 10 BC. The count proceeds under existing rules thus:

A	20	−10	10
B	10		10
C	-		-
NT	-	+10	10

But if the surplus of A is transferred to the next preference B, the count proceeds:

A	20	−10	10		10
B	10	+10	20	−10	10
C	-		-	+10	10

The 30 electors with three quotas of votes have now elected three representatives.

The practical difficulty with this desirable procedure is that if part of the surplus of a candidate A is transferred to a candidate B, who is already elected, or may thereby be elected, part of B's surplus may be transferred to A, and then part of A's surplus to B, and so on indefinitely.

Brian Meek examined the problem in some detail in EQUALITY OF TREATMENT OF VOTERS AND A FEEDBACK MECHANISM FOR VOTE COUNTING, papers published in 1969 and 1970 in *Mathematiques et Sciences Humaines* (English language versions available).

Douglas Woodall also discusses the problem in COMPUTER COUNTING IN STV ELECTIONS in the current issue (Winter 1982-83 issue) of *Representation*.

To illustrate the effect of transferring votes between elected candidates, suppose that in an election with quota 12, candidate A has 18 papers, and candidate B has 10 papers. The papers for candidate A are marked: in case (i) 18 ABC (ii) 15 ABC, 3 A

(iii) 6 ABC, 12 A In each case the 10 papers for B are marked BAD.

Under existing rules, except for non-transferable differences, the result in each case is the same. The consequential surplus of B is transferred entirely to C, and D receives nothing:

A	18	−6	12		12
B	10	+6	16	−4	12
C	-		-	+4	4
D	-		-		-

If voting papers are transferred between A and B however, D receives votes in each case; fewest votes in case (i) when most papers show a (third) preference for C; most votes in case (iii) when fewest papers show a preference for C. In case (iii) the transfers soon terminate, but in the other two cases there is a theoretically unending alternation of transfers as the votes credited to A and B gradually converge to the quota. In practice, the calculations are terminated when a desired degree of accuracy is attained.

Details are appended. In case (iii) the transfers are worked out fully. In cases (i) and (ii) only the early alternations are shown [1].

It may be noted that I have followed principles which differ in some respects from both Meek and Woodall.

(5) If STV counts are to be computerised, it would be foolish not to include remedy (A), since to recommence the count after each exclusion requires only a little more computer time. If satisfactory computer programs can be devised, it would also be appropriate to include remedy (B), incorporating the procedures as illustrated.

A manual STV count is already immensely superior to any other method of election, votes being of nearly equal effect. Remedies (A) and (B) are designed to treat voting papers equally, and to ensure that votes are of exactly equal effect.

(6) This paper makes no suggestion to change the apparently obvious criterion of successively excluding candidates with fewest votes. I know of no better criterion.

The procedures described above will ensure that at most a quota of voters is not represented. Different criteria for exclusion would merely result in the non-representation of a different quota of voters.

[1]These details have been omitted here because Newland changed his mind later. When the members of ERS Technical Committee were arguing between three alternative ways of doing the job: Newland, Meek and Warren, he had another look at it and switched to supporting the Meek method as better than what he had proposed in this paper, so it is fairer to him to ignore his proposed method.

Comments on the STV Rules Proposed by British Columbia

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1 Introduction

In May 2005, the Canadian province of British Columbia conducted a referendum to decide whether to adopt the single transferable vote (STV) to elect the members of its legislative assembly. Although 57% of the electorate voted in favor of adopting STV, the measure was not adopted as a super majority of 60% was required for adoption. A Citizens' Assembly drafted a proposed set of STV rules, which will henceforth be called BC-STV. These rules are set forth in pages 17-20 of a Technical Report drafted by the Citizens' Assembly [1] and are also included as an appendix to this article.

The purpose of this article is to clarify the details of the BC-STV implementation and provide some insight into the rationale underlying the rules. Much of the information presented in this article has been gleaned from email conversations with James Gilmour, Jonathan Lundell, Brian Wichmann, and Joe Wadsworth. I have implemented the BC-STV rules in the software package called OpenSTV.[6]

2 Unitary and Inclusive Philosophies

The primary difference between different STV rules is in how surplus votes are transfered. The different methods for transferring surplus votes can be grouped into two different categories, what I call the unitary and inclusive philosophies of transferring surplus votes.

Before describing these two categories, a distinction must be made between an initial surplus of votes and a secondary surplus of votes. An initial surplus arises when a candidate has more than a quota of first choices, i.e., a surplus after the first stage of counting. A secondary surplus occurs when a candidate does not have an initial surplus but later goes over the quota after receiving votes from other elected or excluded candidates.

