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Voting matters

for the technical issues of STV

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Editorial

In recent years *Representation* has tended to shy away from articles of a technical nature and restrict itself to the non-technical. While there may be some advantages in this course of action, it has left those with technical things to say on voting systems without a suitable outlet for their ideas and arguments. The members of the Electoral Reform Society's Technical Committee, and others, have been unhappy about this. Hence this new venture, which it is intended to circulate to those Society members who request it.

In this first issue, we reprint some earlier articles that deserve a wider circulation. Those by B L Meek, originally published over 20 years ago in French, have been available in English only as a typed and duplicated version containing many errors. These are classic papers which have led to much discussion in recent years. Whether one agrees with Meek's conclusions or not, it cannot be denied that those who argue about his method need to know what he did actually say.

The article by D R Woodall was also printed with an error originally and this reprint includes the necessary correction. Although Woodall's method is basically the same as Meek's, it was entirely independently derived and it is interesting to see his different approach.

The article by C H E Warren has not been published before. It is a slightly rewritten version of a paper first submitted in 1983, but not then accepted. Warren's method is similar in spirit to the other two, but differs in the way it performs. Each of the two counting methods has an advantage over the other in some circumstances so, although a majority of the ERS Technical Committee prefer the Meek/ Woodall formulation, the Warren alternative is worth bearing in mind. The final paper discusses the differences.

I D Hill

Chairman, ERS Technical Committee

A New Approach to the Single Transferable Vote

Paper I: Equality of Treatment of voters and a feedback mechanism for vote counting.

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With some differences in presentation, the paper was originally published in French in Mathématiques et Sciences Humaines, No 25, pp13-23, 1969.

Abstract

It is shown that none of the counting methods so far used in single transferable vote elections satisfies the criterion that all votes should, as far as possible, be taken equally into account. A feedback method of counting is described which does satisfy this criterion within the general limitations imposed by the STV system. This counting method, though very laborious for manual counting, would be feasible in automated elections.

1. Introduction

While the preferential voting system known as the Single Transferable Vote (STV)[1] has been criticised on various grounds, the following advantages claimed for it do not seem to have been seriously challenged:

(A) The number of 'wasted' votes in an election (i.e, which do not contribute to the election of any candidate) is kept to a minimum.

(B) As far as possible the opinions of each voter are taken equally into account.

(C) There is no incentive for a voter to vote in any way other than according to his actual preference.

It is the purpose of this and a subsequent paper to consider (A), (B) and (C) from a decision-theoretic viewpoint, within a single constituency; it will be shown that (A), (B) and (C) in fact do not hold in present STV procedures, but may be made to hold, within certain overall limitations, by appropriate modification of the counting method.

2. The wasted vote

An essential feature of an STV election is the 'quota'. If there are s vacancies to be filled, the quota q is the *smallest* number such that, if s candidates have q votes each, it is not possible for an $(s+1)$th candidate to have as many as q votes. Thus if the total votes are T, then $T-sq < q$, but $T-s(q-1) \geq q-1$, whence $q = [1+T/(s+1)]$, where the square brackets denote 'integer part of'.

Candidates with more than q votes are elected, and have their surplus votes transferred according to the next preferences marked; if there are no such candidates, the bottom candidate is eliminated and all his votes so transferred. Repeated application of these rules ensures that at the end of the count s candidates have at least q votes each and so the total wasted vote w satisfies $w < T/(s+1)$.

Given s and T, it is clear from the definition of q that condition (A) is satisfied provided the next preference at each transfer is always given. It is possible for the above inequality, and hence condition (A), to be violated, if w is increased by the addition of votes which are non-transferable because no next preference has been indicated. In this paper we shall assume that this does not occur; it will be shown in a second paper that it is possible still to satisfy (A) in such cases by modifying the definition of q.

3. Equality of treatment

The discussion of condition (A) shows that, in general, there will be some wasted votes, except in the trivial cases when $s \geq T$. It is therefore not possible under STV to guarantee that *all* votes will be taken equally into account (e.g. votes with first preferences for runner-up candidates), although all are taken indirectly into account when calculating the quota.[2]

Within this obvious limitation, attempts have been made to eliminate possible sources of inequity of treatment by various modifications of the counting rules. Such sources include:

(i) the choice of which votes to transfer from the total for a candidate who has exceeded the quota

(ii) errors introduced by taking whole-number approximations to fractions of totals for transfer − particularly in elections with small total vote

(iii) calculation of the proportion for transfer from an elected candidate on the basis of the last batch of votes transferred to him, and not on his total vote.

The common way of overcoming difficulties (i) and (ii) is to use the variant of STV known as the Senate Rules. Each vote is divided into K parts (usually K = 100 or 1000) and each part treated as a separate vote (of value $1/K$) with identical preference listings.

Difficulty (i) is overcome by transferring the appropriate proportion of each divided vote, while the method clearly reduces the errors involved in (ii) by the factor $1/K$. If $K=10^n$ this is simply working to n decimal places. The value of K has only to be increased until the errors are too small to affect the result of the election.[3] The method is equivalent to transferring the *whole* vote at an appropriately reduced value, and it is this interpretation we shall use from now on.

Difficulty (iii) is slightly more technical, and warrants further explanation. Suppose at some stage a candidate has obtained x ($<q$) votes. By transfer from another (elected or eliminated) candidate he now acquires a further y votes, where $x+y \geq q$. His surplus is now $z=x+y-q$. It would appear that his $x+y$ votes should now be transferred, with value reduced by the factor $z/(x+y)$.

It is, however, common practice for only the y votes to be transferred, with value reduced by the factor z/y. The reason for adopting this procedure is simply the practical one, in a manual count, of reducing as much as possible the rescrutiny of ballots for later preferences. However, neither this nor the argument that 'the difference is unlikely to affect the result' are particularly relevant to a decision-theoretic discussion, though we shall return to the practicability problem later.

Of more importance here is the argument 'in STV a vote only counts for one candidate at a time, and should count for the first preference where possible'. If accepted, this would of course also render difficulties (i) and (ii) irrelevant, and the Senate Rules unnecessary; the first part of it is in fact sometimes used as a 'proof' that STV satisfies condition (B). But even without the Senate Rules the statement is false; however the surplus votes are chosen for transfer, *it is the existence of the untransferred votes which makes the transferred votes surplus.* A vote not only counts directly for one candidate; it can indirectly affect the progress of the count, the pattern of transfers, and ultimately the *election or non-election of other candidates.*[4]

It is this fact which is at the root of the failure of STV to satisfy condition (B).

In the specific situation described above, the candidate achieves election not only because of the accession of the y new votes, but because of the existence of the x previous votes; hence for condition (B) to be satisfied, all $x+y$ votes should be transferred at the appropriate reduced value.

However, there is yet a fourth difficulty, one which does not seem to have been recognised hitherto.

(iv) In determining the next preference to which a vote is to be transferred, *elected as well as eliminated candidates are ignored.*

Let us suppose that of y votes to be transferred, $y/2$ are marked next to go to candidate A, and $y/2$ to candidate B. Let us further suppose that A has already been elected; under STV the $y/2$ votes which would otherwise go to him are transferred to the next candidate marked (assumed C in every case) provided that that candidate is not also already elected. Thus $y/2$ go to B, and $y/2$ to C. The inequities are plain; the votes for A which enabled the $y/2$ to go to C rather than A had no say in their destination, while C obtains these votes at the same value as B receives his. Suppose these y votes were originally first-preference votes for a candidate D, now eliminated; those who voted for A next and then C at least have had their second choice elected, while those who voted next for B have not − yet these votes go, under STV, to both B and C at full value.

In section 6 we shall describe a counting mechanism which overcomes all these difficulties.

4. Making the most of one's vote

Any system which contains wasted votes contains at least some element of incentive to vote in other than his preferred way; the case for (C) in STV is that it is difficult for a voter to be sure (rightly or wrongly) that his vote will be wasted, both because the number of wasted votes is relatively small, and because the wasted votes are those for the non-elected but non-eliminated candidates − i.e. of the stronger, not the weaker, runners-up. However, it is also possible for voters to take advantage of the features of STV described in section 3, provided they are sufficiently well informed, by voting in a sophisticated manner. This is most easily shown by an example:

Let T=3599, s=3, q=900, and the unsophisticated first-preference votes for the six candidates A, B, ... F be as follows:

A	B	C	D	E	F
1020	890	880	589	200	20

In this case the 120 surplus votes of A divide 60 to B, 20 to C, 40 to D and the elected candidates are A, B and C.

Suppose there are 170 voters who above voted A, D, C ... It is known that the second-preference votes of F will go to C, and of E to D. Then the sophisticated way for these 170 to vote is F, A, D, C,... *in order to prevent A from being elected on the first count.*

A	B	C	D	E	F
850	890	880	589	200	190

On the elimination of F, his original 20 votes go to C, and the 170 sophisticated votes return to A. However, the 120 surplus is now taken entirely from this batch (see (iii) in section 3) and goes to D. C having no surplus, E must be eliminated and D is elected.

A different type of sophisticated voting is given below:

T=239, s=2, q=80.

Unsophisticated case: C and A elected:

C,A,B...	C,B,A...	B,A....	A,B.....
120	80	31	8

Sophisticated case: C and B elected:

C,A,B...	C,B,A...	E,B,A...	B,A....	A,B.....
120	50	30	31	8

It seems to be a new result that sophisticated voting is possible in STV, though it is well-known that it can occur in other voting systems and considerable work has been done on decision processes using a games-theoretic approach. Black[5] in his discussion of STV does mention the possibility of ‘an organised minority (perverting) the use of the system’ but only in connection with a candidate with just the quota on first preferences who is rated last by the rest of the electorate. STV supporters would claim that if a candidate can obtain a quota this *ipso facto* entitles him to be elected, particularly if he gets the quota on first preferences, and it is certainly difficult to understand what Black means by ‘pervert’ in this context.

5. Other considerations

At this point we shall mention some other aspects of STV, mainly in order to define the limitations of the present discussion. Proper treatment of the points raised in this section are well outside the scope of the present work, and is the subject of a projected further, more general paper.

The conditions (A), (B), (C) discussed so far were chosen simply because they seem to be specific to STV among constituency-type systems in parliamentary elections. However, other conditions could be applied, notably those specified by Arrow in his General Possibility Theorem.[6]

As STV elections are multi-vacancy, the preferences between candidates listed by the voters do not as they stand represent an ordering of *independent* alternatives, and so Arrow's analysis is not directly applicable. The deduction from the voter’s ordering of candidates of his ordering of the actual independent alternatives (the possible subsets of the set of all candidates who might actually be elected) is by no means straightforward. Nevertheless, at some stage of the count the process reduces to electing one candidate to one remaining vacancy, and so the consequences of the theorem, and the Condorcet paradox, cannot be escaped. Using the alternatives as they stand, even though they are not independent, STV clearly satisfies Arrow's conditions 1, 4, and 5. The condition 3 of independence of irrelevant alternatives is not satisfied, nor is condition 2 (the positive association of social and individual values). This can be seen from the above analysis.

A related point, and probably the strongest decision-theoretic argument against STV, is the fact that a candidate may be everyone's second choice but not be elected. This difficulty is not overcome by the feedback method, and it does not seem to the author to be possible to do so while retaining a system which would be recognisably a ‘single’ transferable vote.

Virtually all other discussion of STV, both for and against, seem to have been about political and not decision-theoretic considerations.

For example, Black[5] does discuss STV from what he terms the ‘statical’ point of view, but although he does express some disquiet about the ‘heterogeneity’ involved in STV (basically, that some votes count for first preferences, others for second or later preferences), he does not go into the problem in detail and concludes ‘in spite of those drawbacks (STV) has merits ... it is not difficult to see why many people, *regarding it purely as a statical system*, (Black's italics) should hold (it) in esteem’. The italicised phrase is to introduce other, ‘dynamical’ arguments against STV.[7] Black does not discuss the conditions mentioned here; though the germ of the idea of inequity is contained in the word ‘heterogeneity’; in fact as section 3 shows, the heterogeneity which worries him is more apparent than real, and the feedback method described in section 6 eliminates what there is. Nor − oddly − does the ‘everyone's second choice’ problem, even though this is closely connected with the doubts mentioned at the end of the last section.

6. The feedback process

One of the criticisms of STV which is often made is that its rules are too complicated, and are not derived from principles which can be simply stated. The above discussion shows that this is not surprising; the rules are in many cases little more than rules of thumb, designed for practical convenience rather than theoretic merit. The feedback process, however, is derived from simply-stated principles:

Principle 1. If a candidate is eliminated, all ballots are treated *as if that candidate had never stood.* [8]

Principle 2. If a candidate has achieved the quota, he retains a fixed proportion of every vote received, and transfers the remainder to the next *non-eliminated* candidate, the retained total equalling the quota.

Principle 1 is the one which leads to the feedback mechanism. For, suppose a voter marks his ballot A, B, C,.. and A is eliminated, the ballot, by Principle 1, is henceforward treated as if it read B, C,.. on the assumption that if A had not stood at all, the voter would have ordered the other candidates as before and B would have been first preference[9]. But suppose that B has at an earlier count reached the quota. Then this ballot must now be treated as an original first preference for B; that is, according to Principle 2, the same proportion of this vote must be retained by B as for the others, passing the rest to C (instead of the *whole* vote going to C as in previous methods). However, this will mean that the total retained by B is now greater than the quota. Thus the proportion of B's votes to be retained must be recalculated, and will in fact drop − in other words we must go back to the beginning, with A now eliminated. This is the feedback process.

Note that the proportion of each of B's votes to be transferred is increased by this accession of support; B's supporters have a say in the transfer of the extra surplus, since it is their existence which has made it surplus. All support for B is now treated equally, being divided proportionately to leave him with exactly the quota.

Consider now the effect of Principle 2. The transfer of B's vote may lead to another candidate, D, being elected. All votes, new and old, for D, have now to be divided, leaving D with the quota and distributing the rest to the next *non-eliminated* candidate. Some ballots may have B, another elected candidate, as next candidate. Under previous rules, only continuing (i.e. non-eliminated and *non-elected*) candidates can receive transfers. Now these votes are regarded as extra support for B: he takes the proportion allotted him by D, *retains the proportion that he keeps of all he receives*, and transfers the rest − now the *third* marked candidate. Formerly the third candidate would get all of the proportion transferred by D (see (iv) section 3).

It can be seen that B will once more have more than the quota if he does not again reduce the proportion which he retains. However, the increased proportion transferred may in part go to D who will therefore have to reduce the proportion *he* retains. This will react back on B, and it is clear that we have an infinite regression. However, it is also clear that the proportions for transfer do not increase without limit, there being only a finite total surplus available from B and D, who must each retain a quota. The problem is in fact a mathematical one of determining the proportions to be retained by each which will leave them both with a quota, taking into account the extent of mutual support. If p_B is the proportion B transfers, and p_D that which D transfers, supporters of both B and D have their votes transferred to third preferences at value $p_B p_D$. Those putting B first have $1-p_B$ retained by him and $p_B(1-p_D)$ retained by D; those putting D first have $1-p_D$ retained by him and $p_D(1-p_B)$ retained by B.

We now, as examples, give the formulae for the proportions for transfer in the cases of 1, 2, 3 and 4 elected candidates:

One candidate

$$t_1(1-p_1)=q$$

This is the same formula as before, except that t_1 now contains all effective first-preference votes for the candidate, including those obtained from eliminated candidates, who by Principle 1 are now ignored. The proportion p_1 is recalculated every time t_1 is increased by the elimination of a candidate.

Two candidates

The first elected candidate has t_1 first preference votes, of which t_{12} have the second elected candidate as second preference. Hence $p_1 t_{12}$ are passed on to that candidate. Similarly $p_2 t_{21}$ are received from the second candidate. Thus

$$(t_1+p_2t_{21})(1-p_1)=q$$

$$(t_2+p_1t_{12})(1-p_2)=q$$

Three candidates

The votes received by candidate 1 are now his first-preference t_1, second-preference p_2t_{21} from candidate 2 and p_3t_{31} from candidate 3, and third-preference $p_2(p_3t_{321})$ from candidate 3 (1st), 2 (2nd) and $p_3(p_2t_{231})$ from candidate 2 (1st), 3(2nd).

Thus:

$$[t_1+p_2t_{21}+p_3t_{31}+p_2p_3(t_{321}+t_{231})](1-p_1)=q$$

Two similar formulae hold, obtained by cyclic permutation of the suffices.

Four candidates

The formula now is:

$$[t_1+\sum_{i=2}^{4}p_it_{i1}+\sum_{i=2(i\neq j)}^{4}\sum_{j=2}^{4}p_ip_jt_{ij1}+p_2p_3p_4\Sigma' t_{(234)1}](1-p_1)=q$$

where Σ' indicates summation over all permutations of (234); there are three similar formulae.

The extension to any number of candidates is straightforward. It should be noted:

(i) The formulae for n candidates may be reduced to those for $n-1$ candidates by eliminating the nth equation and putting $p_n=0$ in the others;

(ii) Full recursion is not necessary on the elimination of a candidate if none of the totals or subtotals in the formulae in use at that stage are changed as a result.

7. Calculating the proportions

It can be seen that one of the difficulties involved in the feedback process arises from the need to calculate the proportions for transfer. However, a simple iterative procedure enables this to be done to any required accuracy. We shall take as the simplest example the position with two elected candidates, where the equations to be solved are, as above:

$$(t_1+p_2t_{21})(1-p_1)=q \qquad (1)$$

$$(t_2+p_1t_{12})(1-p_2)=q \qquad (2)$$

In these equations only the p_i are unknown. Suppose we guess a value of p_2 which is too low; then $(1-p_1)$ will be too large in equation (1), that is p_1 will also be too small. If we substitute this in equation (2) it will similarly give a value of p_2 which is too low.

The total vote for the two candidates is t_1+t_2; for them both to be elected $t_1+t_2 \geq 2q$. Suppose the strict inequality holds; in a non-trivial case t_{12}, t_{21} are both non-zero. Further, at least one of t_1, t_2 is greater than q; assume it is t_1. If we put p_2=0 in (1) we can solve for p_1, giving a value p_1>0. This p_1 is the proportion to be transferred if candidate 1 were the only elected candidate; thus $t_2+p_1t_{12} \geq q$ or candidate 2 would not be elected. If the equality holds, candidate 2 only just gets the quota and so p_2=0 from equation (2); thus the equations are solved.

If the strict inequality holds, we get a value of $p_2 > 0$ which is too small. Substituting in (1) increases the coefficient of $(1-p_1)$ and hence increases p_1; the new value of p_1 is increased (but is still too low). Substitution in (2) gives similarly an increased, but too low, value of p_2. Thus the iterative process gives monotonically increasing sequences of values p_1, p_2 bounded above, which hence tend to limits which are the solutions of the equations. A cycle of iterations which leads to two successive sets of values the same to the given accuracy is taken as the approximate solution required. Note that the approximate values may be slightly smaller than the exact ones, but this is exactly what we want; otherwise too much of the support for the candidate concerned would be transferred and he would be left with less than the quota. The process can also be easily shown to work in the limiting case, t_1+t_2=2q.

It is clear that the success of this iterative procedure depends on the fact that all the quantities in the totals (the coefficients of $(1-p_i)$ in each equation) are non-negative, and that therefore it will work for any number of equations provided they are solved cyclically in order of election − this condition being necessary to avoid getting negative values of p_i. Since the counting process can only increase the totals of support for elected candidates, it is also clear that the p_i for those candidates can only increase as the count progresses;[10] thus it is safe to take as starting values of the p_i the ones obtained at a previous stage, putting p_i=0 initially for newly-elected candidates only (in which case, as mentioned above, the equations reduce to the ones at the previous stage and hence will yield, at the beginning of the iteration, the same answers).

It can be shown fairly simply that the convergence rate of the iterative process is likely to be unsatisfactory only when both of the following conditions hold; that all the p_i are small, and the cross-totals t_{ij} etc, are as large as possible. This would not cause difficulty even on the rare occasions on which all these conditions were satisfied, since the occurrence of slow convergence can be detected in advance and allowed for, while at a later stage in the count some at least of the p_i are likely to rise sufficiently to accelerate to the true convergence satisfactorily.

8. Conclusions

It is obvious even from the above example that the feedback process is a much more laborious method of arriving at a result than any at present in use; in a full-scale election with thousands of ballots to scrutinise, it would be very lengthy indeed. However, even the present methods are sufficiently lengthy to make it worthwhile using computers to help in the counting,[11] and if this is done, then complex counting methods are no problem.

It may be argued that the actual results of any election would be different so infrequently that the additional complication is unnecessary. This is a matter for conjecture, or preferably, for further investigation. However, the method has been tried out in two cases, once using figures obtained by a quasi-random process, and once in an actual STV election. In both, there were differences in the candidates elected.[12] Particularly since STV supporters lay such emphasis on the criterion of equality of treatment (condition (B)), it would seem worthwhile in automated counting to adopt the feedback method.

To sum up, the feedback method does satisfy the criterion, subject to the limitations imposed by the basic STV system − i.e. the theoretical minimum of wasted votes, and the elimination of candidates. There is one further limitation not so far discussed, imposed by the voters themselves if they take advantage of the possibility allowed by STV of listing only some of the candidates in preference order. The extension of the feedback method to cover this is dealt with in Paper II; it turns out that the extension also, as a bonus, allows voters to express their views much more accurately than under previous STV methods.[13]

References and Notes

1. For a complete description of STV see E Lakeman and J Lambert: Voting in Democracies (Faber and Faber 1955). *(The current edition in 1994 is E Lakeman: How Democracies Vote (4th edition, Faber and Faber 1974).)*

2. This is nevertheless more than can be said for some common voting systems, such as the simple majority system.

3. This cannot, of course, cope with the case of exact equality, where some other method has to be used, if only drawing of lots.