Consider an election where the quota is 100. Suppose candidate A has 140 votes after the first stage and thus an initial surplus of 40 votes. Suppose candidate B has 90 votes after the first stage and 110 votes after the second stage, after receiving 20 votes of A's surplus. At the second stage, candidate B has a secondary surplus of 10 votes.

Under the unitary philosophy of surplus transfers, only whole votes are transferred. With candidate A, 40 of her votes transfered at full value, while the other votes remain with A at full value. Similarly with candidate B, 10 votes are transferred at full value. A common practice is to take these 10 votes from the 20 that B received during the second stage.

Under the inclusive philosophy of surplus transfers, a portion of each of a candidate's votes is transferred. With candidate A, each of her votes will be transferred to their second choices at a transfer value of 40/140. The total value of the votes transferred is 40. The transfer is inclusive because each of A's votes takes part. With candidate B, the idea is the same, except that one could (and should) account for the fact that some of the votes that B received in the second stage could already have a value of less than one.[1]

Some STV rules can be clearly classified as exemplifying one of these two philosophies, while others employ a hybrid of these two philosophies. I will now consider several STV rules in addition to BC-STV: Cambridge STV (Massachusetts, USA), Dail STV (Ireland), Northern Ireland STV, Malta STV, Tasmania STV (Australia), Australian Capital Territory or ACT STV, and Meek STV (New Zealand).

Cambridge and Dail STV are examples of the unitary philosophy. With Cambridge STV, the votes selected for transfer are chosen at random. With Dail

[1] Under a method used in Australia, all votes are treated the same even if some of them were received at less than full value. In contrast, BC-STV appropriately weights the votes received at less than full value [4].

STV, the votes selected for transfer are chosen in a manner that proportionally represents the following choices on the ballots but does not seek to proportionally represent later choices on the ballots. Both of these methods are ballot order dependent – the outcome is not guaranteed to be the same if the votes are recounted with the ballots in a different order – a fact that some people find highly undesirable. David Robinson has proposed an interesting unitary STV rule that is ballot order independent (or nearly so).[5]

Northern Ireland, Malta, Tasmania, and ACT STV employ a hybrid of the two philosophies and each is an example of the long-established Gregory method of STV counting. The idea underlying these methods appears to be to exemplify the unitary philosophy to the extent possible but to also ensure that the rules are ballot order independent. With these rules, the method of surplus transfer is different for an initial surplus and a secondary surplus. An initial surplus is transfered according to the inclusive philosophy. While not impossible, it is difficult to transfer an initial surplus in a unitary fashion that is also ballot order independent. The method for transferring secondary surpluses is still hybrid, but much closer to being unitary. For secondary surpluses, only the last batch of received votes is considered. This last batch could arrive from a previous transfer of surplus votes or from the exclusion of a candidate. For example, consider candidate B from above. The last batch of votes has a total value of 20 and the surplus is 10. Each of the votes in this last batch is transfered to the next candidate on the ballot with a transfer value of 10/20.[2] The transfer is thus inclusive among the last batch but much more unitary than a completely inclusive transfer.

BC-STV and Meek STV are examples of the inclusive philosophy. For both initial and secondary surpluses, a portion of each vote is transfered to its next choice. The primary difference between BC-STV and Meek STV is the following: with BC-STV votes are transfered only to unexcluded candidates with less than a quota while with Meek STV votes are transfered to all unexcluded candidates. Meek STV is clearly a better method than BC-STV, but Meek STV requires a computer program to count the votes while BC-STV can be counted by hand.

3 Provenance of the BC-STV Rules

Over the years, rules similar to the BC-STV rules have been considered in numerous places. The Proportional Representation Society of Australia urged Australia to replace an existing STV method with a method similar to BC-STV[4]; Douglas Amy's book includes a method similar to BC-STV[2]; and the model statute on the website of the Center for Voting and Democracy (a United States organization) is similar to BC-STV. Rules similar to BC-STV rules have likely been independently derived numerous times, and I present two possible derivations.

Among people familiar with the different STV rules, Meek STV is generally regarded as the "best" set of rules for STV. The greatest difficulty with Meek STV is that it cannot be counted by hand. The most obvious simplification to Meek STV to make it hand countable is to not allow vote transfers to elected candidates. With this modification, Meek STV becomes very similar to BC-STV.

The Gregory method is another well-known method for counting STV elections, which has been used for more than a century. As described above, for secondary surpluses with the Gregory method only the last received batch of votes is considered. Some may regard this as unfair since the last batch of votes may be quite different from previous batches of votes.[4] Intuitively, it seems desirable to change the transfer of secondary surpluses so that all of the candidate's votes are considered and not just the last batch. With this modification, the Gregory method becomes very similar to BC-STV.

Farrell and McAllister used the term "weighted inclusive Gregory method" to refer to rules like the BC-STV rules, and the drafters of the BC-STV rules also used this terminology.[3] While this terminology is perhaps descriptively correct, I find it misleading in that it overstates the relationship between the BC-STV and Gregory methods. Using only the last batch of votes in transferring secondary surpluses is a distinctive feature of the Gregory method. Without last-batch transfers, the similarity with the Gregory method is mostly lost. The BC-STV rules could also be described as "hand-countable Meek" or "Meek without transfers to elected candidates." A more accurate description of the BC-STV rules is simply "inclusive STV."

[2] For the sake of simplicity, I am assuming that each of the votes has a valid next choice.

[3] Farrell and McAllister appear to have coined this terminology.[4]

4 Corrections to the BC-STV Rules

Several people have pointed out ambiguities and errors in the BC-STV rules. I believe that they are all straightforward to address, and I will briefly do so.

First, the BC-STV rules necessarily entail computations with fractions. The rules do not say if these computations are to be performed exactly or through precisely-specified rounding techniques. While this is an important detail, it is one that can easily be resolved. In my implementation of the BC-STV rules, I round to eight decimal places to approximate an exact solution [6].

Second, there is one clear error in the rules, but this error has a simple and obvious fix. In the appendix, the underlined text has been added to fix this error.

Third, in two places, the rules need to be generalized. First, in part 8 of "Counting procedure rules," the rules acknowledge that it is possible for one candidate to be elected with less than a quota of votes. In reality, it is possible that multiple candidates could be elected with less than a quota of votes. One possible correction would be to delete the second sentence in part 8 and replace it with the following: "When the total number of elected and remaining candidates is equal to the number of members to be elected, then all the remaining candidates are elected even if they have less than a quota of votes." Second, part 3 of "Provisions for tied votes" explains how a tie between two candidates is to be broken, and this needs to be generalized to break a tie among three or more candidates.

Fourth, the BC-STV rules do not precisely specify how to transfer surplus votes. Suppose that two candidates have a surplus on the first count, that after transferring the largest first-count surplus a third candidate is elected, that after transferring the second first-count surplus a fourth candidate is elected, and that the fourth winner has a larger surplus than the third. The rules do not indicate which of the two remaining surpluses is to be transferred first. One could choose the largest surplus (that of the fourth winner) or the earliest surplus (that of the the third winner). In accordance with common practice, I chose to always transfer the largest surplus.

5 Advantages and Disadvantages of the BC-STV Rules

I see four advantages of the BC-STV rules: (1) the rules are very simple, (2) votes can be counted by hand, (3) the rules employ the inclusive philosophy, and (4) the rules avoid the unfairness of transferring only the last batch for secondary surpluses. Only the fourth advantage requires more explanation. Consider candidate B, described above. He received 90 first place votes and later received 20 votes that had been transfered as part of candidate A's surplus. It is quite possible that the latter 20 papers represent quite different views than the first 90 papers, yet only the latter 20 papers have further effect. This hardly seems fair to the 90 voters who ranked B first. Farrell and McAllister cite such a dispute arising from an Australian election where the Gregory method was used.[4]

I see one main disadvantage of BC-STV rules. The outcome of the count is not continuous in the sense that changing only one vote can dramatically affect the outcome. For example, consider the following two sets of ballots for electing three candidates:

Set 1	Set 2
4501 ABC	4500 ABC
2499 BD	2500 BD
1200 C	1200 C
1800 D	1800 D

The quota is 2500, and the two sets of ballots differ by just one vote. I now count these ballots using BC-STV rules.

With Set 1, candidate A is elected and has a surplus of 2001 votes. Since candidate B is second on all of these ballots and candidate B has less than a quota, candidate B receives all of these 2001 votes. Now B has a total of 4500 votes and a surplus of 2000 votes. For these 4500 votes, 2001 rank C next (the ballots transfered from A) and 2499 rank D next. Thus,

$$\frac{2000}{4500} \times 2001 = 889.3$$

ballots of the surplus go to candidate C, and

$$\frac{2000}{4500} \times 2499 = 1110.7$$

ballots of the surplus go to candidate D. Candidate D is elected with 2910.7 votes and candidate C loses with 2089.3 votes.