4. To argue, in connection with a *transferable* system, that a vote should where possible not be transferable, seems inconsistent, particularly in view of the strong arguments put forward by STV supporters against the single non-transferable vote system, where an elector may choose only one from a list of candidates even though more than one are to be elected. See Lakeman

and Lambert, *op, cit.*[1]

5. Duncan Black: Theory of Committees and Elections (2nd edition, Cambridge, 1963, pp 80-83).

6. K Arrow: Social Choice and Individual Values (2nd edition, Wiley 1962).

7. The case for the other side may be found in Lakeman and Lambert, *op cit* [1].

8. The similarity of this principle to Arrow's condition of independence of irrelevant alternatives is obvious. However, the interdependence of the alternatives here means that the condition is not in fact satisfied.

9. This innocent-looking assumption is open to major criticism. Full discussion is outside the scope of this paper; it is hoped to include this in the projected more general paper mentioned in section 5.

10. Clearly Arrow's condition 2, the positive association of individual and social values, is now satisfied by the non-independent alternatives.

11. For a feasibility study in general terms, see P Dean and B L Meek: the Automation of Voting Systems; Paper I; Analysis (Data and Control Systems, January 1967, p16); Paper II; Implementation (Data and Control Systems, February 1967, p22), and B L Meek: Electronic Voting by 1975? (Data Systems, July 1967, p12) − the date in the last source referring to the UK. For a description of the actual use of computers in STV elections in the United States, see Walter L Pragnell: Computers and Conventions (The Living Church, 20th August 1967, p12).

12. For obvious reasons the work on the actual election cannot be made public!

13. These papers are the result of a problem posed by Miss Enid Lakeman, Director of the Electoral Reform Society, London; the author wishes to thank her for her encouragement in the progress of the work. Thanks are due also to Professor W B Bonnor, Mr Robert Cassen, Mr Peter Dean, Mr Michael Steed and Professor Gordon Tullock for valuable discussions, correspondence and advice.

A New Approach to the Single Transferable Vote

Paper II: The problem of non-transferable votes

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The original version of this paper was dated 21 March 1968 and was published in French in Mathématiques et Sciences Humaines No 29, pp 33-39, 1970. This note, and note 8 in its present form, have been added in this reprint.

Abstract

The feedback counting method used for Single Transferable Vote elections, developed in an earlier paper, is extended to cover situations in which there are non-transferable votes. It is shown that present counting methods, on the other hand, may not satisfy the condition that the number of wasted votes be kept to a minimum in such situations. The extension of the method to permit voters to give equal preferences to candidates is also described.

1. Introduction

In an earlier paper[1] (hereafter referred to as Paper I) the Single Transferable Vote (STV) system of voting was considered from the point of view of certain conditions, the main one being that as far as possible the opinions of all voters are taken equally into account; it was shown that present STV counting methods do not satisfy this condition. A 'feedback' counting mechanism was suggested which would overcome this problem. In Paper I, however, we confined ourselves only to the cases where, whenever a vote is rescrutinised for transfer, a next preference is always given. In this paper we shall show how the feedback method can be extended to cope with situations where no such preference is available. We shall here adopt the reverse procedure to Paper I; we shall consider the application of the feedback mechanism to these cases first, and only then discuss present counting methods in the light of the conditions.

2. Rules for vote-casting

Even within the same voting system major differences can be made simply by changing the rules governing what constitutes a valid ballot. For example, in a multiple-vacancy election by simple majority where each voter has one independent vote for each vacancy, the result can be totally different if the voter is forced to use all of his votes (in effect to vote against his favourite candidates) instead of using only some.[2] In STV the equivalent requirement would be that all candidates should be listed in preference order. However, in the simple-majority case distortions can arise in that some votes may not be genuine, having only been added in order to make up the correct number; in STV a voter may only wish to express his preferences for a few candidates, being indifferent to the remainder. Normal STV practice is in fact to accept as valid any ballot showing a unique first preference; thereafter the voter may, optionally, give further preferences for as many or as few of the remaining candidates as he wishes. In STV the feedback mechanism could be applied as it stands simply by declaring as invalid any votes which do not give preferences for all candidates or (relaxing this somewhat) declaring invalid during the progress of the count any vote encountered for which a next preference is required but not available, and then restarting the count. However, it would clearly be more satisfactory not to impose additional restrictions on the voter if this can be avoided.

3. Extension of the feedback method

We recall here the two principles of the feedback mechanism stated in Paper I:

Principle 1. If a candidate is eliminated, all ballots are treated *as if that candidate had never stood*.

Principle 2. If a candidate has achieved the quota, he retains a fixed proportion of every vote received, and transfers the remainder to the next *non-eliminated* candidate, the retained total equalling the quota.

Since transfers are only made from eliminated or elected candidates, non-transferability only arises when all the marked candidates are eliminated or elected. The simplest case to consider is that when all the marked candidates are eliminated. By Principle 1, such a ballot has to be treated as if those candidates had never stood; and hence as if the ballot is invalid. This implies that the total T of valid ballots is reduced; this in turn implies that, on the elimination of any candidate, if non-transferable ballots occur the feedback should include the recalculation of the quota, using the reduced value of T.

The case of a ballot with marked candidates who are elected is less straightforward. Suppose an elected candidate C receives a total x of votes with no further preferences marked on them (any marked eliminated candidates can, by Principle 1, be ignored). By Principle 2, C must pass on a fixed proportion p of these, as all other, votes and retain the rest as part of his quota. The difficulty arises because it is not clear to whom these votes should be transferred.

If the difficulty were to be avoided by increasing the proportion transferred of votes for which a next preference is marked, to enable all x votes to be retained by C, this would clearly reintroduce inequities of the kind Principle 2 was designed to eliminate. Not to transfer the proportion at all would mean leaving C with more than the quota (see also section 4). The two possible ways of strictly obeying Principle 2 are

(a) to divide the otherwise non-transferable proportion equally between the remaining (i.e. unmarked and uneliminated) candidates; or

(b) to subtract this quantity from the total T of votes cast, and recalculate the quota with the new value.

Method (a) is based on the view that the voter regards the unmarked candidates as of equal merit, which is why he has not given preferences. The second method is based on the view that the voter's action is a partial abstention; he has not sufficient knowledge of these candidates to judge between them, and prefers to leave the choice to the other voters. It should be noted that the two methods are not equivalent; in the first the totals of the unmarked candidates, in particular the non-eliminated ones, are raised equally, whereas in the second the quota increases the proportions transferred from the elected candidates, and the increase in the votes of non-elected candidates will vary according to these values.

For the moment we shall resolve the (apparent) dilemma by making the (apparently) arbitrary decision to adopt the second method. The *prima facie* case for this is that in general some unmarked candidates will be elected candidates, and hence the adoption of the first method will in any case involve the recalculation of the quota. However, the real justification will appear in section 6, when it will be shown that the dilemma need not, in fact, exist at all.

4. Current STV practice

Current STV procedure in dealing with non-transferable votes involves different rules in different circumstances. The main rules are

(i) If a vote is not transferable from an eliminated candidate, it is set aside; such votes play no further part in the count.

(ii) If the number of votes non-transferable from an elected candidate is not greater than the quota, those votes are included in the quota and only the transferable votes determine the distribution of the surplus. If the number is greater than the quota, then the transferable votes are transferred (at unreduced value), the difference between the non-transferable votes and the quota increasing the non-transferable total.

In Paper I we considered STV from the point of view of three conditions. Condition (C) we shall discuss later; the others were

(A) The number of wasted votes in an election (i.e. which do not contribute to the election of any candidate) is kept to a minimum.

(B) As far as possible the opinions of each voter are taken equally into account.

It is clear at once that, when there are non-transferable votes, condition (B) cannot be satisfied even by the feedback counting method unless recalculation of the quota is included, for otherwise candidates at a later stage of the count, when a number of non-transferable votes have accumulated, need less that the original quota to be elected. Indeed, if as many as q votes become non-transferable, it is impossible for the last elected candidate to achieve a full quota.

We saw in Paper I that condition (A) is satisfied when there are no non-transferable votes. When votes do become non-transferable these have to be added to the 'wasted' total W, and the formula in Paper I becomes

$$W \leq T/(S + 1) + T_0$$

where T_0 is the non-transferable total. However, this is derived from a quota calculated on the total T and not on the total available vote $T' = T - T_0$. Thus with recalculation of the quota we have

$$W' < T'/(S + 1) + T_0 = W - T_0 /(S + 1) < W$$

i.e. condition (A) is violated unless the quota is recalculated[3].

It is clear that rule (ii) above is an attempt to satisfy condition (A), but it only does so at the cost of violating condition (B); for example, if a candidate E is elected with $q + x$ votes, q of which are non-transferable, the x remaining votes will be transferred at unreduced value to the next preference even though their earlier preference for E has been satisfied. Further, the present rule that votes cannot be transferred to an elected candidate (see Paper I) means that both by rule (i) and by rule (ii) many whole votes may be declared completely non-transferable, thus swelling T_0 and W above, whereas the feedback method allows each vote to count partly for the elected candidates marked and only a fraction becomes non-transferable.

Thus, on two grounds, current STV counting methods violate condition (A). It could perhaps be argued that the feedback method cannot satisfy condition (A) unless method (a) rather than method (b) of section 3 is used when dealing with unmarked candidates. We shall discuss this point in section 6.

5. Recalculating the quota

It can be seen that in recalculating the quota and having to apply it in retrospect to candidates already elected, the same difficulties occur as in the simple feedback situation, without non-transferable votes, described in Paper I. We consider first the case of an elected candidate. If some of his votes are non-transferable, the appropriate proportion is subtracted from the total vote, and the quota recalculated. The reduction in the quota makes more of the elected candidate's votes surplus, which increases the proportion for transfer; this increases the non-transferable proportion to be subtracted from the total, which further reduces the quota, and so on. The equations to be solved are

$$q = [(T - p_1 t_{10})/(S + 1) + 1] \qquad (1)$$

$$\mathrm{t}_1(1 - p_1) = q \qquad (2)$$

where, as in Paper I, S is the number of vacancies, T is the total votes (now ignoring any which mark only eliminated candidates), t_1 the total for the elected candidate, p_1 the proportion he transfers, t_{10} the total vote for the candidate not transferable to others, and q is the quota.

These two equations can be solved easily for p_1 and q by equating the expressions for q; however, if there is more than one elected candidate the iterative method of finding the p_{i}, described in Paper I, will be needed, and it is convenient to discuss the extension of the iterative process to include the recalculation of the quota in terms of the simplest case, above. Equation (1) with $p_1 = 0$ gives the original value of q. Equation (2) then gives a first value of $p_1 > 0$. Substitution of this value in (1) gives a new value of q smaller than before; use of the new q in (2) gives a larger p_1, and so on. Thus we have a monotone increasing sequence of values for p_1, bounded above by 1, and a monotone decreasing sequence of values of q bounded below by 0; these sequences must therefore tend to limits which are the solutions to the equations. The convergence rate is satisfactory; simple analysis shows that the errors are multiplied in each cycle by a factor which is at most $1/(S + 1)$.

The process is extended to the case of n elected candidates by adding to the equations in Paper I the equation

$$q = [T_{\mathrm{n}} /(S + 1) + 1]$$

which must be evaluated for q first in each iterative cycle. $T_{\mathrm{n}} = T_{\mathrm{n}}(p_1,p_2,....,p_{\mathrm{n}})$ is the total available for transfer in each case; for $n = 1, 2, 3$ it is given by

$$T_1 = T - p_1 t_{10}$$

$$T_2 = T - \{p_1 t_{10} + p_2 t_{20} + p_1 p_2 (t_{120} + t_{210})\}$$

$$T_3 = T - \{\Sigma_1 p_i t_{i0} + \Sigma_2\, p_i p_j t_{ij0} + \Sigma_3\, p_1 p_2 p_3\, t_{(123)0}\}$$

In these formulae $t_{ij \cdots k0}$ is the total transferable from candidate i to candidate j, to ..., to candidate k but not further; $\Sigma_1$denotes summing over i; Σ_2 denotes summing over all i, j, $i \neq j$; Σ_3 denotes summing over all permutations of (123).

The reader will easily derive equivalent formulae for higher values of n; putting $p_n = 0$ in the expression for T_n gives the expression for T_{n-1}.

6. Equal preferences

In section 2 we discussed briefly the effect of different validity rules on otherwise identical voting systems. The usual STV counting procedures depend on the existence at each stage of a unique next preference, the only deviation allowed being, as we have seen, that the absence of further preferences does not make the vote as a whole invalid. It is standard practice to accept as valid a vote with a unique first preference, and to accept further preferences provided one and only one is marked at each stage; if no, or more than one, next preference is given at any point, all markings at and past this point are ignored.

For the simplest form of STV counting, involving the physical transfer of ballot papers from pile to pile, the need for a unique next preference is obvious. However, with the feedback method such a restriction is no longer necessary, and indeed it is not necessary even with Senate Rules counting. A vote can be marked A1, B1, C2, ... with A and B as equal first preferences and credited at 0.5 each to A and B. If A is elected or eliminated the 0.5 is transferred at reduced or full value to the next preference − which of course is B and not C. In effect, such a vote is equivalent to two normal STV votes, of value 0.5 each, marked A,B,C... and B,A,C... respectively. Similarly, if A, B, C are all marked equal first, this is equivalent to 6 (= 3!) votes of value 1/6 each, marked A,B,C...; A,C,B...; B,A,C...; B,C,A...; C,A,B...; and C,B,A... . It is easy to see that this can be extended to equal preferences at any stage, and that K equal preferences correspond to $K!$ possible orderings of the candidates concerned, each sharing $1/K!$ of the value at that stage.

Such an extension of the validity rules enables us to resolve the dilemma between the methods (a) and (b) in section 3 of dealing with non-transferable votes. A voter who, at a certain stage, wishes his vote, if transferred, to be shared equally between the remaining candidates, can simply mark those candidates as equal (i.e. last) preferences. Thus the dilemma does not after all exist; both of the methods can be used, and the voter himself can determine which is to be used for his own ballot by the way that he marks it; failure to rank a candidate indicates a genuine (partial) abstention.

This extension of the validity rules also enables condition (C) of Paper I to be satisfied more closely. The condition was:

(C) There is no incentive for a voter to vote in any way other than according to his actual preference.

Here we are interpreting this condition in a particular way not discussed in Paper I: the STV voting rules not merely encourage but force a voter to vote other than according to his preference in the restricted sense that, e.g. if he rates two candidates as equal first he is not allowed to vote accordingly, but must assign a preference order between them which may well be arbitrary. In view of the importance of first preferences in STV, this is undesirable. A voter is similarly forced to make an unreal ordering of candidates to which he is indifferent if, for example, he has listed his real preferences but wishes to give the lowest ranking to a candidate he particularly dislikes. This kind of voting is very common.

Permitting equal preferences thus gives much greater flexibility to the voter to express his ordering of the candidates, and is thus a desirable reform whether the feedback method is used for counting or the Senate Rules retained.[4]

7. Concluding remarks

Two distinct problems arise in the development of a voting system; the information with regard to the choices which is required from each voter, and the way in which this information is to be processed to arrive at "the social choice".

The first problem is mainly outside the scope of these papers, but has been touched on in the last section. It is a basic assumption of STV that the individual preference orderings of each voter is sufficient information[5] to obtain the social ordering, and the voting rule extensions described above follow naturally from this principle, and indeed bring STV more closely into line, in a certain sense, with the work of Arrow.[6]

The possible development of (preferential, transferable) voting systems which use further relevant information is the subject of continuing work.[8]

The second problem is the classical problem of decision theory. Assuming the basic STV structure, these papers have shown that the feedback method of counting is needed to satisfy the declared aims of STV as a decision-making procedure more consistently.

This improvement can be made without causing any more difficulty to the voter, and allows the counting procedure to be described by two simple principles instead of by a collection of rules, some of which are rules of thumb.

The disadvantage of the method is the need for many repetitive calculations, which for reasons of sheer practicality rules it out for manual counting except when the numbers of vacancies, candidates and votes are small. However, as pointed out in Paper I, an STV count is already a sufficiently tedious process for it to be worthwhile to use a computer, and the additions to the feedback method described in this paper would be simple to add to the computer program.

As E G Cluff has pointed out,[7] one advantage of election automation is that one is not restricted in the choice of voting system to what is practically feasible in a manual count. The feedback method can lead to different results from the Senate Rules in non-trivial cases, and is therefore a choice to be considered when the automation of STV elections is being implemented.

References and notes

1 B L Meek, A new approach to the Single Transferable Vote I: Equality of treatment of voters and a feedback mechanism for vote counting, Mathématiques et Sciences Humaines No 25, pp 13-23, 1969.

2 It can in fact lead to the defeat of a candidate who is first choice of a majority of the electorate. It is depressing to note that a public election in England was held under precisely these rules as recently as 1964.

3 This kind of inequity can be found most often in elections with large numbers of candidates and vacancies − e.g. for society committees − and can lead to disillusion with STV as a voting system which has little relation to its merits or demerits.

4 The possibility of a voter sharing his first preference other than equally between a number of candidates would take us too far afield, into the realm of multiple transferable voting systems − the subject of continuing work on more general preferential voting systems. In STV the task of the voter is in comparison a straightforward one, in some ways made easier by allowing equal preferences.

5 And, indeed, necessary information!

6 K Arrow, Social choice and individual values, 2nd edn, Wiley 1962. For what is meant by "in a certain sense" see Paper I.

7 See B L Meek, Electronic voting by 1975?, Data Systems July 1967, p 12.

8 See also note 4. This further work was later published (in English) as: B L Meek, A transferable voting system including intensity of preference, Mathématiques et Sciences Humaines No 50, pp 23-29, 1975.

Computer counting in STV elections

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The Single Transferable Vote is by far and away the fairest form of electoral system. Nevertheless, when the counting in STV elections is carried out by hand, rather arbitrary decisions have to be made in order to simplify the count, and these introduce anomalies. Although small in comparison with anomalies present in other electoral systems, these anomalies may affect the result, and are certainly annoying to the purist.

The biggest anomaly is caused by the decision, always made, not to transfer votes to candidates who have already reached the quota of votes necessary for election. This means that the way in which a given voter's vote will be assigned may depend on the order in which candidates are declared elected or eliminated during the counting, and it can lead to the following form of tactical voting by those who understand the system. If it is possible to identify a candidate W who is sure to be eliminated early (say, the Cambridge University Raving Loony Party candidate), then a voter can increase the effect of his genuine second choice by putting W first. For example, if two voters both want A as first choice and B as second, and A happens to be declared elected on the first count, then the voter who lists his choices as 'A B ...' will have (say) one third of his vote transferred to B, whereas the one who lists his choices as 'W A B ...' will have all of his vote transferred to B, since A will already have been declared elected by the time W is eliminated. Since one aim of an electoral system should be to discourage tactical voting, this seems to me to be a serious drawback.

If, on the other hand, one agrees that surpluses will be transferred to candidates who have already reached the quota, then one has to do something to avoid the never-ending transfer of progressively smaller surpluses between two candidates. Whatever strategy one adopts, it is bound to introduce other anomalies, albeit smaller than the one already described.

If the counting is carried out by computer, however, no such arbitrary decisions are necessary, as the never-ending transfer can be carried out to completion, or at least until the surpluses remaining to be transferred are less than (say) a millionth of a vote. The resulting procedure is described in the next paragraph in a different way. It is comparatively simple in concept, and the undoubtedly long calculations are all safely hidden inside the computer.

The counting is divided into rounds, in each of which one candidate is eliminated. In each round of the elimination, a scaling factor is assigned to each candidate, representing the proportion that will actually be credited to him out of the votes potentially available to him, in such a way that:

1) a candidate who has already been eliminated in a previous round is assigned scaling factor 0, so that no votes will be credited to him in the current round;

2) a candidate whose fate is undecided at the end of the current round is assigned scaling factor 1, so that all the votes potentially available to him are credited to him; and

3) a candidate who by the end of the current round has at least the quota of votes necessary for election (and so is certain to be elected) is assigned a scaling factor less than or equal to 1 so that the number of votes credited to him is brought down exactly to the quota.

The candidate with the smallest number of votes is then eliminated, and the process is repeated until the number of candidates remaining is equal to the number of places to be filled.

For example, suppose that, in a given round of the counting, candidates A and B are certain of election and have scaling factors of two thirds and three quarters respectively, and candidates C, D and E have already been eliminated in previous rounds, whereas the fates of the remaining candidates remain undecided. Then a voter who lists the candidates in the order C, A, D, B, E, F will, in the current round, have none of his vote assigned to C. The whole of his vote will be passed down to A, who will retain two thirds of it. The remaining third of his vote will be passed over D and down to B, who will retain three quarters of it (that is, one quarter of a vote). The twelfth of a vote that is still unassigned will be passed over E and down to F, who will retain all of it.

The calculation of the scaling factors, which would be prohibitively long to do by hand, could be carried out quite easily by computer. However, once the computer had done the work, it would be possible to check by hand that the computer was correct; certainly this would take no longer than carrying out the whole count by hand as at present.

(This situation is not unusual in mathematics. Suppose, for example, that you were asked to find a number x between 1 and 2, accurate to seven places of decimals, such that (say)

$$x^5 + x^4 - 4x^3 - 3x^2 + 3x + 1 = 0.$$

You would find it very tedious to do so by hand, even with the aid of a pocket calculator. Suppose, however, that a computer were to do the work and tell you that the answer is 1.6825071; then it would take you only a few minutes to check that the computer was correct.)