Now consider Set 2. Candidate A is elected and has a surplus of 2000 votes. Since candidate B is also elected, A's surplus of 2000 votes goes directly to candidate C. Thus, candidate C wins with 3200 votes and candidate D loses with 1800 votes. Although there is only one different ballot in these two sets, the outcome differs by more than 1000 votes.

In comparison, with all of the other STV counting methods mentioned in this paper, there is no such discontinuity with these two sets of ballots. For example, let us count the two sets of ballots with the Gregory method. With Set 1, A's surplus of 2001 votes goes to candidate B. B now has a surplus of 2000 votes. Only votes from the last batch are further transfered, so 2000 votes are now transfered to candidate C who wins with 3200 votes. With Set 2, A's surplus of 2000 votes goes directly to candidate C who again wins with 3200 votes. Here, the change in one ballot produced a similarly small change in the outcome.

6 Conclusions

In considering the relative merits of BC-STV and Gregory methods, there is no clear winner. With the Gregory method, one can argue that it is unfair to use only the last batch of received votes in transferring secondary surpluses. With BC-STV, the outcome is not necessarily continuous with small changes in the ballots. The clear solution to this conundrum is to use Meek STV, assuming that computer counts are possible, which does not suffer from either of these disadvantages.

7 References

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[2] Douglas J. Amy, *Behind the Ballot Box* (2000).

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[4] David M. Farrell and Ian McAllister, *The 1983 Change in Surplus Vote Transfer Procedures for the Australian Senate and its Consequences for the Single Transferable Vote*, Australian Journal of Political Science 38:479-491 (2003).

[5] Dave Robinson. Courageous STV. Description available via McDougall web site.

[6] OpenSTV – Software for computing the Single Transferable Vote. Available at http://stv.sourceforge.net.

Appendix: The Recommended BC-STV Electoral System

[Author's note: James Gilmour pointed out a small but important error in the counting rules. This has been fixed with the addition of the underlined text. I have also corrected the incorrect numbering in the section "Provisions for tied votes."]

This section describes the recommended BC-STV system. It provides guidelines to be used in drafting a new election act and in making changes to the current Electoral Boundaries Commission Act.

In addition to choosing an electoral system that incorporates its basic values, the Citizens' Assembly on Electoral Reform wanted a system that is open to public scrutiny and whose results can be reviewed and validated. Consequently, BC-STV is designed to use paper ballots which are available for recount, if required.

General

1. BC-STV is a system of proportional representation by the single transferable vote (STV) method.

2. The members of the Legislative Assembly of British Columbia will be elected from multi-member electoral districts.

3. The number of members in each district will vary from two (2) to seven (7). Given that achieving proportional electoral outcomes is a primary reason for recommending BC-STV, using larger rather than smaller numbers of members per district should always be preferred when drawing district boundaries. While some very sparsely populated areas may require districts with as few as two members, the principle of proportionality dictates that, in the most densely populated urban areas, districts should be created at the upper end of the range.

4. The "Droop quota" will be the formula for calculating the number of votes required by a candidate for election in a district. The quota formula is:

$$\left(\frac{\begin{array}{c}\text{total number of valid}\\ \text{ballots cast in the district}\end{array}}{1 + \begin{array}{c}\text{number of members}\\ \text{to be elected}\end{array}} \right) + 1$$

Fractions are ignored.

5. The method of distributing surplus votes from those candidates with more than the minimum number of votes needed to be elected will be the “Weighted Inclusive Gregory method” (see below, as well as Appendix: Glossary [Author's note: the Glossary is not included.]).

The ballot paper

1. The ballot paper will display the names of all the candidates contesting seats for a district. The names will be grouped according to party affiliation.

2. Candidates who do not indicate a party affiliation, and candidates who do not indicate that they are running as an independent, will be grouped together.

3. Parties with only one candidate, and each candidate running as an independent, will each have their own group.

4. Groupings with more than one candidate in a district will have the rank order of the candidates' names rotated at random so that each candidate has an equal chance of being placed in every position within the grouping.

5. The rank order of groupings appearing on the ballot will be rotated at random so that each grouping has an equal chance of being placed in every position on the ballot paper.

6. The ballot paper will not provide the option of voting for all the candidates of one group by marking a party box (this is the so called “above the line” option used in some Australian elections).

Valid ballots

1. Voters will indicate their preference for the candidates listed on the ballot paper by putting the numbers 1, 2, 3, 4, etc. next to candidates' names.

2. A ballot paper must include a first preference for the ballot to be counted as a valid ballot. The number of subsequent preferences marked on the ballot is at the discretion of the voter.

3. In the case of a ballot paper with gaps or repetitions in the sequence of numbers beyond a first preference, the preferences are valid up to the break in the sequence.

4. If a voter puts a mark next to only one candidate's name, and that mark makes the voter's intention clear, the mark will be accepted as the expression of a single preference for that candidate and the ballot will be counted as a valid ballot.

Counting procedure rules

1. Once the total number of valid ballots is established in each multi-member district, the minimum number of votes required for a candidate to be elected is calculated using the Droop quota formula.

2. All ballots are counted and each ballot is allocated as a vote to the candidate against whose name a first preference (i.e., “1”) is shown on the ballot.

3. If a candidate(s) on the first count has a number of first preference votes exactly equal to the minimum number of votes needed to be elected, then that candidate(s) is declared elected and the counted ballot papers indicating that candidate(s) as a first preference are put aside and the other preferences recorded on the ballots are not examined.

4. If a candidate on the first count gains more than the minimum number of votes needed to be elected, the candidate is declared elected, and the number of votes in excess of the number of votes needed to be elected (the surplus) is recorded. All of the elected candidate's ballots are then re-examined and assigned to candidates not yet elected according to the second preferences marked on the ballots of those who gave a first preference vote to the elected candidate. These votes are allocated according to a “transfer value.” The formula for the transfer value is:

$$\frac{\text{surplus votes cast for the elected candidate}}{\text{total number of votes received by the elected candidate}}$$

5. If two or more candidates on the first count gain more than the minimum number of votes needed to be elected, all of those candidates are declared elected. The ballots of the candidate with the largest number of first preference votes will be re-examined first and assigned (at the transfer value) to candidates not

yet elected according to the second preferences marked on that candidate's ballots, or the next available preference, if the second preference candidate has already been elected. The ballots of the other elected candidate(s) will then be re-examined and their surpluses distributed in order according to the number of first preference votes each candidate received.

6. If a candidate reaches more than the minimum number of votes needed to be elected as the consequence of a transfer of votes from an elected <u>or excluded</u> candidate, the number of votes in excess of the number of votes needed to be elected (the surplus) will be transferred to other candidates. This transfer will be to the next available preference shown on all of this candidate's ballots. These ballots now include 1) the candidate's first preference ballots, and 2) the parcel(s) of ballots transferred to the candidate from one or more elected <u>or excluded</u> candidates. The transfer value for the candidate's first preference ballots is:

$$\frac{\text{surplus votes cast for the elected candidate}}{\text{total number of votes received by the elected candidate}}$$

The transfer value for each parcel of ballots transferred to the candidate from one or more elected <u>or excluded</u> candidates is:

$$\left(\frac{\text{surplus votes cast for the candidate}}{\text{total number of votes received by the candidate}}\right) \times \left(\text{the transfer value of the parcel of ballots received by the candidate}\right)$$

7. If no candidate has a number of votes equal to or greater than the minimum number of votes needed to be elected, the candidate with the smallest number of votes is excluded. All of that candidate's ballots–both first preference ballots and any parcel or parcels of ballots transferred from other candidates–are transferred to candidates who have not been elected or excluded according to the next available preference shown on the excluded candidate's ballots. The excluded candidate's first preference ballots are transferred to the second (or next available) preferences at full value. Ballots received from previously-elected (or excluded) candidates are transferred at the transfer value at which the ballots were received.

8. Counting continues in the described sequence: the surplus of elected candidates is assigned until no more candidates are elected, then the ballots of excluded candidates are assigned until another candidate is elected. When all but one of the candidates to be elected from the district have been elected, and only two candidates remain in the count, the candidate with the most votes is declared elected, even though the candidate may not have reached the minimum number of votes (the quota) needed to be elected.

9. If, during the transfer of preferences, a ballot paper does not indicate an available preference, the ballot is put aside as "exhausted." This can occur because:

- the voter only indicated one, or a small number of preferences;
- all the preferred candidates have already been elected or excluded; or
- there are gaps or repetitions on the ballot in the sequence of numbering preferences.

Provisions for tied votes

1. Where two or more candidates have the same number of first preference votes at the end of the first count, and this number is more than the minimum number of votes necessary for election, then the candidate whose surplus is distributed first will be decided by lot.