The size of computer required would depend on the size of the electorate, on the number of places to be filled and, to a lesser extent, on the number of candidates. In the case of an election with both a very large electorate and a large number of places, it might even be impossible to carry out the calculations in a reasonable time with the present generation of computers.

However, for parliamentary elections, there would be no problem: the calculations could be done quite easily even on a mini-computer.

Since proposing the above method, I have learnt that it is not new; a differently worded but exactly equivalent method was proposed by Brian Meek in 1969.[1,2] I hope it will be possible to agree that, whenever computer counting is used in STV elections, this method should be used.

References

1 B L Meek, Une nouvelle approche du scrutin transférable, Mathématics et Sciences Humaines 25 (1969), 13-23.

2 B L Meek, Une nouvelle approche du scrutin transférable (fin), Mathématics et Sciences Humaines 29 (1970), 33-39.

Counting in STV elections

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Introduction

Whatever criticisms may be levelled against First-Past-The-Post as a system of voting, at least the system has the merit that, although the count may be conducted in many ways, all ways give the same result. The Single Transferable Vote is demonstrably a better system of *voting*, but the system has the disadvantage that the result depends upon how the *counting* is conducted.

Counts have been done in many ways, and in some peculiar ways by some well-meaning, but unversed, enthusiasts for STV. One of the commonest methods of conducting the count, and indeed the method that the Electoral Reform Society uses, is that given by Newland and Britton.[1] Their paper tells one *how* to conduct a count by their method, but not *why* they make many of the arbitrary decisions that they do. Woodall[2] has suggested that they are made for expediency − to simplify the count − and he goes on to propose another method, which he advocates whenever computer counting is used. As Woodall points out, his method would be prohibitively long with human counting. As Woodall also states, a differently worded but an exactly equivalent method to his had been proposed by Meek in 1969.[3,4]

The object of this paper is, first, to consider some of the principles that are felt to be important in deciding upon a method for conducting the count, and then to go on and propose a method that meets these principles.

Principles

The first principle of the STV system is that election is by quota. A candidate is deemed elected when the vote assigned to him attains a given quota. The quota is chosen as the minimum vote which will not allow more than the required number of candidates to be elected. This is the Droop quota, and is the total valid vote divided by one more than the number of candidates to be elected.

The second principle concerns the transference of a voter's vote to the preferences later than his first preference. The voter needs to be assured that his later preferences will in no way upset the voter's earlier preferences. Equally a voter's later preferences should not be considered unless, in regard to each earlier preference candidate, either the voter has borne an equal share with other voters who have voted for that candidate in giving him the necessary quota, or that earlier preference candidate has been eliminated. The way in which Newland and Britton conduct a count does not meet this principle.

The third principle concerns the elimination of candidates. Unfortunately no-one appears to have proposed a principle in this regard. So what is usually done is that, when no candidate has a surplus above the quota, in order to allow the count to continue, the candidate whose vote is least is eliminated.

Method

If, after counting the first preference votes, the votes for one or more candidates exceed the quota, then the essential feature of the method proposed here is that these candidates are allowed to retain only part of the vote that had been expressed for them such as will give each candidate just the necessary quota. The part of the vote that the candidate retains is called the 'amount retained'. The voters who have voted for one of these candidates, for whom the amount retained is x_1, say, then have an amount remaining of $(1–x_1)$, which is then transferred to the voters' expressed second preferences. If an expressed second preference has an amount retained of x_2, say, and if x_1+x_2 is less than unity, then the voter still has an amount remaining of $(1–x_1–x_2)$, which is then transferred to the expressed third preference, and so on. Proceeding in this way, the end of the first stage of the count is reached when some candidates have just the quota, whereas the remainder have varying amounts of vote less than the quota.

The candidate whose vote at the end of the first stage is least is eliminated. This means that, wherever his name appears on a ballot paper, it is 'passed over', and, in effect, all the later preferences are 'moved up one'. Elimination of a candidate will usually cause the votes for some other candidates to exceed the quota. The amount to be retained by each candidate is then reduced to such lower value as will give each candidate just the necessary quota. Voters who have voted for these candidates with reduced amount retained will then find that they have more vote remaining for transference to later preferences. Proceeding in this way, at the end of each stage of the count, some candidates will have just the quota, whereas the remainder will have varying amounts of vote less than the quota.

Eventually the number of non-eliminated candidates will be reduced to one more than the number to be elected. When the amounts to be retained are now recalculated so as to reduce each candidate's vote to the necessary quota, all candidates will have just the quota, so the one candidate who has an amount retained of just 1 is the one eliminated. The remaining candidates are deemed elected.

If at any stage a ballot paper does not contain sufficient preferences for transference to be made, then the balance of vote is ascribed 'non-transferable', and the quota is recalculated excluding the non-transferable vote.

The main question that the proposed method of conducting the count poses is: how does one decide upon the amount to be retained by each candidate at each stage? From what has been said, the amounts retained have to be such that, when the count is made, each candidate to whom an amount to be retained of less than 1 has been assigned achieves just a quota. The problem of finding the amounts retained, and the associated quota, is a mathematical one which is relatively straightforward, even if protracted, but which a computer can help to solve. Here we are concerned only with the principle, not with precisely how the task be done. However, it is not necessary for everyone to know *how* to assign the amounts retained. As Woodall[2] has exemplarily pointed out, it is only necessary for anyone to be able to *check* that the assigned amounts retained do in fact achieve the desired result.

References

1 R A Newland and F S Britton, How to conduct an election by the Single Transferable Vote, second edition, Electoral Reform Society of Great Britain and Ireland (1976).

2 D R Woodall, Computer counting in STV elections, Representation, Vol.23, No.90 (1982), 4-6.

3 B L Meek, Une nouvelle approche du scrutin transférable, Mathématics et Sciences Humaines 25 (1969), 13-23.

4 B L Meek, Une nouvelle approche du scrutin transférable (fin), Mathématics et Sciences Humaines 29 (1970), 33-39.

Meek or Warren counting

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The Meek system and the Warren system for counting an STV election are very similar, but whereas the fractions of a vote retained by successive elected candidates are multiplicative under Meek, they are additive under Warren. For example, if candidate A is keeping 1/2 of everything received and candidate B is keeping 1/3, a vote reading AB... will, under Meek, give 1/2 of a vote to A and 1/6 of a vote to B (i.e. 1/3 of the remaining 1/2), leaving 1/3 of a vote to be passed on further. With those same fractions under Warren, a similar vote will give 1/2 of a vote to A and 1/3 of a vote to B, leaving 1/6 of a vote to be passed on further. (It should be noted, though, that in any actual case the fractions will not usually be the same under the two systems). The Warren system will often lead to the situation where not enough vote remains for the fraction required; in such a case all that remains is taken and nothing remains to go any further.

There is no difference in the ease of writing a computer program to satisfy the one system or the other; the choice can be made solely on which is regarded as better in principle. It should also be reported that in real examples of STV elections, as distinct from artificially constructed examples, no case has yet been found where the two elect a different set of candidates, so the difference for real life seems to be slight.

There has been much argument over which system is to be preferred. In the end, we have settled on a particular example which demonstrates that each system can be said to suffer from a difficulty that the other one solves. It must therefore be a matter of judgement which difficulty is regarded as the more serious, rather than a firm decision of one always being better than the other.

The Meek rationale is that all transfers from a surplus should be in proportion to the 'votes-worth' put into that surplus. Thus 5 identical votes, each of current value 1/5, should have the identical effect to that of 1 complete vote for the same preferences. The Warren rationale is that no voter should be allowed to influence the election of an additional candidate until having contributed as much as any other voter to the election of each candidate who has already been elected and is named earlier in the voter's preferences. Thus the 5, each of value 1/5, are to be treated as 5, not as the equivalent of 1.

The example that shows the differences has 5 candidates for 3 seats and 32 votes, leading to a quota of 8.0. The votes are:

12 ABC, 12 BE, 7 C, 1 D.

Meek supporters can point out the Warren anomaly that A and B each had a substantial surplus on the first count, yet the 12 ABC votes are given by the Warren system entirely to A and B and, in consequence, C fails to get the 1 extra vote needed for election and E takes the third seat. Under Meek, C easily beats E.

Warren supporters can point out the Meek anomaly that if the 12 ABC voters had voted BAC instead, the Meek system would have behaved exactly like the Warren system, and E would have beaten C. It seems illogical that the choice of C or E should depend upon the ordering by those 12 voters as ABC or BAC when A and B were both elected anyway.

Deciding between the two systems must therefore remain a matter of personal preference.

It may be of interest to see exactly how each of the two systems would treat this example. Each would note that A and B are both elected on the first count, each having 12 first preferences for a quota of 8.

The Meek system would calculate that A needs to keep 2/3 of everything received whereas B needs to keep 1/2, these fractions being derived so that each of A and B keeps exactly a quota. The 12 ABC votes would be allocated as 2/3 of 12 = 8 to A, 1/2 of the remaining 4 = 2 to B, the remaining 2 to C. The 12 BE votes would be allocated as 1/2 of 12 = 6 to B, the remaining 6 to E. At the next count the current votes would therefore be A 8, B 8, C 9, D 1, E 6. The third seat is thus assigned to C and no more needs to be done.

The Warren system would calculate that A's amount retained needs to be 2/3 and B's 1/3, again derived such that (under the different counting method) each of A and B keeps exactly a quota. The 12 ABC votes would be allocated as 2/3 of 12 = 8 to A, 1/3 of 12 = 4 to B. The 12 BE votes would be allocated as 1/3 of 12 = 4 to B, the remaining 8 to E. At the next count the current votes would therefore be A 8, B 8, C 7, D 1, E 8. The third seat is thus assigned to E and no more needs to be done.

Issue 2, September 1994

Editorial

Voting matters is concerned with the implementation of the Single Transferable Vote. However, STV is merely one method of analysing ballot papers in which preferential voting is used. In consequence, other methods of analysis could provide some insight into STV. In this issue, one particular problem of STV is highlighted, namely that of the elimination of a popular candidate with few first-preference votes. David Hill and Simon Gazeley provide algorithms to ‘overcome’ this problem and discuss the consequences. Due to the impossibility of satisfying apparently simple requirements, Douglas Woodall has shown that overcoming the above problem is bound to introduce other anomalies.

Brian Wichmann.

STV with successive selection — An alternative to excluding the lowest

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The problem with current STV systems

A feature of STV which is not shared by other preferential voting systems is election on attaining a certain number of votes (the ‘quota’). If the number of candidates who have a quota of first preference votes is insufficient to fill all the seats being contested, those which are left are filled by candidates whose quotas contain votes which have been transferred from other candidates. These transfers take two forms: of surpluses above the quota for election from candidates who are already elected, and of all the votes previously standing to the credit of candidates who have been excluded in accordance with the rules.

When it is necessary to withdraw a candidate from contention, all versions of STV currently in use exclude the one who has fewest votes at that time. It is contended that the consequences of this rule in conventional STV formulations can be haphazard and therefore unjust in their effect. Consider the following count:

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AD  35
BD  33
CD  32
```

There are here 3 separate and substantial majorities: against A, against B and against C. The only thing that all the voters agree on is that D is preferable to two out of the other three candidates; yet STV excludes D first, however many seats are being contested. Unfairness and anomalies such as this arise because candidates are excluded before the full extent of the support available to them has been investigated. Even though every ballot-paper may have the same candidate marked as the next available preference, that candidate will not survive if they do not have enough votes now.

An even more serious consequence of the ‘exclude the lowest’ rule is that it is possible for voters to assist their favoured candidates by withholding support rather than giving it. Consider the following election for one seat:

```
AC  13
BC   8
CA   9
```

Having been excluded, B's votes go to C, who now has an absolute majority and gets the seat. But suppose that two of A's supporters had voted BC instead:

```
AC 11
BC 10
CA  9
```

Now C is excluded first and A gets the seat.

Is it possible, then, to remove this anomaly without introducing another? The answer, unfortunately, is ‘no’. Woodall[1] proposed that every count under any reasonable electoral system should have the following four properties:

1. Increased support, for a candidate who would otherwise have been elected, should not prevent their election;

2. a. Later preferences should not count against earlier preferences;

   b. Later preferences should not count towards earlier preferences;

3. If no second preferences are expressed, and there is a candidate who has more first-preference votes than any other candidate, that candidate should be elected;

4. If the number of ballots marked X first, Y second plus the number marked Y first, X second is more than half the total number of ballots, then at least one of X and Y should be elected.

He then proved that no such system can be devised.

We have already noted that current STV systems can (but usually do not) fail on Woodall's first property; this is the failure that in Dummett's[2] eyes precludes consideration of STV as a possible option for public elections in the UK. As no system can have all four properties, a price for having one has always to be paid in terms of lacking at least one other. Under the system proposed below, some counts (but by no means all) may fail to have Woodall's first or second property, but all will have the other two. Whether the price is worth paying is a question to which no definitive answer can be given: it is ultimately a matter of personal preference.

STV by successive selection (SS)

The object of exclusion in current STV formulas is to release votes from one candidate to be transferred to others so that one or more of them will get a quota. STV(SS) retains the transfer of votes from candidates who are not yet elected, but differs from present STV systems in that no candidate is permanently withdrawn from contention. When it becomes necessary to release a candidate's votes, that candidate is ‘suspended’ (withdrawn temporarily) after being identified as the one whose election to the next vacant seat would be least appropriate.

Manual STV systems need to keep within reasonable bounds both the time taken to count an election and the scope for human error and this need can give rise to anomalies. Meek[3] and Warren[4] have devised schemes without these anomalies for distributing votes which would be impracticable using manual methods. STV(SS) is designed (but not yet programmed) to be run on a computer using either of these schemes, but only one should be used in any one election.

In addition to Woodall's four properties, every count under a reasonable system would have the property that of a set of *d* or more candidates to which *d* Droop quotas of voters are solidly committed, more than (*d*-1) should be elected; if the set contains fewer than *d* candidates, all of them should be elected. According to Dummett, a group of voters are ‘solidly committed’ to a set of candidates if every voter in the group prefers all candidates within the set to any candidate outside it. STV(SS) and other STV formulas achieve proper representation of sets of candidates by withdrawing from contention candidates who have less than a quota of votes and by transferring surplus votes from those candidates who have more than a quota.

The principle underlying STV(SS)

STV(SS) is predicated on the proposition that when no surpluses remain to be transferred, there is only one candidate (barring ties) who is the most appropriate occupant of the next seat. Appropriateness depends among other things on who has been elected already: if Candidate X is the ‘most appropriate’ and Candidate Y is the ‘next most appropriate’ at any given point, it does not follow when X is elected that Y is now the ‘most appropriate’. The next candidate to be elected is the one who can command a quota and for whose election the other non-elected candidates need to sacrifice the smallest proportion of their votes.

Under STV(SS), each non-elected candidate in turn is tested to see what proportion of the votes of the other non-elected candidates have to be passed on in addition to the surpluses of the elected candidates to give them the quota. Of those who can command a quota, the candidate who requires the smallest proportion of the others' votes is the ‘most appropriate’ to be elected next. The process is best illustrated by an example. Consider the following votes for one seat:

```
A  49
BC 26
CB 25
```

No candidate has a quota, but instead of excluding the lowest we test each candidate in turn to see which is the ‘best buy’. Let us test A first. The quota is 50 and B and C have 51 votes between them; we therefore change their Keep Values (KVs: see the Annex for further details) from 1.0 to 50/51 (0.9804). At the second distribution the votes look like this:

```
A   49.0000
B   25.9708
C   25.0096
```

The new total of votes is 99.9804, making the quota 49.9902. A still has not got the quota, so the count proceeds. The final distribution looks like this:

```
A   49.0000
B   24.8216
C   24.1784
```

At this point, we record the fact that the common KV of B and C is 0.8020. If we now test B, we find that the final common KV of A and C is 0.5152; when we test C the common KV of A and B is 0.5050.

At first sight, A seems the obvious choice to get the seat: however, if A were to be successful, Woodall's fourth property would be lacking. No candidate should be elected who cannot command a Droop quota of the votes which are active at the time of their election. If we remove C from contention (C is ‘least appropriate’ as the other candidates had to give up the greatest proportion of their own votes to secure C's quota) and redistribute C's votes, B now secures a Droop quota and is elected.

But why make the selection on the basis of the other candidates' final KVs? The reason is that these represent the degree of support that exists for the proposition that a given candidate should be added to the set of elected candidates. Suppose that some of the votes in an election were cast as follows:

```
AC   54
BC   45
```

(there may be other candidates and other votes, but these need not concern us) and that it is necessary for 33 of these votes to be passed from A and B to C. This is achieved by setting the common KV of A and B at 0.6667 − A and B have to pass on 0.3333 of the current value of each incoming vote to secure C's quota. But suppose the votes had been

```
ABC 54
BAC 45
```

the other votes and candidates being the same. This time, to give 33 votes to C, the common KV of A and B has to be 0.4226 i.e. 0.5774 of the current value of each incoming vote has to be passed on, over 1.7 times as much. The lower a candidate is in the order of preference of the average vote being considered at any point, the lower the common KV of the other non-elected candidates has to be in order to give that candidate a quota.

How STV(SS) works

STV(SS) has two parts: detailed instructions to the computer are given in the Annex. What follows is a general description and explanation of their functions.

The first part

In the first part, the non-elected candidates are ranked in ‘order of electability’, which forms the basis on which candidates are elected or suspended. All the non-elected candidates are sub-classified at the start as ‘contending’. There are two further sub-classifications, namely ‘under test’ and ‘tested’; only one candidate at a time is under test. The object is to ascertain for the candidate under test what proportion of the votes of the contending and tested candidates it is necessary to pass on to give them the current quota. Each non-elected candidate in turn is classified as under test. If a candidate under test is classified as elected, the first part is repeated.

When the candidate under test and the elected candidate have Q or more votes each, the candidate under test has recorded against their name the common KV of the contending and tested candidates: this is that candidate's ‘electability score’. When all the non-elected candidates have been tested, they are ranked in descending order of electability score: this ranking is for use in the second part. An electability score of 1.0 indicates that the candidate needs to take no votes from other unelected candidates to get the quota, so there is no reason not to classify that candidate as elected at once.

The second part

In the second part, the next candidate to be elected is identified on the basis of their ranking from the first part and their ability to command a Droop quota of votes. The highest candidate in the ranking is elected as soon as it is shown that they can command a Droop quota of currently active votes. If the highest candidate cannot, the second highest non-suspended candidate gets the seat instead. In this part, non-elected candidates are sub-classified as ‘contending’, ‘protected’ (contending candidates become protected when they get a quota) and ‘suspended’; they are all classified as contending at the start. Suspended candidates have a KV of 0.0. At the end of the procedure, all the candidates' KVs are reset at 1.0.

Contending candidates are suspended in reverse order of ranking: protected candidates cannot be suspended before the next candidate is classified elected. The fact that a candidate has a Droop quota of currently active votes now does not necessarily indicate that they will achieve one at a subsequent stage and vice-versa. The rankings obtained in each pass through the first part are crucially dependent on which of the previously contending candidates was elected in the preceding second part.

An example

Let us see how STV(SS) works on the examples on page 1:

Count 1		Count 2	
AC	13	AC	11
BC	8	BC	10
CA	9	CA	9

In Count 1, the ranking is A (the common KV of the other two candidates would be 0.7962), C (0.7143) and B (0.2023), so B is suspended first and C gets the seat. The Count 2 ranking is C (0.7143), A (0.6311) and B (0.2929); B is once more the first to be suspended so C again gets the seat.

Conclusion

As specified above, the system appears to be long-winded: there are possible short-cuts, but these would obscure essentials and have been excluded.

STV(SS) is a logical system which is submitted as a contribution to the continuing debate on what the characteristics of the best possible system might be. Refinements are necessary (for instance, a way of breaking ties has to be devised), but there is here the basis for a debate.

References

1 D R Woodall, *An Impossibility Theorem for Electoral Systems*, Discrete Mathematics 66 (1987) pp 209-211

2 Michael Dummett, *Towards a More Representative Voting System: The Plant Report*, New Left Review (1992) pp 98-113

3 i. B L Meek, *Une nouvelle approche du scrutin transférable*, Mathématiques et Sciences Humaines, No. 25, pp 13-23 (1969)

ii. B L Meek, *Une nouvelle approche du scrutin transférable (fin)*, Mathématiques et Sciences Humaines, No. 29, pp 33-39 (1970).

4 C H E Warren, *Counting in STV Elections*, Voting matters, No. 1, pp 12-13 (March 1994)

Annex

STV(SS) — Detailed Instructions

The first part

1. If there is any candidate for whom no voter has expressed any preference at all, treat every such candidate as having withdrawn. If fewer than (N+1) candidates remain, end the count; otherwise, set the ranking of every remaining candidate to equal first.

2. Classify every non-elected candidate as contending and repeat the following procedure until there are no contending candidates left:

a. Set every candidate's KV at 1.0 and select a contending candidate to be the candidate under test.

b. Examine each ballot-paper in turn and distribute the value of the vote in accordance with the voter's preferences and the KVs of the candidates as follows:

Either

i. *The Meek Formulation*. Offer the vote to each candidate for whom the voter has voted in order of preference expressed. Multiply the fraction of the vote which has not yet been allocated by the KV of the candidate to whom it is being offered, and allocate that proportion of the vote to that candidate. Any part of the vote left over after all the candidates for whom the voter has voted have received their share is non-transferable.

or

ii. *The Warren Formulation*. Offer the vote to each candidate for whom the voter has voted in order of preference expressed. Award to each candidate in turn a fraction of the vote equal to that candidate's KV; if the fraction of the vote remaining is less than the KV of the current candidate, award all that is left to that candidate. Any part of the vote left over after all the candidates for whom the voter has voted have received their share is non-transferable.

c. Calculate the quota according to the formula $Q=V/(N+1)$, where V is the total number of votes credited to all the candidates and N is the number of seats being contested.

d. If the elected candidates and the candidate under test have at least Q votes each, go to Step e. Otherwise, calculate new KVs for all the candidates as follows:

i. For all the elected candidates and the candidate under test, multiply the current KV by Q and divide the result by that candidate's current total of votes.

ii. Multiply the common KV of the contending candidates and the tested candidates by $(V–(E+1)Q)/T$, where E is the number of candidates elected so far and T is the total of the votes credited to the contending and tested candidates.

If any new KV exceeds 1.0, reset it at 1.0. Go to Step b.

e. Record the common KV of the contending and tested candidates against the name of the current candidate under test; let this be that candidate's ‘electability score’. Classify that candidate as tested.

3. If no tested candidate has an electability score of 1.0, rank the tested candidates in their existing order within descending order of electability score and go to Step 5. Otherwise, classify as elected every tested candidate whose

electability score is 1.0.

4. If there are N elected candidates, end the count. Otherwise, go to Step 2.

The second part

5. Classify every non-elected candidate as contending and set every candidate's KV to 1.0. Repeat the following procedure until either the highest-ranked contending or protected candidate and the elected candidates have Q or more votes each, or there are only N non-suspended candidates.

a. Examine each ballot-paper in turn and distribute the value of the vote in accordance with the voter's preferences and the KVs of the candidates as follows:

Either

i. *The Meek Formulation*. Offer the vote to each candidate for whom the voter has voted in order of preference expressed. Multiply the fraction of the vote which has not yet been allocated by the KV of the candidate to whom it is being offered, and allocate that proportion of the vote to that candidate. Any part of the vote left over after all the candidates for whom the voter has voted have received their share is non-transferable.

or

ii. *The Warren Formulation*. Offer the vote to each candidate for whom the voter has voted in order of preference expressed. Award to each candidate in turn a fraction of the vote equal to that candidate's KV; if the fraction of the vote remaining is less than the KV of the current candidate, award all that is left to that candidate. Any part of the vote left over after all the candidates for whom the voter has voted have received their share is non-transferable.

b. Calculate the quota according to the formula $Q=V/(N+1)$, where V is the total number of votes credited to all the candidates and N is the number of seats being contested. Classify any contending candidate with Q or more votes as 'protected'.

c. If any candidate has more than Q votes, calculate a new KV for each such candidate by multiplying their present KV by Q and dividing the result by their present total of votes. Otherwise, suspend the contending candidate who is ranked lowest.

6. Classify as elected the highest-ranked contending or protected candidate.

7. If N candidates are elected, end the count: otherwise, go to Step 2.

Sequential STV

I D Hill

David Hill is a retired statistician. He is a member of Council and Chairman of the Technical Committee of the ERS.

The Meek system for counting an STV election overcomes most of the troubles encountered in using older systems designed for counting by hand, but the problem of premature exclusion remains. Premature exclusion of a candidate occurs when someone is the lowest because hidden behind another who, in the end, is also not going to succeed. If A, who would otherwise have been elected, fails because B stood and was elected instead, it is bad luck for A but there is nothing disturbing about it in principle. If, however, A fails because B stood, but then B does not get in either, that is disturbing.

Exclusion of the lowest candidate, when an exclusion is necessary, is the trouble. After all, if the so-called first past the post is not necessarily the right person to elect, then neither is the last past the post necessarily the right one to exclude. Is there some other way of handling things that would do better? What is needed is a mechanism to discover initially which candidates have some hope of election and which have virtually none, and to get rid of the 'no-hopers' at the start of the count. Others cannot then suffer from their presence.

Let the election be to fill k seats from n candidates, and let $m = n - k$. Sequential STV then consists of a number of main-phases and sub-phases, each being an STV election for k seats but with varying selections of candidates. The STV elections are preferably conducted using Meek-style counting but other rules could be used.

Main-phase 1. All n candidates, but instead of dividing into elected and excluded, divide them into probables and others respectively. Set all n candidates to unmarked.

Sub-phase 1.1. The k probables plus any other one candidate. Set the winners to marked.

Sub-phase 1.2. The same k probables plus any other one candidate not yet tested. Set any unmarked winners to marked.

etc.

Sub-phase 1.*m*. The same k probables plus the last candidate not yet tested. Set any unmarked winners to marked.

If at any sub-phase there is a tie that has to be settled using random selection, then all $k + 1$ of the candidates involved are set to marked.

Main-phase 2. All marked candidates, dividing into probables and others. If the resulting set of probables is the same as a previous set, those candidates are elected and the process finishes. Otherwise reset all *n* candidates to unmarked and continue.

Sub-phases 2.1 - 2.m. As 1.1 - 1.m but using the new probables.

Main-phase 3. As main-phase 2.

etc. etc.

It may be noted that anyone getting a quota of first preferences on the original count is, in fact, certain to be elected in the end, but to be classified for the time being as probable does no harm.

The process must terminate because there is only a finite number of sets of *k* that can be formed from *n*. Usually it will terminate with two successive main-phases showing the same set of *k* probables. In that case the result is firmly established. If, however, the two showing the same set are not successive it will mean that the system is cycling in Condorcet-paradox style. In that case it may be that a better rule could be devised than taking the first set to occur twice but it has to be recognised that a totally satisfactory answer is impossible.

Each candidate is given a fair chance by being tested against each new set of probables and since each sub-phase consists of only *k* + 1 candidates for *k* seats, exclusion is never necessary during the sub-phases so the 'exclude the lowest' rule is not operative there.

Example

With 5 candidates for 2 seats, suppose the votes

```
104 AEBCD
103 BECDA
102 CEDBA
101 DEBCA
  3 EABCD
  3 EBCDA
  3 ECDBA
  3 EDBCA
```

It is evident that E is a strong candidate, in that if any one of A, B, C or D were to withdraw, E would be the first elected. Yet under simple STV the first action is to exclude E, and B and C are elected. Under sequential STV we find

Phase	Candidates	Winners	Probables	Marked
1	ABCDE	BC	BC	
1.1	BCA	BC		BC
1.2	BCD	BC		
1.3	BCE	BE		E
2	BCE	BE	BE	
2.1	BEA	BE		BE
2.2	BEC	BE		
2.3	BED	BE		
3	BE	BE	BE	

B and E are consequently elected. It will be noted that some elections may be repeats of ones already done (main-phase 2 and sub-phase 2.2 in the above example are both repeats of sub-phase 1.3). The result may of course merely be copied down without actually repeating any calculations.

Should it be used?

If any scheme is to be adopted to get rid of (or at least to ease) the problem of premature exclusion, I believe that this is about as good as can be devised. Yet, after much consideration, I do not recommend it for general use, because it breaks the rule, which simple STV always obeys, that a voter's later preferences ought not to interfere with that voter's earlier preferences.

The following example to demonstrate this trouble is derived from those that Douglas Woodall devised to prove his 'impossibility' theorem. Let there be 3 candidates for 1 seat and votes

```
25 A
17 BC
16 C
```

Phase	Candidates	Winner	Probable	Marked
1	ABC	A	A	
1.1	AB	A		A
1.2	AC	C		C
2	AC	C	C	
2.1	CA	C		C
2.2	CB	B		B
3	BC	B	B	
3.1	BA	A		A
3.2	BC	B		B
4	AB	A	A	

So A is elected. But if the A voters had put in C as a second preference, we get

```
25 AC
17 BC
16 C
```

Phase	Candidates	Winner	Probable	Marked
1	ABC	A	A	
1.1	AB	A		A
1.2	AC	C		C
2	AC	C	C	
2.1	CA	C		C
2.2	CB	C		
3	C	C	C	

and C is elected. So the A voters have failed to elect A because they gave C as a second preference.

Even if this is a rare event, it still means that we cannot assure voters that their later preferences cannot upset their earlier preferences. I believe that this is too high a price to pay. There is not much point in reducing the frequency of one type of fault if, in doing so, you introduce another fault as bad.

Only one seat

The system is really intended, as is STV in general, for situations where there are several seats to be filled, but it can also be used in place of Alternative Vote for a single seat. Trying it out on many examples suggests that, for realistic voting patterns, it is almost certain to elect the Condorcet winner if there is one, but artificial examples can be devised to demonstrate that there is no guarantee that it will do so.

For example, let there be 4 candidates for 1 seat and votes

```
98 ADCB
98 CDBA
99 BDAC
 3 ACBD
 2 CBAD
```

Phase	Candidates	Winner	Probable	Marked
1	ABCD	A	A	
1.1	AB	B		B
1.2	AC	A		A
1.3	AD	D		D
2	ABD	B	B	
2.1	BA	B		B
2.2	BC	C		C
2.3	BD	D		D
3	BCD	C	C	
3.1	CA	A		A
3.2	CB	C		C
3.3	CD	D		D
4	ACD	A	A	

So A is elected, even though D would be the Condorcet winner (for the results of AD, BD and CD are all D). It should be emphasised, though, that this is not likely in practice but only with carefully devised artificial examples.

Acknowledgement

I acknowledge that, since I first produced this scheme, Dr David Chapman has produced an almost identical scheme entirely independently.

Two STV Elections

B A Wichmann

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I believe two STV elections may be of interest to the readers of *Voting matters*, due to the implications of the results on the properties that an ideal STV algorithm should (perhaps) have.

The first election is the Eurovision Song contest for 1992 which is an interesting election to analyse since the votes are publicly available, in spite of the voters not knowing of the other votes. Each country votes for the songs of other countries by awarding 12, 10, 8, 7, 6, 5, 4, 3, 2 and 1 points, which can be transcribed into STV preferences.

The points system gave for those over 100: Ireland (155), UK (139), Malta (123), and Italy (111). Since the points total is given after each country has voted, the commentator (Terry Wogan) reported that Ireland was unbeatable by the UK before the last few countries voted. An analysis of the votes by other means is quite different.

The ERS hand counting rules declare the UK as the winner, as does the Meek STV algorithm. However, more countries preferred Ireland to the UK than the contrary (by 12 to 11, rather close). Indeed, by the Condorcet rules, Ireland would be the winner, since Ireland is preferred to any other country by a majority. The reason that the ERS rules elect the UK is that Ireland is eliminated earlier, leaving the last contest between Malta and the UK, which the UK wins. The Meek algorithm is similar, but with Italy being the last to be eliminated.

One STV algorithm due to Tideman considers all possible pairs of results. In the case of a single seat, Tideman will elect a Condorcet winner (assuming there is one) and hence chooses Ireland in this case. One is therefore left to wonder if an ‘ideal’ STV algorithm should always elect a Condorcet winner, assuming there is one.

The second election is one for which I acted as returning officer for a rather unusual ‘election’ at my place of work.

The research institute at which I work has had a library for a group of about 60 scientists for at least 30 years. As the research has changed over the years, new journals have been ordered. However, except in obvious cases, it has not been clear which journals should be cancelled — especially since a complete ‘run’ of a journal will be lost. I therefore proposed that an STV election be run to determine which journals should be cancelled and which new ones to order.

The management agreed to this proposal and hence I ran the election as follows: A list was obtained of the (about) 200 journals, which were assigned a code. The scientists were asked to place up to 40 journals in preferential order, being given about a month to place their ballot.

Quite a bit of effort is necessary to fill in the ballot paper. Nobody attempted more than the 40 preferences, the average being about 20. About half of those eligible voted, which I thought was quite reasonable, since quite a few would have no direct use for the library.

The ballot revealed that 4 journals were in the library but not on the list provided. Eight journals were written in by electors which were not in the library.

The analysis of the results proved very interesting. With 31 people voting for a total of 198 journals, the quota is a lot less than 1. This implies that about the first six preferences would be selected for any reasonable number of journals. However, there was not a fixed number of ‘seats’, and hence I had to decide what threshold to set. Due to the difficulty for the electors, I did not interpret the ballot papers according the usual ERS rules. In one case in which one preference was unclear, I omitted that preference but did not ignore subsequent preferences. In two other cases in which a journal was selected twice, I merely ignored the second choice.

An initial analysis showed that 27 Journals did not appear in any position on the ballot papers. This gave an instant selection of journals to cancel. I ran the ballot with the option to cancel 10 and 20 further journals.

I have several STV algorithms available on my home computer which I used to compute the result. I had decided in advance that I would use the Meek algorithm for the election, but the other versions could be used to see what difference it made.

The first problem was that the programs I had, required a trivial modification to handle as many as 200 ‘candidates’. After having made that modification, it was found that the programs would not work on my PC because the full results over-filled my floppy discs! A further modification was needed to output only the final table and a summary of the eliminations and elections.

The three versions of STV were:

1) The Meek algorithm, as published in the Computer Journal (1987, Vol 30, p277)

2) The ERS hand-counting rules (as programmed by David Hill)

3) The Tideman algorithm, as approximated by my program.

The ERS results were quite unacceptable which shows that the hand-counting rules do not seem to have been used upon such an election. The problem is that if the election is run with the same number of seats as those selected in any preference, the algorithm does not select just those selected by the electors! This problem can be expected of any algorithm that does not see subsequent preferences.

The other two algorithms produced virtually identical results. With the reduction to 20 fewer journals than those selected, one difference was found between Meek and Tideman. A manual inspection of the results with the two journals in question, showed no clear distinction.

After producing the result, I computed for each of the 31 ballots, the way in which the final stage of the ballot had divided up the vote. This information was given to each elector. It created further interest in the STV algorithm. Those who had given more preferences had, in general, a lower non-transferable loss. However, the variations were very large. For instance, a person would gave the largest number of preferences (36) had a small loss, while a person would gave 15 preferences had no non-transferable loss.

I conclude from this election that STV can be used for such selections, but that the ERS hand-counting rules are not appropriate. Also, any STV algorithm approved by ERS in future should not suffer from this noted defect. Namely, if only N candidates are represented in the preferences and N is the number of seats, then the algorithm should elect those N. This requirement does not seem to lead to additional problems. It appears that the STV algorithms which recompute the quota can satisfy this requirement, since in the particular circumstances the entire ballot papers are then processed.

An STV Database

B A Wichmann

Since we know that no single algorithm for STV can have all the properties one might like, it appears that some statistical analysis may be needed to select an optimal algorithm. People do not vote at random and therefore any effective analysis must take into account voting patterns. For instance, if voters always voted strictly along party lines, proportional representation among such parties would be an important factor.

Collections of ballot papers from real elections would be useful for any practical analysis. There is a *de facto* standard for the representation of ballot papers in a computer, being the form used by the Meek algorithm. Hence collection of such data is practical and useful. Both David Hill, Nicholas Tideman and myself had such collections, accumulated informally over several years. I have now put this collection into a consistent framework so that the material can be provided to anybody who would like it — merely post a floppy disc to me, and I can return the disc with this data.

The data available has been classified in a number of ways as follows:

Real: Data here is that from real elections, with the possible exception that a statistical sample of the total ballot papers would be acceptable. The reason for this is that it presents a means of providing ‘real’ data without providing the total information. There are potential dangers in analysis of real data, since an alternative algorithm could elect a different person, giving rise to concerns about the election itself, rather than the principles involved. Another reason for accepting a subset of all the votes is that this is all that may be feasible for a large election. Obviously, this data is provided in a form which precludes the identification of the election involved. There are currently 46 data sets in this class.

Mock: This is data from genuine elections, except that no position or office is at stake. Mock elections are often used to educate people into the principle of STV. There are currently 2 sets in this class.

Semi: Elections in this class are not genuine elections, but are clearly related to real elections. Examples in this class are ‘ballot’ papers derived from published STV elections (from Northern Ireland), elections from the Eurovision Song Contest and elections in which there was no fixed number of ‘seats’. There are currently 21 data sets in this class.

Test: Data in this class are not derived from any election but have been constructed to demonstrate the difference between some algorithms, show a bug in a computer algorithm, or some similar purpose. There are currently 129 in this class.

I would very much welcome additional data, especially from real elections in which some ‘party’ aspect is involved. The data can be provided in a form in which the origin cannot be traced. I have analysed an Irish election to produce a single data set in the **Semi** class, but this is very time consuming and has to make a number of assumptions to produce anything like the actual ballot papers. Hence real data is much superior.

Is a feedback method of calculating the quota really necessary?

R J C Fennell

Robin Fennell is a retired radar technician with the RAF and more recently, a Customs Officer. He has been a member of ERS since an abortive attempt to introduce STV into his union elections. He is currently active in transport and defence campaigning as well as electoral reform.

The March issue of *Voting matters* reprinted papers by B L Meek, [1,2] D R Woodall [3] and C H E Warren.[4] In this paper I will propose that their feedback method of calculating the quota is not necessary. To do this I will consider some of the basic principles of the Single Transferable Vote (STV) system.

One problem identified[5] is that if a candidate is elected any further preferences for that candidate are passed over. The question to be considered is ‘are elected candidates continuing in the election or should they be considered as no longer available to receive votes’?

In other words is the purpose of a vote in the Single Transferable Vote system to try to elect candidates in the order the voter wishes or to place candidates in popularity order and have this order respected whatever the outcome of the rounds of the count? I suggest that it is the former. Once a candidate has been elected he has achieved the aim of participation in the election and, henceforth should take no further part in the election. Under these circumstances the manual counting method is satisfactory.

We will take a voting paper that shows preferences A,B,C,D and assume that B was elected on the first round. The transfer of B's surplus elects A on the second round. The question now arises on our paper, should the transfer of A's surplus go to B or C. Let us assume that our voter had future vision when deciding the preferences and knew that B would be elected in the first round; would our voter put B as the second preference? I suggest that anyone so gifted would select the preferences A,C,D thus maximising the transfers to the candidates they wished to see elected. Of course this foresight is not available to voters so to cover all possibilities the voter will elect to keep to the original selection knowing that the counting system will not waste any part of a vote by transferring it to a candidate already elected.

In practice few voters would take the risk of excluding a candidate on the grounds that they are certain to be elected. If too many did then B would not be elected. Voters can be expected to behave in a rational fashion and vote for the candidates of their choice in the order they wish. When a candidate has been elected they have achieved the aim of both the candidate and the voter. The voter will now wish any surplus votes to be concentrated on the unelected choices.

Another problem identified by Meek[6] is how to treat unmarked candidates. He suggests that they should be considered either as being of equal merit, or that the voter wishes to leave the ordering of these candidates to others. Meek ignores the third possibility that the voter does not wish these candidates to have any part of the vote. The omission of the third alternative in Meek's paper is possibly due to the voting instructions that take a form similar to ‘place the candidates in order until you can no longer differentiate between them’. If the instructions were changed to a form similar to ‘place the candidates in order until you no longer wish the remaining candidates to have your vote’ it would be clear how the voter required unmarked candidates to be treated. Under these circumstances the manual counting method is satisfactory.

If STV is to be used in local or parliamentary elections many voters will only want to vote for their particular party. They will not wish any proportion of their vote to go to candidates of a party with an opposing view to theirs. If votes are apportioned to all non-selected candidates, voters will have no way of ensuring that they do not vote for candidates of a party whose policies they cannot agree with.

The other problem foreseen by Meek[7] that I will consider is the possibility of voters indicating the same preference for two or more candidates. He suggests that this should be allowed and the counting system modified to accommodate it. The Electoral Reform Society (ERS) supports the Single Transferable Vote system, not the Transferable Multi-vote of Unity Value System. This second system may exist but it is not that supported by the Society and therefore should not be considered. The Single Transferable Vote system requires voters to cast a single vote, all or part of which may be transferred. That a multiple vote may have unitary value is irrelevant, it is a single vote which must be utilised.

D R Woodall[8] raises a different problem, that of the tactical voter. He postulates a situation where there are several Sensible Party candidates, say A,B,C and one Silly Party candidate, W. The tactical voter decides that W will be excluded and in order to maximise the transfer of votes after the first round he will vote W,A,B,C rather than A,B,C which is the real preference. The problem for the tactical voter comes when several voters take the same line. Assume in this election that the quota is 200. If 201 voters vote tactically and put W first then W will be elected reducing the vacancies available for Sensible Party candidates. In these circumstances the tactical voters will be as silly as W's party. The only way to avoid this is to place preferences in the order the voter wishes the candidates to be elected and not to attempt to vote tactically.

One of the main advantages of STV is that attempts to vote tactically are likely to end in a result that will not suit the tactical voter. The situation above could happen irrespective of the number of candidates or the size of the quota. The only safe way for voters to use their vote successfully is to vote according to preference.

The three works printed in the March issue of *Voting matters* may be mathematically rigorous but are they required? My contention is that if the basic principles of the Single Transferable Vote system are carefully considered then the feedback method of counting is unnecessary. The manual method used to date is satisfactory to ensure the correct result.

There is one further matter to be considered. If the feedback method is to be used, the constant recalculations necessary will require computers to be used. It is recognised in the papers supporting the method that it is too laborious to use hand counting. While the ERS has voted to use both computer and manual counting for its internal elections, I doubt if a system which cannot reasonably be counted by hand will be accepted by the general public. Computers are quick but they rely on the integrity of their programming. Computer technology is not yet at a state where incorrect programming, whether by accident or intent, will always be exposed. While it is not possible to say that the currently accepted Newland/Britton hand counting rules will always produce the correct result, they will produce a satisfactory result. I can see no reason to change the current system of counting.

References

1. B L Meek, A New Approach to the Single Transferable Vote, Paper I, *Voting matters*, March 1994.

2. B L Meek, A New Approach to the Single Transferable Vote, Paper II, *Voting matters*, March 1994.

3. D R Woodall, Computer Counting in STV Elections, *Voting matters*, March 1994.

4. C H E Warren, Counting in STV Elections, *Voting matters*, March 1994.

5. As 1, section 3, item (iv).

6. As 2, section 3, penultimate paragraph.

7. As 2, section 6.8. As 3, second paragraph.

8. As 3, second paragraph.

Issue 3, December 1994

Editorial

In this issue we have a mixture of papers. There is a continuing debate about revisions to the ERS rules, which arose from Fennell's paper in the last issue.

Hill and I, in separate papers, consider the effect of small changes — steadiness or stability. Global properties and local properties are the topic of Woodall's paper which I hope could be used as a basis for terminology and analysis in further issues of *Voting matters*.

It would be nice to automate all suggested algorithms for STV and compare them against a library of test cases. Unfortunately, the effort involved often precludes this which means that choices are being made on less than perfect information (not unlike elections themselves).

Brian Wichmann.

Comparing the stability of two STV algorithms

B A Wichmann

Brian Wichmann is the Editor of *Voting matters* and a visiting professor of The Open University

The problem of stability

This note does not consider the usual properties of STV algorithms that have been the subject of Woodall's analysis, but that of *stability*. For a mechanical system modelled by continuous variables, the analysis of stability is an application of differential calculus. We cannot use such an approach with STV algorithms since the system is discrete, and we know that some small changes are bound to produce a discrete change in those elected.

For an STV algorithm, we could have too much stability in that part of the ballot papers are simply ignored — for instance, by only using the first preference. On the other hand, we could have an algorithm which lacks stability in vital respects by changing the result for inconsequential changes to a ballot paper.

One change made to a ballot paper can be regarded as *small*, due to the nature of the preferential system. Since the usual means of balloting does not provide for the voter to give equal preference, when the ballot paper records ABC, this might be because A and B were regarded as equal, but the voter specified A first arbitrarily. Hence the voter could equally have written BAC instead. Hence given the ballot ABC, the voter's true intentions could perhaps have been expressed as BAC or ACB. In general, given n preferences, $n-1$ ballot papers constructed by interchanging neighbouring preferences could be regarded as *small* differences.

Now consider two algorithms for STV which have broadly similar properties (as do all serious contenders). Figures 1 and 2 represent graphically these two algorithms.

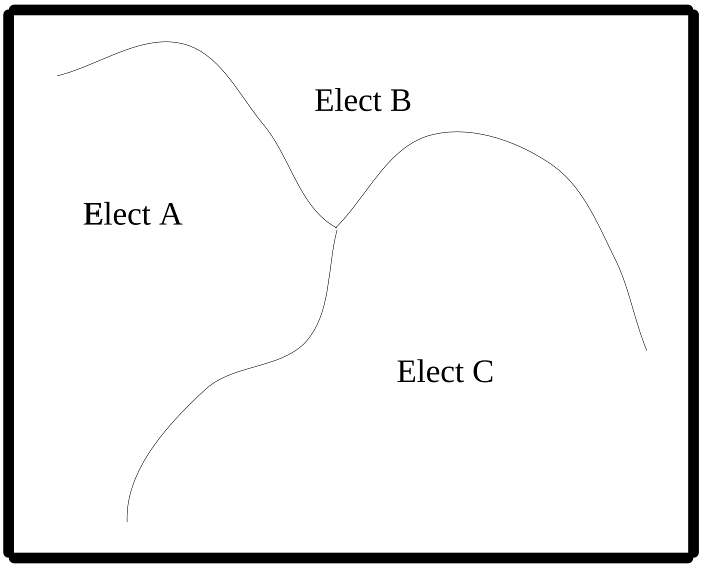

Figure 1

Figure 1 represents a stable algorithm since small changes are unlikely to change the result of an election, while Figure 2 represents an unstable algorithm. If we were operating in two dimensions, then the property of stability could be measured rather like the game of shove-halfpenny: one would measure the probability that a small circle placed at random on the figure crossed one of the dividing lines.

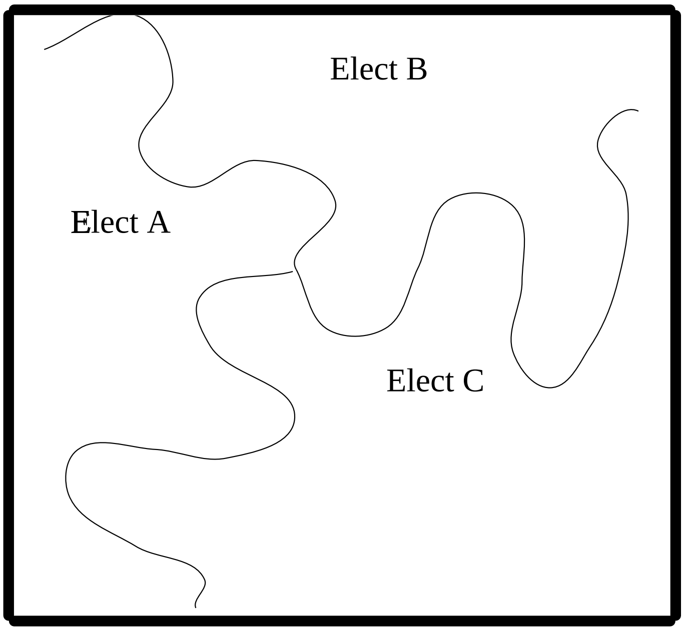

Figure 2

In the case of STV algorithms, we do not have a simple two-dimensional system, and hence the figures are a crude diagrammatic representation. To measure the probabilities we must conduct a suitably controlled experiment. Fortunately, we can use a computer to aid this process so that we can perform the equivalent of shove-halfpenny sufficiently often to obtain results which are likely to be meaningful.

The experimental method

We now specify the experimental method to compare the ERS hand counting rules versus the Meek algorithm. (Any two algorithms could be chosen, but this seems the most interesting pair.)

We select an actual election for which the ballot papers are available. We also choose a number, about 20, which is the number of ballot papers from the full set that is to be selected at random. (We return to the choice of this number n later.)

From each real election, we derive 100 mini-elections by randomly selecting n ballot papers. The experimental method is to analyse the effect of making small changes to these mini-elections. The analysis is as follows. Firstly, we compute the result of the two algorithms from the mini-election. (The result need not be the same for the two algorithms, nor the same as for the full election.) We now consider all the possible similar mini-elections derived by making one small change to one of the ballot papers. (This is potentially hundreds of elections — hence the computer.) This particular mini-election is on the edge if a specific criterion is met, say at least one of the small changes produces a different result.

The choice of n is important. If n is very small (say 1), then it is clear that the mini-election will not be representative of the real election. On the other hand, if n is large (say the full election), then the computation of the ‘edge’ becomes too large, and also the number of possible mini-elections becomes too small (in this case only 1). Care must be taken over the specific criterion for being on the edge. If one takes something like the ERS council elections (i.e., several posts to fill with no parties, so that small changes are likely to make a difference to the outcome), with the criterion that *any* small change resulting in a difference implies being on the edge, then there is a danger that *all* mini-elections are on the edge!

For the 100 random mini-elections we perform a different analysis in each of the three experiments given here. If one could assume statistical independence, then it would be a simple matter to undertake a χ^2 test to see if the result is significant. Unfortunately, we do not have elections with a large enough number of ballot papers to ensure the independence, and therefore we must be content with a non-statistical treatment.

The programs and test data

Two programs have been written, one for using the ERS algorithm and the other the Meek version. Apart from the STV algorithm in use, both work in an identical fashion. They read 100 mini-elections in the conventional format. Firstly, the result is computed for this election, then every possible small change is made, and for each such change, the number of changes to those elected is recorded.

The number of changes to those elected for one small change is usually 0 (no change), but is sometimes 1, rarely 2 and very rarely 3. Hence for each ballot paper in the mini-election, $n-1$ integers are output, representing the number of changes arising from each of the $n-1$ possible interchanges of adjacent preferences, where n is the number of preferences marked on the ballot paper. This implies that the output is of similar length to the input — an important consideration, since if complete results were printed for each election result computed, hundreds of pages of material would be produced.

The analysis is most easily seen by considering an example. A mini-election from election R038 is as follows:

```
 17 5
1 11 9 10 0
1 10 17 5 9 11 16 0
1 6 16 2 1 14 17 10 9 11 5 8 4 12 13 15 0
1 4 8 12 15 13 0
1 17 5 11 1 16 10 2 0
1 5 9 10 11 17 0
1 3 7 9 14 17 0
...
```

```
1 8 10 13 11 17 0
1 9 5 6 17 0
1 11 10 5 17 9 0
1 6 4 15 14 16 8 1 0
1 6 14 16 1 2 4 13 12 8 15 0
1 13 4 15 12 8 0
0
"A.1 " "B.2 " "C.3 " "D.4 "
"E.5 " "F.6 " "G.7 " "H.8 "
"I.9 " "J.10 " "K.11 " "L.12 "
"M.13 " "N.14 " "O.15 " "P.16 "
"Q.17 "
"1R038: H3H "
```

The above data is for an election with 17 candidates for 5 seats, in which the first ballot paper selects candidate 11 (K) as the first preference, then 9 and lastly 10. The names of the candidates are the letters A-Q, a convention used throughout.

The program computes the effect of making all possible interchanges of adjacent preferences, which for Meek gives:

```
v1 +F-L-B-O-P-H-G-N-E-M-C+I+Q-K+J+A-D 68
0 1 1 1
2 1 1 1 1 1 1 0 0
2 1 1 0 0 1
0 0 1 1
...
2 1 1 1
0 1 0 1
1 0 0 1
1 0 0 0 0 0
2 1 0 1
1 1 0 2 0 0 1 1 0 1 0 0 1 1
1 1 0 1 1
0 1 m
```

The first line gives the result (with Meek) for this mini-election, where `+F-L` means F is elected and L is excluded, etc. (The v1 and 68 are not relevant.) Then, starting with the last ballot paper and working back towards the beginning, the number of differences to the result is printed for each possible interchange. Hence the last ballot paper has four possible interchanges, the first one giving no difference, but the last three each making a single difference. So in this case, interchanging the first two preferences makes no difference, but interchanging the 2nd and 3rd preferences does change the result by one candidate. The 'm' relates to the third experiment and is explained later.

One other program is needed which selects *n* ballot papers at random from a real election, and repeats this 100 times. This program is fast and straightforward.

For the main election data, six real elections have been chosen from the data already available (see *Voting matters*, Issue 2). The statistics from these elections are as follows:

Identifier	Papers	Candidates	Seats	*n*
R006	239	9	2	20
R008	261	10	3	25
R010	270	9	5	27
R017	479	8	1	15
R033	196	14	7	25
R038	177	17	5	20

Unfortunately, none of the elections in the data base are from elections involving parties, and so such elections could not be selected for this study.

We can now summarise the results obtained by example. For election R017, 100 mini-elections are computed by selecting 15 ballot papers from the actual 479. For each of these mini-elections, we compute what difference (if any) would be made by a single transposition of a preference. This is repeated for each possible transposition, which in this case, involves the analysis of 4585 elections!

Experiment 1

We now consider the issue raised initially — that of the 'size' of the edge dividing the line between different election results. We therefore need to devise a criterion for being on the 'edge', and compare the results for the six elections with the two algorithms.

Criterion: Some change for any transposition

Election	ERS edge	Meek edge
R006	74	65
R008	80	74
R010	95	87
R017	69	74
R033	99	95
R038	100	100

This table means, for instance, that for the 100 mini-elections derived from R006, 74 are on the 'edge' for ERS and 65 for Meek — which implies that there were 26 or 35 elections for which no change was made by any transpositions. Hence a very high proportion of the mini-elections are on the 'edge', over three quarters in almost all cases. However, even the most optimistic assumption shows that there is not much difference between the two algorithms.

We now change the criterion for being on the edge so that a lower proportion are on the edge.

Criterion: More than three transpositions make a change

Election	ERS edge	Meek edge
R006	41	38
R008	55	46
R010	76	57
R017	32	49
R033	91	75
R038	94	91

We again conclude that there is not much difference between the two algorithms.

We need to look at aspects other than the actual size of the edge to see significant differences.

Experiment 2

In this experiment, conducted with the programs and data as before, we look at properties of the edges rather than their actual magnitude.

Given a mini-election which is on the edge, then we know at least some transpositions of the preferences will change the result. It is therefore natural to ask which specific transpositions can change the result. Clearly, it is more likely that transposing the first two preferences will alter the result, but what about the subsequent transpositions? We therefore analyse the number of times a transposition makes a change, against the position of the transposition (p_i).

Combined results

	p1	p2	p3	p4	p5	p6	p7	p8	p9	p10	p11	p12	
R006													
ERS	283	31	14	6	4	0...							
Meek	310	147	61	39	23	16	14	0...					
R008													
ERS	452	56	11	8	5	4	0	1	3	0...			
Meek	393	161	70	42	25	13	12	11	0...				
R010													
ERS	668	173	36	21	9	4	2	0...					
Meek	423	174	82	34	21	18	13	0...					
R017													
ERS	214	27	4	5	2	1	0...						
Meek	279	210	123	119	104	94	0...						
R033													
ERS	979	227	78	31	17	8	3	2	1	0	0	1	
Meek	1876	392	225	144	117	91	69	61	57	41	41	34	37
R038													
ERS	734	203	44	31	17	6	3	2	0	1	1	0	
Meek	723	502	376	346	157	138	107	97	91	44	36	33	

In the table above, for each of the six elections, the number of times a transposition makes (at least) one change to the result is tabulated against the preference position for all the 100 mini-elections. The difference between ERS and Meek is now obvious. The number of changes for the first preference between the two algorithms is similar and is surely not significant. However, in all subsequent preferences, many more changes arise from Meek than from ERS.

In examining the subsequent preferences, there is no natural scale to work to, since a change in preference *n* is more significant if there are *n* candidates than 2*n* candidates. The number of seats is also relevant to this scale. Hence in analysing the table above, both the number of candidates and seats must be considered.

We can add up the results from each election for those positions beyond the number of seats (*s*) for each election, giving the following results:

	Position			
	s +1	*s*+2	*s*+3	>*s*+3
ERS	61	21	15	11
Meek	530	364	293	596
ratio	8.7	17	19	54

Hence we conclude that transposing preferences beyond the number of seats has virtually no effect with ERS as compared with Meek.

Experiment 3

In a paper in this issue of *Voting matters*, Woodall defines the property mono-raise. For the elections analysed by the experiments undertaken here, we can determine the extent to which a weaker property than mono-raise is violated. Since our analysis determines the effect of a single interchange in the preferences, given a preference pair A,B which is replaced by B,A, the raising of the order of B should not disadvantage B. This implies that if the election with A,B elects B, then that with B,A should also elect B. If this condition is not satisfied, then mono-raise is violated, and is marked by 'm' in the output files.

We can now compare the violation rate for ERS and Meek, which is as follows:

Election	ERS violations	Meek violations
R006	0	32
R008	2	29
R010	5	8
R017	5	78
R033	5	70
R038	0	141

Hence there is no question that Meek violates mono-raise much more than ERS. This is likely to be due to the increased sensitivity of Meek to the effects of late preferences.

Conclusions

The analysis undertaken in this paper has led to the following conclusions:

1. There is no evidence that the ERS and Meek algorithms are any different with respect to the size of the boundary between the election of different candidates.

2. Making small changes by transposing preferences later than the number of seats makes virtually no difference with ERS but a substantial difference with the Meek algorithm.

3. Meek violates mono-raise much more than ERS.

Point 2 indicates that the Meek algorithm is much more sensitive to the voter's wishes than ERS, and moreover this sensitivity is not at the expense of making the algorithm less stable. However, the fact that Meek violates mono-raise so much more than ERS might question the extra sensitivity of Meek. It would appear that an ideal algorithm would have the sensitivity of Meek, but would only violate mono-raise with the same frequency as ERS. I suspect that it is actually the extra sensitivity of Meek that gives rise to the mono-raise violations, so that the best of Meek and ERS is not possible.

It appears that the results presented here have some limitations. Firstly, the mini-elections necessarily have a small number of ballot papers and so the results need not apply to larger elections. Secondly, a consequence of the small number of ballot papers is that in many cases, random choices are made by both the ERS and Meek algorithms.

The comparative steadiness test of electoral methods

I D Hill

David Hill is a regular contributor to *Voting matters.*

In comparing one electoral method with another it is useful to examine their comparative steadiness. It should be noted that it is only a comparative test and does not give a "goodness" score for any individual method on its own but only for one method relative to another. Nor does the fact that any method comes out as the better of the two by this test indicate that it is necessarily better in any other way.

To use it, first run each method for the same number of seats and the same given set of votes and see whether they both elect the same candidates. If they do, this test is not applicable. Otherwise, see whether there is one or more candidate whom neither method elects. If there is no such candidate, again the test is not applicable. In particular, the test can never be applicable if the number of candidates is only 1 greater than the number of seats, but the fact that it is often not applicable does not destroy its value in those cases where it does apply.

If the test is applicable, then treat all candidates who failed to be elected by either method as withdrawn, and re-run each method. If each method continues to elect the same candidates as before, then there is nothing to choose between them on this test for this particular set of votes. If, however, one method makes no change in whom it elects while the other makes a change, then the no-change method gains a point in comparison with the other.

For example, if there are 5 candidates for 3 seats, and the votes are:

```
51 ABC
44 ABD
 5 EABD
```

the current ERS rules will elect A, B and D whereas the Meek rules will elect A, B and C. They agree that E is not elected, so the comparative steadiness test treats E as having withdrawn and re-runs the election. Now the Meek rules still elect A, B and C, but the ERS rules switch to electing A, B and C too. Meek therefore shows greater steadiness for this particular set of votes.

While such artificial elections are important as illustrations, what most matters is which rules are steadier for real elections. Taking the 57 real elections that I have available, I find the test to be applicable for only 10 of them. In 4 of those, these two systems are both steady, neither changing its result when the relevant candidates are withdrawn. In the other 6, however, the Meek system remains steady but the ERS system changes. By this test, the Meek system seems to be superior, so far as the evidence goes, though a few more results in the same direction would help to make more certain that the difference is not just a chance effect.

It should be noted, of course, that discovering a lack of steadiness must not be used to change the result of a real election, which must always be in accordance with the rules as laid down for that election. The test is only for research purposes, not to interfere with a result.

Editorial Note: It is possible to apply the steadiness test even when an election gives the same result. This can be done by selecting random ballot papers from the election in the manner of the mini-elections in the previous paper. With the 100 mini-elections from the real election R006, 17 of these elect different candidates so that the steadiness test can be applied. Of these 17, none were steady for the ERS rules, while 13 were steady according to Meek. One mini-election could not be considered since a random choice was made. For the remaining 3 mini-elections, neither were steady, and in one case, the removal of the no-hope candidates causes the two algorithms to interchange the results!

Response to the paper by R J C Fennell

P Dean

Peter Dean is a Trustee of the ERS Ballot Services.

I was surprised to see the existing manual system defended by R J C Fennel as being beyond reproach.

The basic flaw in the manual method is that it allows for the election of candidates receiving less than the quota. This has led to Tasmania requiring at least 7 preferences, combined with a rotated ballot paper since 1973. Even in our own elections some are elected with 4 fewer votes when there are 8 non-transferable votes.

There is a refinement which could easily be introduced in manual elections. This is that when the stage is reached when some candidate(s) fail to reach the quota, a recount takes place with only those remaining taking part. This means that votes previously wasted on candidates with no chance can then influence the result by being allotted to a lower preference for a candidate previously elected. The result will then be demonstrably fair. Taking an actual mock election in the Solent area in 1989 as an example, in which there were 20 candidates for 5 seats. The manual result gave a lead of 4.88 to the last elected — although short of the quota. The Meek system elected the runner-up instead by a margin of 2.01 votes. If a further 5 counts has been added the manual system would have come to a similar result, but by an even larger margin of 7.42. The new result is demonstrably fair — with the last candidate having 2.53 over the quota.

Sometimes the unfair result is even obvious to the public. Such a case occurred in Cork East in 1954. The two Fine Gael candidates received 153 more votes than the two Fianna Fail candidates (1162 non-transferable) yet Fianna Fail won 2 of the 3 seats. Such results discredit the whole system.

The current mechanised system is quite unsuitable for small elections. For instance, with 9 votes and 18 candidates for 3 seats it proceeds to eliminate 7 candidates with 1 vote completely at random. It is quite clear that a different order of exclusion would give a different result. Personally I favour a points method based upon the preferences expressed which would give some form of ranking order to be used instead of the random method.

Are better STV rules worthwhile?

A reply to R J C Fennell

I D Hill

David Hill is the Chairman of the ERS Technical Committee.

R J C Fennell's article, in *Voting matters* issue 2, raises a number of matters that deserve reply.

Taking a voting paper naming ABCD in order of preference, where B has been elected on the first count and A is elected on the second count as a result of transfers from B, he asks whether that paper's surplus should go to B or to C. He appears to have failed to notice that, in the current ERS rules, it is totally immaterial whether the vote is taken as if it were ACD instead of ABCD, because that paper's surplus does not go anywhere. The voter's second and subsequent preferences are completely ignored, the whole paper remains with A, while only the new votes that A has received are redistributed.

Let us look instead at the point that he was trying to make. Suppose, in that same election, that C was also elected on the first count, and we have a paper naming BACD. That paper will pass from B to A and will be further redistributed, at a suitable value. Should it go to the next choice C, or jump over C straight to D as currently happens? He suggests that such a voter with future vision would not have put C into the list, so it is right to jump to D. But all voters ought to be treated alike, and therefore, if we are to treat one as if future vision existed, we must do so to all others too, and most voters would wish to change their votes if they knew what was going to happen; nobody would vote for the runner-up, of course. But such a change would make sense only if nobody else changes; if we treat everybody as though allowing them to change, the assumed future vision would collapse, no individual could then know how to change and the whole system would become wildly unstable. There is only one satisfactory way out, and that is to treat each vote in strict accordance with what it says, and not by what we assume that it might have said if only the voter had known what would happen.

Transferring to a candidate who has already been elected, as in Meek-style STV, does not waste votes, as is suggested, because the same size surplus is passed on in any case. The change is only to whether the surplus is taken fairly, from all relevant groups in proportion to their current totals of votes, or unfairly in some other way. To change the example, suppose that there are 100 AC votes and 10 BAD votes in a situation where the quota is 77. A is elected on the first count giving 77 to stay with A for quota, 23 to be transferred to C.

If, later on, B is excluded then in current ERS rules the 10 all pass to D. The Meek alternative is that only 3 pass to D, while 7 more of the original ACs pass to C. The two methods are