2. Where no candidate has a number of first preference votes equal to or greater than the number of votes necessary for election at the end of the first count, and two or more candidates have the same number of first preference votes, this number being the smallest number of first preference votes gained by any candidate, then the candidate who is excluded first will be decided by lot.

3. If, at any stage of the count other than during the first count, two candidates have the same number of votes, the candidate who is declared elected first, or who is not excluded will be:

a) the candidate with the larger number of votes in the previous or immediately next preceding count where there is a difference in the votes between the two candidates; or

b) the candidate whose name is drawn by lot, where there is no difference in the number of votes between the candidates at any preceding count.

By-elections

The single transferable vote method (preferential voting) is to be used for by-elections where a candidate is to be elected to fill a single casual vacancy in a district. The BC-STV method is to be used where candidates are to be elected to fill two or more casual vacancies in a district.

Developing STV Rules for manual counting to give effect to the Weighted Inclusive Gregory Method of transferring surpluses, with candidates' votes recorded as integer values

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The Local Governance (Scotland) Act 2004 [1] makes provision for councillors in Scotland to be elected by the single transferable vote (STV) from wards returning either three or four councillors. The first elections under these new provisions will be held in May 2007. The Act does not specify any STV counting rules, but requires Scottish Ministers to make such rules by order.

1 Proposal to use WIGM

When the Local Governance (Scotland) Bill [2] was introduced in the Scottish Parliament it included most (but not all) of the STV counting rules used for District Council elections in Northern Ireland [3]. Among those included were the provisions for the transfer of surplus votes by the Gregory Method, applied only to the 'last parcel' of ballot papers for a consequential surplus [4]. During the Stage 1 consideration of the Bill by the Local Government and Transport Committee of the Scottish Parliament, several MSPs questioned the use of the Gregory Method and suggested that the 'last parcel' provision treated some voters unfairly (eg see [5] at col 380). The Committee also discussed the possibilities of using electronic counting which was attractive because the elections for the Scottish Parliament (by a regional version of the Additional Member System) would be held on the same day.

In their Report [6] on the Stage 1 consideration of the Bill, the Committee said, in relation to technical issues surrounding the counting of votes:

> "The Committee: Concludes that the method set out in the Bill is the most appropriate one for local government elections in Scotland at this time, given the currently available counting technology;
> Believes that its preferred alternative, the 'weighted inclusive Gregory method', is, theoretically, the most effective counting method as it ensures that the preferences expressed by all voters are counted; but notes manual counts using this system would be unrealistically time consuming; and
> Recommends that the 'weighted inclusive Gregory method' be introduced to replace the system set out in the Bill when electronic counting becomes available."

Several technical amendments to the STV counting rules were discussed during the Stage 2 debate on the Bill, but the Gregory Method and the 'last parcel' provision were retained for the transfer of surpluses. However, at the Stage 3 debate on the Bill, on the floor of the Parliament Chamber, the Scottish Executive Minister tabled amendments that had the effect of removing all the detailed STV counting rules, and these amendments were passed [7,8].

The second Newsletter of the 2007 Elections Steering Group [9] announced: "Scottish Executive Ministers have agreed that work should go forward on the possibility of introducing e-counting for the 2007 local government elections." The invitation to tender for the provision of e-counting facilities was issued in August 2005 [10]. (The award of this contract to DRS Data Services Ltd was announced in February 2006 [11].)

The tender document issued to interested contractors [12] specified that the STV counting rules were to be based on the "Weighted Inclusive Gregory Method" (WIGM) of transferring surpluses. The tender document included a description of STV rules incorporating WIGM, based on the incomplete and defective description given in the Technical Report of the British Columbia Citizens' Assembly on Electoral Reform [13].

2 Definition of WIGM

The term "Weighted Inclusive Gregory Method" appears to have been coined by Farrell and McAllister [14], where they give the following description of the procedure for determining the transfer value for a candidate's surplus votes:

"For those votes that the candidate has received at full value, TV = *s*/*v*, where *v* is the candidate's total vote. For those votes that the candidate has received from another candidate's surplus, TV = $(s/v)\beta$, where β is the TV that was applied in the transfer of the surplus votes to the previous candidate."

(The definitions of "TV" and "*s*" were given earlier in the paper: "TV" = transfer value; "*s*" = candidate's surplus.)