```
ERS    100 votes    77 stay with A    23 go to C
further 10 votes                      10 go to D

Meek   100 votes    70 stay with A    30 go to C
further 10 votes     7 stay with A     3 go to D
```

In either case 77 have been kept and 33 redistributed, but I do not see how anyone could claim that the first method is satisfactory if we are able to operate the second. The article suggests that the voters ‘will wish any surplus votes to be concentrated on the unelected choices’. That is to say that the BAD voters would like the first alternative. Of course they would; that is not in dispute. But it is not fair to the AC voters to allow it.

The next point addressed by Fennell is the treatment of ‘short lists’, that is to say votes that would be transferred if they had a next choice, but do not show one. He mentions the two possible treatments discussed in detail by Meek, but says that Meek's papers ignore a third possibility, and it is evident that he is thinking of something like the current ERS rule. He is wrong to say that Meek ignored this; his paper said ‘If the difficulty were to be avoided by increasing the proportion transferred of votes for which a next preference is marked, to enable all *x* votes to be retained by C, this would clearly reintroduce inequities of the kind Principle 2 was designed to eliminate’. I agree with Meek that this possibility does not deserve any more discussion than that, but many people have failed to see that this method is wrong in principle, and a far greater quantity of writing has gone into it in the last few years than can be reproduced here. I can well see that people might take the wrong decision on this at a first quick glance, but the number who continue to do so even after thought and discussion is quite extraordinary.

I disagree with Meek that the voters should be given the choice between the two methods he discusses in detail. This would have to mean explaining to them the different effects of each, a task that I would not wish on anyone. Meek points out that the two can give different results; usually they do not but, in the few cases we know of where they do, to give the relevant surplus to ‘non-transferable’ and reduce the quota to compensate is always the preferable option.

Fennell suggests that these voters may not wish other candidates to have any part of the vote. I agree with that — indeed I insist that, whatever those voters wish, we have no right to assume what their wishes are, but only to obey what their ballot papers say, namely that if they become entitled to a further choice they wish to abstain from making one. It is true that, in the current ERS (and most other) rules, the ballot papers are not physically transferred to any other candidate, but what matters is not what is done with pieces of paper, but the effect of the rule. Consider the simple case, with 4 candidates for 2 seats, and votes

```
40 AB
17 CD
 3 DC
```

The quota, in current ERS rules, is 20. So 20 votes go to A and A's surplus of 20 goes to B, and A and B are elected, but the situation is ‘on a knife-edge’ for, if D were to withdraw before the count, A and C would be elected. Now with a knife-edge situation any relevant change in one direction must settle the matter, and it is certainly a relevant change if half B's support is lost, to give

```
20 AB
20 A
17 CD
 3 DC
```

Yet the current ERS rules take no notice whatever, but still give 20 to A and 20 to B.

It is sometimes argued that if the AB voters had had pre-vision, they would have gone straight to B but, as argued above, we cannot allow that without allowing pre-vision to other voters too. Given pre-vision, the DC voters would have voted for C. Given pre-vision, the A plumpers need not have bothered to vote at all. The only fair thing to do is to take what the ballot papers actually say, and everything in proportion to the numbers involved. That means that half A's surplus must go to B, and half to non-transferable, which gives C the second seat.

Provided that the quota can be changed to allow for the non-transferables, as in Meek's method, it can be shown that this does not waste any extra votes at all. What one method wastes in non-transferable, the other wastes on leaving more votes with elected candidates than they now need to be sure of election. With hand-counting methods, where true quota-reduction is not practicable, it could be the case that the present rule does more good than harm, but I know of no evidence to support such a view.

Turning to the discussion of whether voters should be allowed to express equality of preference if they wish, rather than a strict ordering, I cannot agree that it ought not to be considered. It is undoubtedly the case that the absence of this feature is regarded by many as a major disadvantage of STV. There are some difficulties of implementing it, and it would complicate the instructions to voters. I believe that it is something that we ought to consider introducing one day, but that there are more important things to be done first.

Fennell then discusses the ‘Silly Party candidate’ method of tactical voting discussed in Woodall's paper. He is right, of course, that trying to utilise it may be to the voter's disadvantage if a wrong guess is made, and would certainly be troublesome if too many voters tried to do it. It is a bad feature though that it should be possible at all. Furthermore it is not necessary for there to be a silly party candidate or tactical voters for the effect to occur. It always happens to those voters who put as first preference the first candidate to be excluded, no matter how sincere their choice, causing a distortion that should be avoided if possible.

Fennell is correct that it is not practicable, save for very small elections, to use methods such as those proposed without doing the count by computer but, in this electronic age, can that really be an adequate reason for putting up with second-best results? He queries whether computer-generated results would be trusted and this is certainly something to which attention has to be given. There are two distinct ways in which things might go wrong. The first is in the input of the data from the ballot papers, but this could be subject to repetition if a recount is requested.

The second possibility of error is in the program to calculate the results but, in a public election it could be arranged that, once the data input has been agreed as correct, each candidate would be given a copy of the data on floppy disc. Each party would have its own program, each independently written from the rules specified in the Act of Parliament, and its own computer near at hand. Within a few minutes each could have checked that the official result is agreed. Such a system would lead to much greater protection against errors than anything that could be done with hand-counted STV.

The article concluded that ‘the currently accepted Newland/Britton hand counting rules ... will produce a satisfactory result. I can see no reason to change the current system of counting’. What is meant by a satisfactory result? In comparing the results of real elections by hand-counted rules and by Meek rules the result is different more often than not, and all the indications are that the result is not merely different but better, in more accurately representing the voters' wishes. Now that the ability exists to do something better than can be done by hand, it would be absurd to try to exist in the past. Does it not matter to the Electoral Reform Society (or others) whether we get the best result or not?

Properties of Preferential Election Rules

D R Woodall

Douglas Woodall is Reader in Pure Mathematics at Nottingham University. In this article, he argues that more attention should be paid to properties of electoral systems, and less to procedures. He lists many properties that a preferential election rule may or may not have, and discusses them with reference to STV.

1. Introduction

I have often been struck—and never more than in the last year—by how much the types of argument used by the supporters of the Single Transferable Vote (STV) differ from those used by its opponents. When it comes to the details of the count, the supporters of STV almost invariably try to defend its procedures directly, on the grounds that they follow certain principles, or that they do with each vote exactly what the voter would want done with it, if the voter were able to be present at the count and to express an opinion. Unfortunately, there is no guarantee that adopting sensible procedures, at each stage of the count, will lead to a system with sensible properties, and the opponents of STV often emphasize its less desirable properties. In particular, it is now well known that STV is not monotonic: that is, that increased support, for a candidate who would otherwise have been elected, can prevent that candidate from being elected. It was ostensibly because of this and related anomalies that the Plant Report rejected STV.

Properties of electoral systems can be thought of as "performance indicators", and like any other performance indicators they need to be used with care. If one chooses a set of performance indicators in advance, it may well be possible to manufacture a high score on those indicators in an artificial way, which does not represent good performance in any real sense. Nevertheless, it seems to me that the Electoral Reform Society needs to pay more attention to properties if it is not to be sidelined in the electoral debate. In particular, since different desirable properties often turn out to be mutually incompatible, it is important to discover which sets of properties can hold simultaneously in an electoral system. Only then will it be possible to decide whether there are electoral systems that retain what is essential in STV while avoiding some of the pitfalls.

The purpose of this article is to introduce a long list of technical properties that an election rule may or may not have, to invent snappy descriptive names for them all, and to discuss them with special reference to STV. Except where otherwise indicated, statements made about STV apply equally well to the Newland-Britton and Meek versions of

STV. In a later article I hope to address the question of monotonicity in more detail.

2. Notation and terminology

As is usual in the Social Choice literature, I shall use lower-case letters *a, b, c*,... to denote candidates (or choices). Each voter casts a *ballot* containing a *preference listing* of the candidates, which is written as (for example) *abc*, to denote that the voter places *a* first, *b* second and *c* third, with no fourth choice being expressed. A preference listing is *complete* if all candidates are included in it and *truncated* if some are left out. (A preference listing that leaves out just one candidate will be treated by most election rules, including STV, as if it were complete; but one should not call it complete, since some election rules may not treat it as such.) A *profile* is a set of preference listings, such as might represent the ballots cast in an election. Profiles may be represented in either of the forms shown for Elections 1 and 2 below, indicating either the proportion, or the absolute number, of ballots of each type cast.

The term *outcome* will be used in the sense of "possible outcome" (assuming there are no ties). Thus in an election to fill two seats from four candidates *a, b, c, d*, there are six outcomes, corresponding to the six possible ways of choosing the two candidates to be elected: {*a*, *b*}, {*a*, *c*}, {*a*, *d*}, {*b*, *c*}, {*b*, *d*} and {*c*, *d*}.

```
  Election 1        Election 2
   (1 seat)          (2 seats)
ab   0.17         a   9    ea   4
ac   0.16         b   9    eb   4
bac  0.33         c  10    fc   1
cb   0.34         d  10    fd   1
                           fe   6
```

An *election rule* is usually thought of as a method that, given a profile, chooses a corresponding outcome—or, in the event of a tie, chooses two or more outcomes, one of which must then be selected in some other way (such as by tossing a coin). However, this description is not quite adequate to deal with the complexities of ties. Consider Election 1 above, with the votes counted by STV (or, rather, by the Alternative Vote (AV), which is the rule to which STV reduces in a single-seat election). No candidate reaches the quota of 0.5, and there is an initial tie for exclusion between *a* and *b*. If *b* is excluded then *a* is immediately elected, whereas if *a* is excluded then *b* and *c* tie for election. Thus *a* is elected with probability ½, and *b* and *c* are elected with probability ¼ each.

A similar situation arises in Election 2, again under STV. There are 54 votes cast, so the quota is 18, and there is an initial tie for exclusion between *e* and *f*. If *e* is excluded then *f*, *c* and *d* must also be excluded, and *a* and *b* are elected; whereas, if *f* is excluded, then *a* and *b* must also be excluded, and then *e* is elected and *c* and *d* tie for second place. Thus the outcome {*a*, *b*} is chosen with probability ½, and the outcomes {*c*, *e*} and {*d*, *e*} are chosen with probability ¼ each.

Because of examples like these, I define a (preferential) *election rule* to be a procedure that, given a profile, associates a corresponding non-negative probability with each outcome, in such a way that the probabilities associated with all possible outcomes add up to 1. The "normal" situation is that all the outcomes are given probability 0 except for one, which has probability 1 (meaning that that outcome is chosen unequivocally). If anything else happens, then we say that the result is a *tie* between all the outcomes that have non-zero probability.

3. Axioms

There are so many properties that an election rule may have, that it is useful to categorize them in some way. Four in particular seem sufficiently basic to deserve to be called *axioms*. The first is more or less implicit in the above definition of an election rule; but it has a name, and so for completeness I include it here.

Anonymity. The result should depend only on the number of ballots of each possible type in the profile (and not, for example, on the order in which they are cast, or on extraneous information such as the heights of the candidates).

Neutrality. If some permutation is applied to the names of all the candidates on all the ballots in the profile, then the same permutation should be applied to the result. For example, since STV is neutral, if *a* is replaced by *c* and *c* by *a* on every ballot in Election 2 above, then STV would choose {*b*, *c*} with probability ½ and {*a*, *e*} and {*d*, *e*} with probability ¼ each. One consequence of neutrality is that a tie in a single-seat election cannot be resolved simply by electing the first in alphabetical order among the tied candidates.

A rule that is both anonymous and neutral is called *symmetric*.

Homogeneity. The result should depend only on the *proportion* of ballots of each possible type. In particular, if every ballot is replicated the same number of times, then the result should not change. It is this property that enables us to describe profiles as in Election 1 above, showing the proportion, rather than the absolute number, of ballots of each type cast.

Discrimination. If a particular profile P_0 gives rise to a tie, then it should be possible to find a profile P that does not give rise to a tie and in which the proportion of ballots of each type differs from its value in P_0 by an arbitrarily small amount. This rules out, for example, the following method of electing one candidate from three: elect the candidate who beats both of the others in pairwise comparisons, if there is such a candidate, and otherwise declare the result a three-way tie. For in that case, not only would the profile in Election 3 below give rise to a tie, but anything at all close to it would also give a tie, contrary to the axiom of discrimination.

```
                abc  1/3
Election 3:     bca  1/3
(1 seat)        cab  1/3
```

A *proper* election rule is one that satisfies the above four axioms; that is, one that is anonymous, neutral, homogeneous and discriminating. The term "axiom" is used rather freely in the literature as a synonym for "property", but I shall restrict its use to these four, which I regard as genuinely axiomatic, in the sense that I am not interested in any rule that does not satisfy them.

A word of warning is needed about homogeneity. In any practical election where the count is carried out by computer, there will be a limit to the number of decimal places that the computer can hold accurately. Thus there are bound to be situations in which two numbers that are not really equal are regarded as equal by the computer program, because they become equal when rounded to the appropriate number of decimal places. In this case, if every ballot were replicated the same, sufficiently large, number of times, then the difference between the two numbers of votes would become significant, and the computer might give a different result. However, this is a minor problem, introduced by the practical need to round numbers; the axiom of homogeneity should be applied to the underlying theoretical rule, with no rounding.

With this interpretation, STV is a proper election rule.

4. Global or absolute properties

It is convenient to divide properties into *global* or *absolute* properties on the one hand, and *local* or *relative* properties on the other. The former say something about the result of applying an election rule to a single profile, whereas the latter say something about how the result should (or should not) change when certain changes are made to the profile. Not all properties fall unambiguously into one of these two classes, but sufficiently many do for the distinction to be useful.

The most important single property of STV is what I call the *Droop proportionality criterion* or *DPC*. Recall that if v votes are cast in an election to fill s seats, then the quantity $v/(s + 1)$ is called the *Droop quota*.

> DPC. If, for some whole numbers k and m satisfying $0 < k \leq m$, more than k Droop quotas of voters put the same m candidates (not necessarily in the same order) as the top m candidates in their preference listings, then at least k of those m candidates should be elected. (In the event of a tie, this should be interpreted as saying that every outcome that is chosen with non-zero probability should include at least k of these m candidates.)

In statements of properties, the word "should" indicates that the property says that something should happen, not necessarily that I personally agree. However, in this case I certainly do: DPC seems to me to be a *sine qua non* for a fair election rule. I suggest that any system that satisfies DPC deserves to be called a *quota-preferential* system and to be regarded as a system of proportional representation (within each constituency)—an STV-lookalike. Conversely, I assume that no member of the Electoral Reform Society will be satisfied with anything that does not satisfy DPC.

The property to which DPC reduces in a single-seat election should hold (as a consequence of DPC) even in a multi-seat election, and it deserves a special name.

> Majority. If more than half the voters put the same set of candidates (not necessarily in the same order) at the top of their preference listings, then at least one of those candidates should be elected.

The following rather weak property was formulated with single-seat elections in mind, but it makes sense also for multi-seat elections and, again, it clearly holds for STV.

> Plurality. If some candidate a has strictly fewer votes in total than some other candidate b has first-preference votes, then a should not have greater probability than b of being elected.

The next property has been suggested to me by Brian Wichmann in the light of his experiences reported in the last issue of *Voting matters*[6].

> No-support. A candidate who receives no support at all (that is, who is not listed by any voters in their preference listings) should not be elected unless every candidate who receives some support is also elected.

This is not satisfied by STV with the Newland-Britton rules. For example, if x receives no support at all, and the only support that y receives is on ballots marked *ay*, where a reaches the quota as a result of transfers from other candidates, then x and y will both be recorded throughout as having no votes (since the *ay* ballots are not re-examined when a reaches the quota), and so y is as likely to be excluded as x. It seems that no-support is satisfied by Meek's version of STV, although I do not have a formal proof of this.

The remaining three global properties consist of *Condorcet's principle*, which was proposed by M. J. A. N. Caritat, Marquis de Condorcet (1743-1794), and two modern strengthenings of it. We say that a voter, ballot or preference listing *prefers a* to *b* if he, she or it lists a above (before) b, or lists a but not b. Let $p(a, b)$ denote the number of voters who prefer a to b. We say that *a beats b* (in pairwise comparisons) if $p(a, b) > p(b, a)$; that is, if the number of voters who prefer *a* to *b* is greater than the number who prefer *b* to *a*. We say that *a ties with b* (in pairwise comparisons) if $p(a, b) = p(b, a)$. A *Condorcet winner* is a candidate who beats every other candidate in pairwise comparisons. A *Condorcet non-loser* is a candidate who beats or ties with every other candidate in pairwise comparisons; note that if there is more than one Condorcet non-loser then all the Condorcet non-losers must tie with each other.

Note that there need not be a Condorcet winner, or even a Condorcet non-loser. In the profile shown in Election 3 above, *a* beats *b*, *b* beats *c* and *c* beats *a*, all by the same margin of 2/3 to 1/3. This is the so-called *Condorcet paradox* or *paradox of voting*: even though each voter provides a linear ordering of the candidates, the result when the votes are totalled can be a cyclical ordering. The *Condorcet top tier* is the smallest nonempty set of candidates such that every candidate in that set beats every candidate (if any) outside that set. In Election 3, the Condorcet top tier consists of all three candidates. If there is a Condorcet winner, then the Condorcet top tier consists just of the Condorcet winner. If there is a Condorcet non-loser, then the Condorcet top tier contains all the Condorcet non-losers, but it may possibly contain other candidates as well.

Condorcet's principle and the two strengthenings of it given below were formulated originally for single-seat elections in which every voter provides a complete preference listing; but I have reworded them here so that they make sense (although they are not necessarily sensible) for all preferential elections.

Condorcet[1]. If there is a Condorcet winner, then the Condorcet winner should be elected.

Smith-Condorcet[4]. At least one candidate from the Condorcet top tier should be elected.

Exclusive-Condorcet (see Fishburn[2]). If there is a Condorcet non-loser, then at least one Condorcet non-loser should be elected.

Note that Smith-Condorcet and exclusive-Condorcet both imply Condorcet, and Smith-Condorcet also implies majority. Smith-Condorcet seems a very natural extension of Condorcet. Exclusive-Condorcet is also very natural, but it is of much less importance since it differs from Condorcet only when there is a "tie" for first place under pairwise comparisons, and that will not happen very often.

```
Election 4        Election 5
(1 seat)          (2 seats)
abc      0.30     ad      0.36
bac      0.25     bd      0.34
cab      0.15     cd      0.30
cba      0.30
```

STV does not satisfy Condorcet, and so it certainly does not satisfy either of the above two extensions of it. This can be seen in Election 4 above. Under STV (AV), *b* is excluded and *a* is elected. However, *b* is the Condorcet winner, beating both *a* and *c* by the same margin of 0.55 to 0.45. This example highlights a fundamental difference in philosophy between STV and Condorcet-based rules. Loosely speaking, STV tries to keep votes near the tops of the ballots. Thus the preferences of the *cba* voters for *b* over *a* will not even be considered under STV until *c* is excluded, which means that in this example they are not considered at all, since *b* is excluded before *c*. In contrast, Condorcet's principle requires that, right from the outset, the preferences of the *cba* voters for *b* over *a* should be given equal weight with the similar preferences of the *bac* voters. However, despite this difference in philosophy, Condorcet and majority are not actually incompatible in single-seat elections: if one wishes, one can use AV (or any other system of one's choice) to select a candidate from the Condorcet top tier. Any such rule clearly satisfies Smith-Condorcet, and hence satisfies both majority and Condorcet, although it is a moot point whether it is really any better than AV on its own. In multi-seat elections, Condorcet is undesirable, in my opinion, because it is incompatible with DPC, as shown by Election 5 above. Here the quota is $0.3\dot{3}$, and so DPC requires that *a* and *b* should be elected, whereas *d* is the Condorcet winner.

5. Local or relative properties: monotonicity

Local or relative properties are concerned with what happens when a profile is changed in some way. We shall say that a candidate is *helped* or *harmed* by a change in the profile if the result is, respectively, to increase or to decrease the probability of that candidate being elected.

As we saw in Election 4, under STV the later preferences on a ballot are not even considered until the fates of all candidates of earlier preference have been decided. Thus a voter can be certain that adding extra preferences to his or her preference listing can neither help nor harm any candidate already listed. Supporters of STV usually regard this as a very important property, although it has to be said that not everyone agrees; the property has been described (by Michael Dummett, in a letter to Robert Newland) as "quite unreasonable", and (by an anonymous referee) as "unpalatable". There are really two properties here, which we can state as follows.

Later-no-help. Adding a later preference to a ballot should not help any candidate already listed.

Later-no-harm. Adding a later preference to a ballot should not harm any candidate already listed.

We come now to the different versions of monotonicity. The basic theme is that a candidate *x* should not be harmed by a change in the profile that appears to give more support to *x*; but one gets different flavours of monotonicity if one specifies different ways in which the profile might be changed.

Monotonicity. A candidate *x* should not be harmed if:

(mono-raise) *x* is raised on some ballots without changing the orders of the other candidates;

(mono-raise-delete) *x* is raised on some ballots and all candidates now below *x* on those ballots are deleted from them;

(mono-raise-random) *x* is raised on some ballots and the positions now below *x* on those ballots are filled (or left vacant) in any way that results in a valid ballot;

(mono-append) *x* is added at the end of some ballots that did not previously contain *x*;

(mono-sub-plump) some ballots that do not have *x* top are replaced by ballots that have *x* top with no second choice;

(mono-sub-top) some ballots that do not have *x* top are replaced by ballots that have *x* top (and are otherwise arbitrary);

(mono-add-plump) further ballots are added that have *x* top with no second choice;

(mono-add-top) further ballots are added that have *x* top (and are otherwise arbitrary);

(mono-remove-bottom) some ballots are removed, all of which have *x* bottom, below all other candidates.

There is also the following property, which is not strictly a form of monotonicity but is very close to it. It is an extension to multi-seat elections of a property proposed by Moulin[3] for single-seat elections.

Participation. The addition of a further ballot should not, for any positive whole number *k*, reduce the probability that at least one candidate is elected out of the first *k* candidates listed on that ballot.

These properties are not all independent. For example,

mono-raise-random implies both mono-raise and mono-raise-delete;

mono-raise and later-no-help together imply mono-raise-delete;

mono-raise-delete and later-no-harm together imply mono-raise-random;

mono-sub-top implies mono-sub-plump;

mono-sub-plump and later-no-harm together imply mono-sub-top;

mono-append and mono-raise-delete together imply mono-sub-plump;

mono-append and mono-raise-random together imply mono-sub-top;

mono-add-top implies mono-add-plump;

mono-add-plump and later-no-harm together imply mono-add-top;

participation implies mono-add-top.

Moreover, in single-seat elections,

participation implies mono-remove-bottom.

Also, if truncated preference listings are not allowed, then mono-raise-random implies mono-sub-top.

```
                        ab     10
  Election 6:           bca     8
    (1 seat)            ca      7
```

STV satisfies mono-append but none of the other properties, although in single-seat elections AV satisfies mono-add-plump and mono-add-top. To see that AV does not satisfy mono-raise, mono-raise-delete, mono-raise-random, mono-sub-plump, mono-sub-top or mono-remove-bottom, consider its effect in Election 6 above. As it stands, *c* is excluded and *a* is elected. But if two of the *bca* ballots are removed, or replaced by *a* or by *abc* or by anything else starting with *a*, then *b* is excluded and *c* is elected instead of *a*.

```
Election 7        Election 8
(2 seats)         (2 seats)
ab       30       ac       207
ac       90       bd       198
bd       59       bdac      12
cb       51       cd       105
d        70       dc       105
```

To see that STV does not satisfy mono-add-plump or mono-add-top, consider Election 7. The quota is 300/3 = 100, so that *a* is elected with a surplus of 20. This is divided 5 to *b*, 15 to *c*, and so *b* has 64 votes to *c*'s 66, *b* is excluded, and *d* is elected. Suppose now that we add a further 24 ballots with *d* top. The quota is now 324/3 = 108, so that *a*'s surplus is now only 12. This is divided 3 to *b*, 9 to *c*, and so *b* has 62 votes to *c*'s 60, *c* is excluded, and *b* is elected instead of *d*.

Although all the monotonicity properties look attractive, I do not think that mono-remove-bottom is desirable in multi-seat elections. Consider Election 8. The quota is 627/3 = 209, and so DPC requires that we elect *b* and either *c* or *d*. It seems to me that {*b*, *c*} is clearly the better result (although STV gives {*b*, *d*}). But if we now remove the 12 *bdac* ballots, then the quota drops to 205, so that we must elect *a* and either *c* or *d*. It seems to me that now {*a*, *d*} is the better result (although STV gives {*a*, *c*}). Thus the removal of the 12 ballots that have *c* bottom *should*, in my opinion, harm *c*.

All the monotonicity properties seem desirable in single-seat elections. However, I proved[7] that no rule simultaneously satisfies mono-sub-plump, later-no-help, later-no-harm, majority and plurality. Since I do not think anyone would seriously consider a rule that did not satisfy both majority and plurality, this shows that in order to have mono-sub-plump one must sacrifice either later-no-help or later-no-harm (or both). Whether or not this is desirable may depend on what other properties one can gain at the same time.

Mono-raise-random, mono-sub-top and participation are very strong properties, and it is possible that they are incompatible with DPC. If one could find a reasonable-looking "STV-lookalike" rule that satisfied all the other monotonicity properties (except for mono-remove-bottom when there is more than one seat), then I personally might well prefer it to STV itself. But we are a long way from finding such a rule at the moment.

While on the subject of monotonicity, I should mention one other monotonicity property, if only to dismiss it immediately.

House-monotonicity. No candidate should be harmed by an increase in the number of seats to be filled, with no change to the profile.

This seems to me to be plain wrong. Consider the profile in Election 5, for example, which is a very slight modification of one suggested to me by David Hill. If one were using this profile to fill a single seat, then clearly *d* should be elected (although that is not the result achieved by AV). But if this same profile were used to fill three seats, then clearly *a*, *b* and *c* should be elected; thus *d* is harmed by the increase in the number of seats.

Another property that is related to monotonicity is known in the literature as *consistency*[8] or *reinforcement*[3], but I prefer to call it by its mathematical name:

Convexity. If the voters are divided into two districts and the ballots from each district are processed separately and the results in the two districts are the same, then processing the ballots of all voters together should give the same result.

		(a)	(b)	(a)+(b)
	ab	6	3	9
Election 9:	bc	4	4	8
(1 seat)	cb	3	6	9

STV does not satisfy convexity. Again, I cannot do better than to quote an example of David Hill's (Election 9). In district (a), *c* is excluded and *b* is elected. In district (b), *a* is excluded and *b* is elected. But when the ballots from the two districts are processed together, *b* is excluded and *c* is elected.

Convexity is one of the best-understood of all properties. Young[8] proved that a symmetric preferential election rule for single-seat elections satisfies convexity if and only if it is equivalent to a point scoring rule (in which one gives each candidate so many points for every voter who puts them first, so many for every voter who puts them second, and so on, and elects the candidate with the largest number of points). Since no point scoring rule can possibly satisfy DPC, it follows that convexity and DPC are mutually incompatible. This is a pity, because convexity implies several of the monotonicity properties; but, sadly, it is of no use to us.

Of course, the absence of convexity will hardly ever be noticed in practice, since elections are not counted both in separate districts and together as a whole. But it is worrying inasmuch as it may suggest that something odd is going on.

6. Further properties

A question that is sometimes asked about STV is, is a truncated preference listing treated as if all the remaining candidates were placed equal last? Since STV (in its usual formulation) does not allow for equality of preference, the question does not really make sense. But one can make sense of it as follows. The *symmetric completion* of a truncated preference listing is obtained by taking all possible completions of it with equal weight, chosen so that the total weight is 1. For example, suppose that there are five candidates, *a, b, c, d, e*. Then

the symmetric completion of a ballot marked *abcd* is a single ballot marked *abcde*, with weight 1;

the symmetric completion of a ballot marked *abc* consists of two ballots, each with weight ½, one marked *abcde* and the other marked *abced*;

the symmetric completion of a ballot marked *ab* consists of six ballots, each with weight 1/6, completed in the six different possible ways: that is, *abcde, abced, abdce, abdec, abecd* and *abedc*;

the symmetric completion of a ballot marked *a* consists of 24 ballots, each with weight 1/24, completed in the 24 different possible ways; and so on.

Symmetric-completion. A truncated preference listing should be treated in the same way as its symmetric completion.

It is not difficult to see that AV satisfies symmetric-completion. Although AV is usually described in terms of a quota, it can alternatively be described as follows: repeatedly exclude the candidate with the smallest number of votes, until there is only one candidate left. The effect of replacing truncated preference listings by their symmetric completions is simply that, at each stage in the count, the votes of all non-excluded candidates are increased by the same amount. It follows that the order of exclusions is not affected, nor therefore is the eventual winner.

```
                a       60
Election 10:    ab      60
(2 seats)       b       14
                c       46
```

To see that STV does not satisfy symmetric-completion in general, consider Election 10. The quota is 180/3 = 60, so that *a* is elected with a surplus of 60. Under the Newland-Britton rules, the whole of *a*'s surplus goes to *b*, who is elected. Under Meek's method, the transfer of *a*'s surplus ends with the quota reduced to (180 − 36)/3 = 48, with 36 non-transferable votes going to ‘excess’, and 36 votes transferred to *b*. Either way, *a* and *b* are elected. However, if each ballot is replaced by its symmetric completion, then, of *a*'s surplus of 60 votes, 45 go to *b* and 15 to *c*, and *c* is elected instead of *b*.

```
Election 11     Election 12
(2 seat)        (3 seats)
ab      40      ab      40
ba       2      ba       2
cd      12      cd      12
dc       6      dc       6
                e      180
```

David Hill has sent me an example, which I have modified slightly above, to show that quota reduction is preferable to symmetric completion in STV. In Election 11 the quota is 60/3 = 20, and so *a* and *b* are elected. In Election 12 the quota is 240/4 = 60, so that *e* is elected with a surplus of 120. Under symmetric completion, this would be used to increase the votes of the remaining candidates by 30 each, so that *a* would be elected first, after which *d* would be excluded and *c* would be elected. However, if the quota is reduced to 20 after the election of *e* then *a* and *b* are elected as in Election 11. To paraphrase David's comments slightly, "Election 12 has one extra candidate, one extra seat, and a large number of extra voters whose sole wish (apparently) is to put that extra candidate into that extra seat. It is nonsense that the original 60 voters should get *a* and *c* elected in Election 12 instead of the *a* and *b* they would have got from Election 11."

The remaining properties are all concerned with the avoidance of "wrecking candidates". A "wrecking candidate" is a candidate who is not elected but who, by standing for election and so "splitting the vote", prevents someone else from being elected. One might naïvely hope to avoid wrecking candidates altogether, which would result in the Independence of Irrelevant Alternatives, or IIA:

IIA. If a candidate *x* is not elected, then the result of the election should be as if *x* had not stood for election.

However, it is easy to see that no discriminating election rule can satisfy both IIA and majority. For, consider Election 3 above. By the axiom of discrimination, there must be a profile arbitrarily close to this one that does not give rise to a tie. If this profile results in the election of *a*, then *b* is a wrecking candidate: for, if *b* had not stood for election, then *c* would have been elected (by majority, since roughly two thirds of the voters prefer *c* to *a*); thus the result of the election is not as if *b* had not stood. In a similar way, if *b* is elected then *c* is a wrecking candidate, and if *c* is elected then *a* is a wrecking candidate.

In an attempt to find a property weaker than IIA but expressing a similar idea, I came up with the following.

Weak-IIA. If *x* is elected, and one adds a new candidate *y* ahead of *x* on some of the ballots on which *x* was first preference (and nowhere else), then either *x* or *y* should be elected.

Unfortunately I do not know of any sensible election rule that satisfies even this. Certainly STV does not. For example, if there are 15 ballots marked *x* and 14 marked *z*, then AV (and any sensible rule) will elect *x*; but if 10 of the 15 *x* ballots are now changed to read *yx*, then AV will exclude *x* and elect *z* instead.

An alternative weakening of IIA has been proposed by Tideman[5]. In his terminology, a number of candidates form a *set of clones* if every preference listing that contains one of them contains all of them, in consecutive positions (but not necessarily always in the same order). He says that a single-seat election rule is *independent of clones* if it satisfies the following properties, which I have reformulated here so that they make sense for multi-seat elections as well.

Clone-in. The expected number of candidates elected from any given set of clones should not increase if one member of the set is deleted from every ballot containing it.

Clone-no-help. Replacing a candidate *x* by a set of clones should not help any other candidate *y*.

Clone-no-harm. Replacing a candidate *x* by a set of clones should not harm any other candidate *y*.

```
                xx'a    13
                x'xa    11
Election 13:    abc     10
(2 seats)       bc      12
                c       14
```

It is not difficult to see that AV satisfies all the clone properties. I am fairly sure that STV also satisfies clone-in in multi-seat elections, although I do not have a formal proof of this. To see that STV does not satisfy the other two clone properties, consider Election 13. The quota is 60/3 = 20. Nobody having reached the quota, *a* is excluded and *b* is elected; then *x*' is excluded and *x* is elected. However, if the clones *x* and *x*' are replaced by a single candidate *x*, then *x* has 24 votes initially and so is elected, and the surplus of 4 votes goes to *a*; therefore *b* is excluded, and *c* is elected instead of *b*. So replacing *x* by a pair of clones helps *b* and harms *c*.

Clone-no-harm is actually incompatible with DPC. To see this, note that if only two candidates stand in a 2-seat election, where the voting is (say) *x* 70, *y* 30, then both must be elected. But if *x* is replaced by a pair of clones and the voting is now *xx*' 35, *x'x* 35, *y* 30, then DPC requires that *x* and *x*' should both be elected. This suggests that clone-no-harm is not a desirable property for multi-seat elections—and Tideman never suggested that it was. But clone-in and clone-no-help both look sensible to me, even for multi-seat elections.

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Issue 4, August 1995

Editorial

Readers will have noticed that there has been a significant delay in the appearance of this issue. The reason is very simple — a lack of sufficient material. Also, in reading this issue, you will see many familiar names amongst the authors. The conclusion is that we need a wider base for the authorship than we have currently. Hence could I ask all readers to ensure that friends with similar interests subscribe to *Voting matters*?

In the last paper in this issue, Douglas Woodall uses barycentric coordinates to present the analysis of election results with three candidates. Unfortunately, this elegant method of presentation is regarded by the media as too complex for general use. In consequence, in the recent three-way by-election, the comparison between the previous general election and the by-election was hard to understand. Perhaps this is an advantage to the three party managers who could all claim a 'victory'.

Brian Wichmann.

Progressive Elimination

P Dean

Peter Dean is a Trustee of ERS Ballot Services

In my previous article [Issue 3, page 6] I took the Solent mock election of 1989 to show that electing 5 candidates from a field of 20 gave a different result to choosing 5 from the last 6.

It occurred to me that a computer used in a progressive elimination (19 from 20, then 18 from 19 and so on) could give a different result. Dr Hill proved this to be the case, though he did not favour this method.

Whereas all systems elected candidate Nos 1, 7, 9 and 18; the normal manual method elected No 2, but electing 5 from the last 6 preferred No 20. The progressive elimination finally elected No 19 with No 14 as the runner-up. An examination of the first 5 preferences on each ballot paper revealed that No 19 came 2nd (60), No 20 - 6th (45), No 14 - 7th (37), and No 2 - 8th (34).

This demonstrates that a candidate with considerable secondary support can easily lose out in such an election. No 19 was originally 9th to be eliminated, and No 14 was 13th to go out, being less than a vote behind his running mate - No 2.

Taking only the top 8 based on the first 5 preferences produced that same result as the progressive elimination process.

Meek and monotonicity

I D Hill

David Hill is Chairman of the ERS Technical Committee

In *Voting matters* issue 3, B A Wichmann reported that, using data sampled from real voting patterns, 'Meek violates mono-raise much more than ERS'. Is this something that Meek supporters should worry about?

We know: (1) that all electoral systems have to suffer from some anomaly or other; (2) that STV's anomaly is that it can fail on monotonicity i.e. a change of vote in a candidate's favour can cause that candidate's defeat; (3) that traditional rules do not even look at a voter's second or subsequent preferences if the first preference is elected later than the first count. So the way to make Meek run into an anomaly where traditional rules do not is to find a case where monotonicity trouble occurs among the preferences that such rules ignore.

Although the numbers of such violations reported are indeed considerably greater for Meek, it should be remembered that these arise from examining many thousands of pseudo-elections, and the proportions of occasions are small. For example, the greatest number of Meek violations found was 141 from a data set called R038, but that number comes from 12421 comparisons of one pseudo-election with another. Furthermore each of these pseudo-elections has only 20 voters, which is very few for electing to 5 places from 17 candidates. So the degree of trouble should not be exaggerated, but nevertheless 141 Meek violations were found and no ERS violations in comparisons derived from that particular data set.

It should be borne in mind that the method used to form these pseudo-elections from any given data set involved sampling each time from the same set of votes and thus there are many repetitions, of particular votes being used more than once. This makes it difficult to judge what the results would be from truly independent samples.

I have examined one case in detail to see what it shows and, to avoid all bias in choosing which case to examine, I decided to take the first one found in the data sets available to me. This involved 14 candidates (A − N) for 7 seats, and contained among its votes one for EJICDNG in that order of preference. Those elected are EFGHIJN by Meek rules, but if that one vote is changed to read EIJCDNG (all other votes being unchanged) which should be to I's advantage, those elected become CEFGHJN, and I has lost the seat to C.

The current ERS rules elect EFGHJCN with 25% probability, EFGHJAI with 58% probability and EFGHJCI with 17% probability, depending upon how two random choices come out. That they reach the same result, given the same random choices, irrespective of whether the one vote is as in the original data set or changed, is inevitable because the only vote changed is from EJICDNG to EIJCDNG. At the first count E has 3 votes where the quota is 3.13 and so is not yet elected. At the second count 2 votes starting GE are transferred to E each at value 0.55, to give E a total of 4.10 and a surplus of 0.97, but that surplus is redistributed solely from the 2 newly-received votes. Whether J or I comes next in the vote that is changed is never even looked at.

Using Meek rules with either set of votes GEFHJ are elected and BDKLM are excluded. At that point with the original votes A has 2.145 while C has 2.100, C is excluded, N and I elected and A left as runner-up. With the modified votes, A has 2.053 while C has 2.060, so A is excluded and nearly all A's votes pass to C. This results in C and N elected, I as runner-up. Either way it is a very close-run thing, but who is ahead, of A and C, happens to reverse and the result unfortunately causes the observed lack of monotonicity.

Should all this worry Meek supporters? I think no more than the fact that lack of monotonicity is an upsetting feature of all STV. We could get rid of that feature by abandoning STV altogether and refusing ever to look at preferences beyond the first, but we know that what is lost by so doing far exceeds what is gained. Similarly if we do not look at later preferences some of the time (even when they are relevant) then we can get rid of the feature some of the time, but again, what is lost by doing that far exceeds what is gained. In general, looking at voters' later preferences whenever they are relevant helps to meet those voters' wishes; that it is occasionally troublesome is a pity but cannot be helped. It remains true that the voter concerned could not possibly anticipate such an effect, so it cannot lead to tactical voting, and also that even if such votes were to arise in reality, the lack of monotonicity would never be noticed except by detailed research of the ballot papers such as is hardly ever performed.

In case anyone wishes to examine this data set further, here are the original votes in Wichmann-Hill format. For those not used to this:

14 7 means 14 candidates for 7 seats;

1 5 10 9 3 4 14 7 0 means a vote for candidates 5 10 9 3 4 14 7 in that order, the initial 1 meaning 1 vote and the 0 terminating it, and so on;

Following all the votes there is an extra 0 to terminate them all and then the names of candidates in the order of their reference numbers, and a title for the election.

To get the modified votes, change the first one to start 1 5 9 10 instead of 1 5 10 9, and change the title on the last line.