The Weighted Inclusive Gregory Method has not yet been implemented anywhere in the world and so there is no working legislative language available. However, a legislative description of WIGM was included in the Electoral Legislation Amendment Bill 2003 presented to the Legislative Assembly of the Parliament of Western Australia [15]:

> "Unless all the vacancies have been filled, the surplus votes (if any) of any candidate elected under clause 4, or elected subsequently under this clause, shall be transferred to the continuing candidates as follows —
> (a) the number of surplus votes of the elected candidate shall be divided by the number of votes received by him and the resulting fraction shall be the surplus fraction;
> (b) in relation to any particular ballot papers for surplus votes of the elected candidate, the surplus fraction shall be multiplied by the transfer value at which those ballot papers were transferred to the elected candidate, or by one if they expressed first preference votes for the elected candidate, and the product shall be the continued transfer value of those particular ballot papers;
> (c) the total number of ballot papers for surplus votes of the elected candidate that each
> (i) express the next available preference for a particular continuing candidate; and
> (ii) have a particular continued transfer value,
>
> shall be multiplied by that transfer value, the number so obtained (disregarding any fraction) shall be added to the number of votes of the continuing candidate and all those ballot papers shall be transferred to the continuing candidate,
>
> and if on the completion of the transfer of the surplus votes of the elected candidate to a particular continuing candidate that candidate has received a number of votes equal to or greater than the quota, that candidate shall be elected."

(The Bill received a first and second reading, but was withdrawn in November 2003 for reasons not related to the proposed change to the STV counting rules.)

This legislative description introduces the term "surplus fraction" for Farrell and McAllister's calculated "*s/v*", which is then applied to each parcel of ballot papers with a different current value, Farrell and McAllister's "β", ie the "transfer value" at which those ballot papers were received by the candidate with the current surplus. The Western Australian Bill used the term "continued transfer value" for the value at which the ballot papers would be transferred from the candidate with the current surplus. In UK STV rules we prefer the term "current value" for whatever value a ballot paper may have when a calculation is made and "transfer value" for the value at which the ballot paper will be transferred to the next available preference.

3 Putting WIGM into UK legislation

The terminology of the Western Australia Bill is helpful in that it distinguishes (and names) the two steps in the process of calculating correctly weighted transfer values when a candidate has a surplus and all of that candidate's ballot papers are transferred. This legislative language does not, however, provide 'ballot-paper-by-ballot-paper' handling instructions of the kind usually found in UK rules for the conduct of STV counts (eg [3]). It was with this in mind that I prepared the detailed rules in the document that has been deposited on the McDougall website [16]. That document has been through several drafts and I am grateful to Brian

Wichmann, David Hill, John Curtice and the anonymous Referee of this paper for corrections and helpful comments. It has been made widely available to those who are involved in the preparation of the secondary legislation that will be required for the 2007 elections.

Although the intent was that e-counting would be used for the 2007 elections, and the Local Government and Transport Committee of the Scottish Parliament recommended the use of WIGM only if e-counting were to be introduced, there was nothing to indicate that manual counting by WIGM rules should not be undertaken if this were demanded or necessary. A manual count by WIGM rules would take longer than a manual count by (classical) Gregory Method rules because more ballot papers have to be sorted and counted more times, but it would not be impracticable for a public election as an exceptional requirement. It seemed appropriate, therefore, to devise first the WIGM rules for a manual count. Once these had been determined as coherent and unambiguous, it would be a smaller task to adapt the manual rules for e-counting. As explained in the preamble [16], the rules were written to fit into a more comprehensive legislative document and follow the conventions of UK secondary legislation (eg [3]).

4 Consequential issues

The essential description of WIGM is quite simple, but its adoption raises several issues that affect other aspects of the STV counting rules.

Because surpluses are to be spread across all the ballot papers then held by the candidate from whom the surplus is being transferred, each ballot paper will, in most cases, carry forward a smaller vote value. In the Northern Ireland rules [3], transfer values are calculated to two decimal places and any remainder ignored. The votes transferred to successive preferences are similarly calculated to two decimal places and the totals of votes credited to candidates are shown to two decimal places on the result sheet. If the WIGM calculations were similarly truncated at two decimal places, substantial numbers of ballot papers would quickly have no recordable value. The precision of calculation must, therefore, be greater when WIGM rules are applied. To ensure reproducibility no matter how the count is undertaken, it is necessary also to specify the precision of each step of each calculation. As explained in the preamble to the rules, the precision was set at seven decimal places on pragmatic and practical grounds. (The information about the precision of the transfer value calculations in the STV elections to the Australian Federal Senate taken from the AEC website and given in an earlier paper [17] was incorrect [18]. For those STV elections the precision is not limited at all [19], but this has no consequences because of the 'value averaging' method that is used in those rules to calculate transfer values *de novo* for each surplus.)

As noted in the document deposited on the McDougall website, these rules do not make any provision to overcome the anomaly that arises with WIGM when votes are not transferred to already elected candidates. This will be the subject of a separate paper.

5 Integer vote values

It is a feature of the Australian STV rules that use an 'inclusive' method of transferring surplus votes that only whole numbers of votes are credited to candidates when transfers are made [20]. The Commonwealth Electoral Act 1918 prescribes the flawed "Inclusive Gregory Method" and not the Weighted Inclusive Gregory Method, but the Western Australian WIGM Bill [15] included the same provision (see sub-paragraph (c) in the text quoted above). This approach has much to commend it, as it will simplify the result sheet and so aid public comprehension. (It would probably be of benefit if it were adopted more widely for STV counting rules.) Apart from its presentational advantages, this approach avoids acceptability problems that could arise in WIGM elections from candidates being separated by minute fractions of votes. With integer vote totals, candidates will either be separated by at least one vote or have the same number of votes.

Of course, the fractional parts of the vote totals that are not transferred to the candidates cannot be ignored; they must be accounted for properly. These fractional parts are shown separately on the Australian integer result sheets as 'Lost by fraction'. I prefer the term 'Vote fraction not transferred' because it is more correctly descriptive and does not convey the idea that any votes can be "lost".

This truncation to an integer value is applied only to the total value of all the parcels and sub-parcels being transferred to any one candidate; it is not applied to the values of the individual parcels and sub-parcels before the candidate's transferable total is calculated. There is only one truncation for each candidate to whom votes are transferred in any one stage. That way the 'Vote fraction not transferred' is minimised.

Note that when a multiple exclusion occurs, the 'Vote fraction not transferred' can be negative. This happens when the sum of the values of the ballot papers, **including all the fractional parts**, held by the excluded candidates exceeds the sum of the integer votes credited to the excluded candidates. Thus previously 'non transferred' votes can be brought back into play. This is another reason for preferring a term other than "lost".

6 Non-transferable votes

When an 'inclusive' transfer of a surplus is effected, the transfer values are calculated taking into account **all** the votes then credited to the elected candidate and **all** the ballot papers are transferred. Ballot papers with no 'next available preference' are set aside as 'non transferable' and take with them as 'non-transferable' the proportionate share of the surplus. This approach is wholly consistent with the 'inclusive' concept that is given effect by the requirement to examine and transfer all parcels of ballot papers held by the candidate with the surplus.

7 Deferred surpluses

It could be argued that the 'inclusive approach' that underlies WIGM would require the transfer of **all** surpluses, ie that there should be no provision to defer the transfer of any surplus, no matter how small. However, if there is to be any possibility of manual counting, it would be best to retain the 'deferred surplus' provision so that the handling of large numbers of ballot papers of extremely small values could be avoided except when the votes on those ballot papers would affect what has to happen next.

8 Sub-stages during exclusions

STV counting rules that use the Gregory Method of transferring surpluses usually provide for sub-stages during exclusions, in which the transfer of a parcel of ballot papers of the same value constitutes a sub-stage. The transfer of first preference ballot papers before the transfer of other ballot papers of value 1 vote also constitutes a separate sub-stage in the Northern Ireland rules [3]. If any candidate attains the quota at the end of a sub-stage, that candidate is 'deemed elected' and no further transfers are made to that candidate. This is consistent with the 'exclusive approach' to STV that seeks to keep the voters in discrete, 'exclusive' groups so far as possible. Although it is clearly not directly related to WIGM, the sub-stage approach to handling exclusions seems incompatible with the 'inclusive' approach that underlies WIGM. I have, therefore, made no provision for sub-stages during exclusions.

9 Publication of results

I have taken the opportunity to specify fully what must be published once an STV count has been completed. This rectifies a deficiency in the Northern Ireland rules [3].

10 Casual vacancies

The suggested rules do not include any provisions relating to the filling of casual vacancies because policy decisions on casual vacancies are required before the relevant election rules can be devised. Should it be decided that a by-election must be held when a single vacancy occurs, I would commend the use of the special purpose STV rules published by the Electoral Reform Society [21]. I codified these rules in their present form in 1978, working under the guidance of Frank Britton and Robert Newland.

11 'Inclusive' and 'exclusive' representation

A discussion of the 'inclusive' and 'exclusive' approaches to proportional representation and STV counting rules will be the subject of a separate paper.

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