```
14 7
1 5 10 9 3 4 14 7 0
1 3 5 13 12 7 1 4 8 0
1 8 7 10 12 13 3 6 4 14 11 9 1 2 5 0
1 5 11 14 7 9 0
1 6 7 10 11 12 3 0
1 8 7 5 13 12 14 6 3 1 2 0
1 6 7 10 12 0
1 7 9 5 8 10 14 3 4 1 2 6 11 12 13 0
1 10 7 12 5 8 3 6 9 14 0
1 7 5 11 6 0
1 1 12 3 14 8 6 13 5 0
1 7 5 12 10 14 4 3 9 6 0
1 9 0
1 7 6 10 12 9 14 0
1 1 12 3 8 14 6 5 13 0
1 10 1 12 8 6 3 9 0
1 8 5 12 3 9 1 7 13 10 11 4 6 0
1 3 4 7 10 0
1 7 10 8 12 3 4 9 14 1 13 2 6 11 5 0
1 14 11 5 10 0
1 14 13 2 1 3 9 12 4 5 8 0
1 7 8 9 5 6 0
1 7 12 4 9 8 14 3 11 0
1 5 14 7 0
1 6 7 10 12 0
0
"A""B""C""D""E""F""G"
"H""I""J""K""L""M""N"
"Original"
```

Trying to find a winning set of candidates

I D Hill

In *Voting matters* issue 2, I introduced the idea of Sequential STV and came to the conclusion that it should not be recommended for general use. But there remains something very attractive in trying to find a set of candidates, of the right size for the number to be elected, such that if an STV election were conducted with that set plus any other one candidate, all other candidates being treated as withdrawn, that set would always be the winners.

We know from Condorcet's paradox that in the one-seat case, where the set is of size 1, there may not be any winner who fulfils the criterion, but at least if we can find such a winner, the result is unique.

In the multi-seat case, we can still get results where no set satisfies the criterion. For an example, consider 4 candidates for 2 seats and votes 1 AB, 1 BC, 1 CD, 1 DA. If we choose AB to test we find that ABD leads to AD as winners; testing AD we find that ACD leads to CD; testing CD we find BCD leads to BC; testing BC we find that ABC leads to AB. So round in circles we go.

But now things are far worse for, even where a set to satisfy the criterion is found, it may not be unique. Again consider 4 candidates for 2 seats and votes 6 A, 6 B, 5 C, 5 D, 4 DA, 4 DB, 4 CA, 4 CB, 4 BC, 4 BD, 4 AC, 4 AD. If we choose AB as potential winners, we find that ABC elects AB and ABD elects AB, which would seem to confirm the choice; but if we choose CD we find that ACD elects CD and BCD elects CD, so that choice is also confirmed. Looking at the votes we can see that AB is, in fact, the better choice, but merely to find any set that fulfils the criterion is not adequate.

Can we then say that, having found a potential winning set, we need only look at disjoint sets to see if there are any others? Again things are not as easy as that. Consider 6 candidates for 4 seats, with the same votes as in the last example, with the addition of 20 E, 20 F. Then if we choose ABEF as potential winners, we find that ABCEF elects ABEF and ABDEF elects ABEF, seeming to confirm the choice; but if we choose CDEF we find that ACDEF elects CDEF and BCDEF elects CDEF, so that choice is also confirmed, and the sets are not disjoint as they both contain E and F.

It is clear therefore that there cannot be a universally best algorithm. For everyday practical use, I believe that simple STV by Meek's method should remain the algorithm of choice.

A simple model of voter behaviour

B A Wichmann

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Voting patterns

The additional information provided by preferential voting means that it is difficult to characterise voter behaviour. For instance, one cannot state that a voter supports party A merely because his first preference is for party A. The total information provided in a preferential ballot is very much larger than in X voting, although the result sheet only provides a small fraction of this information.

An obvious question to raise is if the information provided in a ballot can somehow be simplified to provide the essential content. In this paper, a simple model is proposed which appears to provide the essential information from a preferential ballot.

An example

The principle behind the model is most easily understood by means of an example. The model does not depend upon the number of seats to be filled (indeed, should this value alter the voting patterns?).

Hence we consider the case with four candidates: Albert, Bernard, Clare and Diana, with the votes cast as follows:

```
20  AB
15  CDA
 4  ADC
 1  B
```

From this data, we compute the number of each pair of preferences, adding both the starting position and a terminating position. For instance, the number of times the preference for A is followed by B is 20, and the number of times the starting position is ‘followed by’ A is 20+4=24. The complete table is therefore:

	A	B	C	D	e
s	24	1	15	0	-
A	-	20	0	4	15
B	0	-	0	0	21
C	0	0	-	15	4
D	15	0	4	-	0

Obviously, a preference for X cannot be followed by X, resulting in the diagonal of dashes. The entry under `s-e` could represent the invalid votes.

Having now computed this table, we can use it to characterise voting behaviour. For instance, 24 out of 40, or 60% of voters gave A as their first preference. More than this, we can use the table to compute ballot papers having the same statistical properties. For example, if the first preference was A, then the second row of the table shows that the subsequent preference should be B, D or e in the proportions of 20:4:15. Due to the fortunately large number of zeros in the table, we can easily compute the distribution of all the possible ballot papers which can be constructed this way. Putting these in reducing frequency of occurrence we have:

```
AB     30.8%    (50.0%)
A      23.1%
CDAB   16.9%
CDA    12.7%    (37.5%)
C       7.9%
ADC     6.1%    (10.0%)
B       2.5%    ( 2.5%)
```

The figures in brackets are the frequencies from the original data — which can be seen to be quite different.

A number of points arise from this example:

1. The computation of the frequencies needs to take into account the valid preferences. For instance, the frequency of the ballots starting AD is $0.6 \times 4/39 = 6.1\%$; the next preference can only be C, since the other option in the table is A which is invalid.

2. The large percentage that plump for A is due to the combination of the large percentage having A as the first preference, and the large percentage having A as the last preference, even though plumping for A does not occur in the original ballot. One would not expect this to occur in practice.

3. In this example, the table seems to be larger than the original ballot papers in information content. Exactly the opposite would occur with real elections with hundreds or more ballot papers.

4. Note that the number of occurrences of A in the ballot papers is the sum of the column A and also the sum of row A (which are therefore equal).

5. It is clear that the ballot papers constructed this way do not have the same distribution of the number of preferences as the original data. However, the mean number of preferences is similar, but smaller (2.19 for the computed data, 2.45 for the original). Clearly, when all ballot papers give a complete set of preferences, the computed data will rarely, but sometimes, give plumping.

6. If the voters voted strictly according to sex (A,B or C,D), then this characteristic would be preserved by the model. Similarly, the model does characterise party voting patterns.

The conclusion so far is that the model characterises some aspects of voter behaviour, but does not mirror other aspects. However, from the point of view of preferential voting systems, we need to know if the characterization influences the results obtained by a variety of STV algorithms. The property can be checked by comparing sets of ballot papers constructed by the above process against those produced by random selection of ballot papers from the original data.

We take the ballot papers from a real election which was to select 7 candidates from 14, being election R33 from the STV database. From this data, which consists of 194 ballot papers, we select 100 elections of 25 votes by a) producing random subsets of the actual ballots, or by b) the process described above.

For each of the 200 elections we determine 4 properties as follows:

1. Determine if the Condorcet top tier consists solely of the candidate G. This was a property of the actual election.

2. Determine if the Meek algorithm elects candidate C. This was a property of the actual election.

3. Determine if the ERS hand counting rules elects candidate N. This was a property of the actual election.

4. Determine if Tideman's algorithm elects candidate E. This was not a property of the actual election. Unfortunately, computing the result from this algorithm can be very slow, and hence the result was determined for 50 elections rather than the 100 for the other three cases.

The results can be summarised by the following table:

	Subset	Process	Number
Condorcet (G)	75	67	100
Meek (C)	42	34	100
ERS (N)	56	47	100
Tideman (E)	14	20	50

I believe that the four properties above are sufficiently independent, and the elections themselves independent enough to undertake the χ-squared test to see if the two sets of elections could be regarded as having come from the same population. Passing this test would indicate that the statistical construction process is effective in providing ‘election’ data for research purposes.

The statistical testing is best done as a separate 2×2 table test of each line. The first line, for example, gives the table

	Condorcet Analysis (G)	other	
Subset	75	25	100
Process	67	33	100
	142	58	200

The four tables give P = 0.28, 0.31, 0.26 and 0.29 respectively, using a two-tailed test. So, so far as this test goes, these show no significant differences in the two methods.

Acknowledgement

Dr David Hill provided the statistical analysis above.

Monotonicity — An In-Depth Study of One Example

D R Woodall

Douglas Woodall is Reader in Pure Mathematics at Nottingham University.

Here is a fairly typical example of the way in which monotonicity can fail with STV (or, as in this case, AV). Consider the pair of single-seat elections below. In Election 1, no candidate has reached the quota of 15, and so *c*, the candidate with the smallest number of first-preference votes, is excluded. All *c*'s votes are transferred to *a*, and so *a* is elected. However, just before the result is announced, it is discovered that two of the ballots placed in the pile labelled *bca* are not in fact marked *bca* at all, but *abc*, so that the true situation is as in Election 2. Naturally *a* is delighted with this increased support. But now *b* has the smallest number of first-preference votes, and so, when the count is redone, *b* is excluded instead of *c*. All *b*'s votes go to *c*, and so *c* is elected instead of *a*. So the effect of this increased support for *a* is to cause *a* not to be elected.

	Election 1	Election 2
abc	11	13
bca	10	8
cab	9	9
Excluded	*c*	*b*
Elected	*a*	*c*

This is the sort of anomaly that has caused some people to reject the whole idea of STV. The question I want to discuss here is, how serious is it really? Certainly nobody is going to pretend that it is desirable; but is it really as bad as some people have been making out?

The first thing to notice is that nobody has been wrongly elected. One might object that it cannot possibly be the case that *a* is the right person to elect in Election 1 and that *c* is the right person to elect in Election 2, in which *a* clearly has more support. But it does not really make sense to talk about "the right person to elect" in these elections. In Election 1, for example, there are 19 voters who prefer *c* to *a*, and only 11 who prefer *a* to *c*, so that *c* seems a better candidate to elect than *a*. But then there are 21 voters who prefer *b* to *c*, and only 9 who prefer *c* to *b*, and so *b* seems a better candidate to elect than *c*. But then again, there are 20 voters who prefer *a* to *b*, and only 10 who prefer *b* to *a*, and so *a* seems a better candidate to elect than *b*. Whichever candidate you choose to elect, someone else can claim to be better! (Of course, this is just an example of the famous Condorcet paradox.) In this situation one should not talk about which is *the right* candidate to elect, but, rather, about which candidates it would be *permissible* to elect. It seems to me that in either of these elections it would be perfectly permissible to elect any one of the three candidates. In this situation STV really does no more than make a somewhat arbitrary selection from among the permissible candidates. It is certainly unfortunate that it chooses *a* in Election 1 and *c* in Election 2, where *a* clearly has more support; but it is in the nature of such processes that this sort of thing will happen.

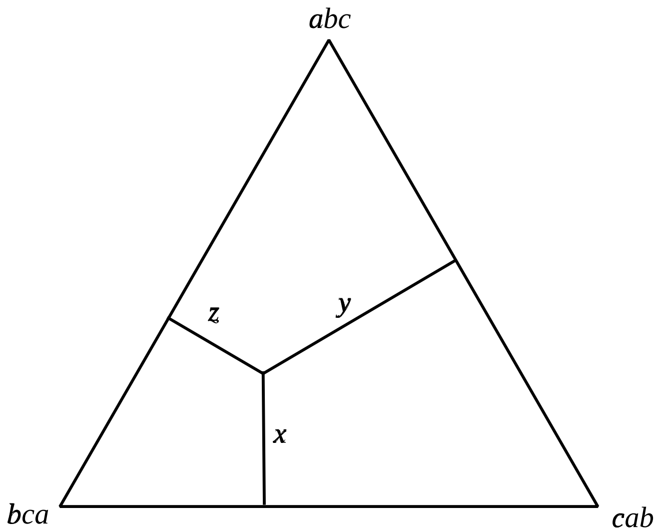

Figure 1

Let us examine more closely what is going on here. Because there are only three different types of ballot present, we can represent the situation diagrammatically, using what are known as *barycentric coordinates* in a triangle. Suppose we draw an equilateral triangle of unit height (Figure 1). If we put a point inside the triangle and drop perpendiculars from it, of lengths x, y and z, to the three sides of the triangle, then it is easy to prove that $x + y + z = 1$, the height of the triangle. So if we label the three corners of the triangle with the three different types of ballot, as in Figure 1, then we can use the point depicted to represent an election in which the proportion of voters voting *abc* is x, the proportion voting *bca* is y, and the proportion voting *cab* is *z*. Thus, for example, the top vertex of the triangle represents an election in which all the voters vote *abc*; the mid-point of the left side represents an election in which half vote *abc* and half vote *bca*; and so on.

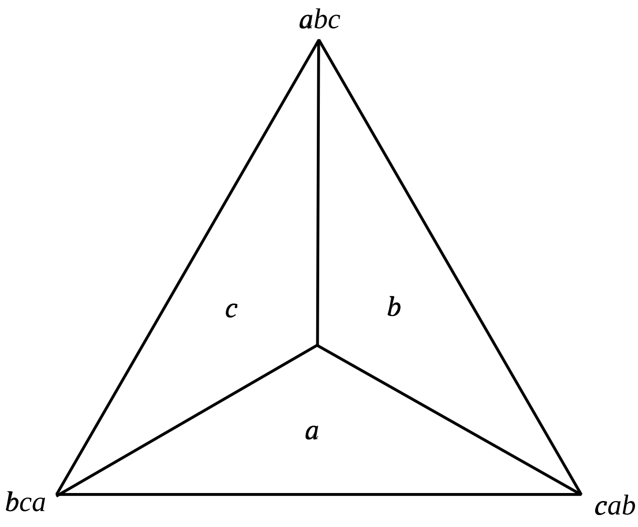

Figure 2: candidate excluded

Suppose now that we exclude the candidate with the smallest number of first-preference votes. Figure 2 shows which candidate is excluded. For example, to the right of the vertical line through *abc* there are more *cab* than *bca* ballots; and above the middle of the three lines through *cab* there are more *abc* than *bca* ballots. So in the region marked *b*, there are fewer *bca* ballots than ballots of either of the other two types, which means that *b* has fewest first-preference votes. Similar remarks apply to the other two regions.

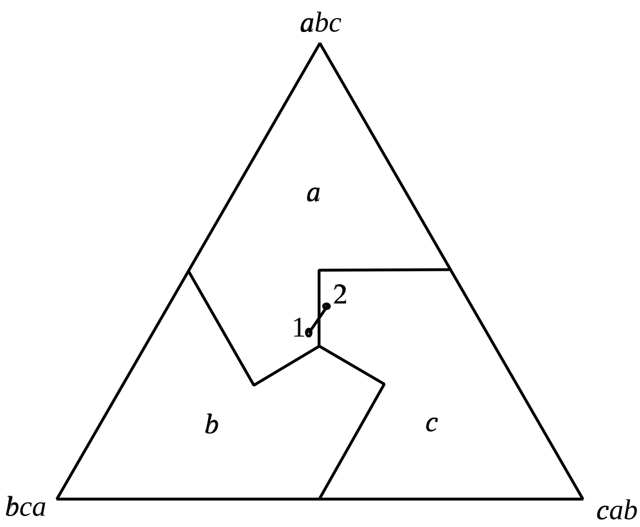

Figure 3: candidate elected

Now consider what happens if *b* is excluded. All of *b*'s votes are transferred to *c*. So the only way that *a* can win is if more than half the ballots are marked *abc*; that is, we are above a horizontal line drawn half way up the triangle. (Of course, in this case *a* will be elected outright — one would not normally exclude *b* first; but it would make no difference to the outcome if one did.) Similar remarks apply to the other two regions, and so the result of the election is as indicated in Figure 3. Figure 3 also shows the points representing Elections 1 and 2. Election 1 is in the region where *a* is elected. Election 2 is obtained from it by converting two *bca* ballots into *abc*, hence by moving parallel to the left edge of the triangle. This takes us into the region in which *c* is elected. Of course, if one continues a bit further in the same direction, then one gets back into the region in which *a* is elected.

The problem is caused, in a sense, by the fact that the regions are not convex. However, one cannot make them convex without violating the spirit of STV. Their convexity is equivalent to the property called Convexity in Woodall[1]; and, as mentioned there, the only election rules that possess this property are point-scoring systems, which do not conform to the spirit of STV.

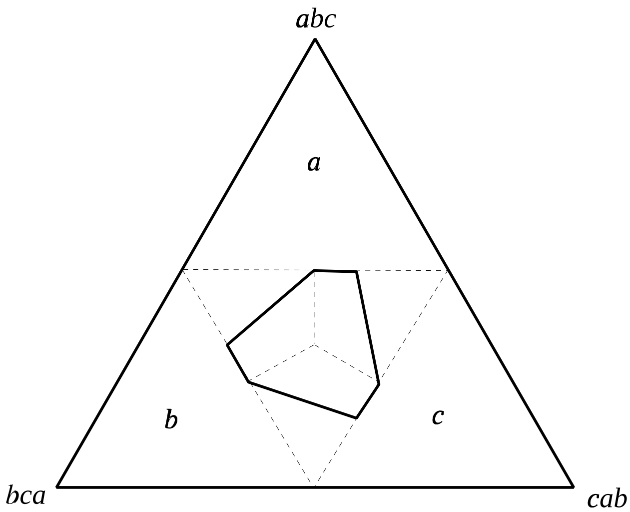

Figure 4: where monotonicity fails

This representation also gives us a way of visualizing where monotonicity fails. If there are two elections (involving only these three types of ballot) that between them show this type of failure of monotonicity, then both elections must lie inside the central region indicated in Figure 4. Note that this region is completely contained within the large dotted triangle, which is where the Condorcet paradox arises. So, in this example, monotonicity does not fail except when there is a Condorcet paradox. However, it is important to stress that, in general, monotonicity can fail even when there is no Condorcet paradox.

Figure 4 suggests the following interpretation. There are certain regions in which it is quite clear who ought to be elected, and in these regions STV elects the candidate that one would expect. But in the middle there is a grey area, where it is not at all clear who ought to be elected, and it is in this grey area that STV behaves in a somewhat haphazard manner; it is really doing no more than making a pseudo-random selection from the appropriate candidates, and it is here that small changes in the profile of ballots can cause perverse changes in the result.

The effect of this is to blur the result of an STV election. Nobody is being wrongly elected, because the problem only arises in the region where one cannot say for certain who ought to be elected anyway. And there is no systematic bias that would, for example, favour one political party rather than another. But the accuracy with which the person or persons elected in an STV election can be said to represent the views of the voters is less precise than it would be if this sort of anomaly did not arise.

The obvious question at this point is whether one can find a system that retains the essential features of STV while avoiding this sort of anomaly. The answer depends on what one regards as the essential features of STV. As we shall see in a later article, it is not possible to avoid this anomaly without sacrificing at least one property that many supporters of STV regard as essential. Nevertheless, I shall describe there a system for single-seat elections that gains so many forms of monotonicity, while sacrificing only one property of STV, that I personally would be willing to recommend it as a better system than the Alternative Vote. Unfortunately, it is not feasible when the votes are to be counted by hand. Also, it is not clear whether it can be extended in any sensible way to multi-seat elections; this is a crucial question, which I have so far been unable to answer.

Reference

1. D R Woodall, Properties of preferential election rules, *Voting matters* Issue 3 (1994), 8-15.

Issue 5, January 1996

Editorial

In this issue, two long and one short article appear which I hope will be of substantial interest to readers. In the first, Crispin Allard produces some estimates of the likely rate of non-monotonicity, based upon a mock election. Secondly, Hugh Warren gives an interesting example of the Condorcet paradox which can only serve to show the inherent complexity of preferential voting. Lastly, I report on a program which attempts to produce plausible election data from STV result sheets.

Estimating the Probability of Monotonicity Failure in a UK General Election

Dr C Allard

Crispin Allard holds a PhD in statistics from the University of Warwick, and is a member of the ERS Council.

1. Summary

Three years ago, the Plant Report rejected STV as a system worth considering for elections to the House of Commons, citing evidence submitted by Michael Dummett (based on an example originating from Reference 2) on the grounds that it could be non-monotonic. In this paper I attempt to estimate the probability of a monotonicity failure which affects the number of seats won by a party. I estimate the probability of this occurring in a multi-member constituency in one election as: 2.5×10^{-4}, equivalent to less than once every century across the whole UK. [*This result was first reported in Reference 1 as 2.8×10^{-4}. I have revised this down as a result of a refinement in the method.*]

2. Representing the problem

Consider an n-member STV constituency, in which n-1 candidates have so far been elected, and the three remaining candidates (denoted A, B and C), one each from the Conservative, Labour and Liberal Democrat Parties are competing for the final place. The conditions for monotonicity failure are as follows:

1. A is ahead of B, and B is ahead of C;
2. When C is eliminated, his transfers put B ahead of A, so that B is elected;
3. If a number of voters switch their relevant preference from A to C, so that both A and C are ahead of B, then when B is eliminated, A is ahead of C, so that A is elected;

for any ordering of A, B and C.

Writing these conditions down in mathematical terms we get:

1. $a > b > c$.
2. $a < b + \alpha c$.
3. There exists x such that: $a - x > b$, $c + x > b$, $a > c + 2x + \beta b$

where

a = the proportion of votes credited to A

b = the proportion of votes credited to B

c = the proportion of votes credited to C

$\alpha = T_{CB} - T_{CA}$

$\beta = T_{BC} - T_{BA}$

T_{ij} = the proportion of i's votes which transfer to j if i is eliminated.

(α and β can be considered as the level of advantage which one party can expect to gain over another as a result of the exclusion of a candidate from a third party).

The following conditions are equivalent to 1-3 above:

M1. $b > c$

M2. $a < b + \alpha c$.

M3. $a > \max\{2b - c,\ (2 + \beta)b - c\}$

Using barycentric coordinates (and denoting each point of the triangle to represent one candidate having all the votes), these conditions are illustrated in figure 1.

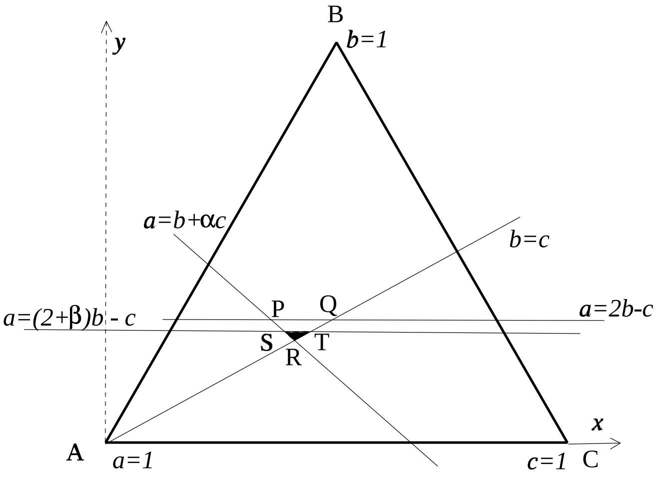

Figure 1

Thus, if we assume a uniform distribution, the probability of this type of monotonicity failure is the ratio of the area of the small triangle (either PQR or STR, whichever is the smaller) to the area of the large one (ABC). To see why we must take the smaller triangle, note that to satisfy condition M3, a point in Figure 1 must be below both the lines:

$a = (2 + \beta)b - c$ and $a = 2b - c$.

Note that if $\beta > \alpha$, conditions M1-M3 cannot simultaneously be satisfied, so in this case we define: Area (STR) = 0.

Switching to Cartesian coordinates,

$x = c + b/2$

$y = \sqrt{3}\, b/2$

the areas of the three triangles are found to be:

$$\text{Area(ABC)} = \sqrt{3}/4$$

$$\text{Area(PQR)} = \frac{\alpha^2}{12\sqrt{3}(3+\alpha)(1+\alpha)}$$

$$\text{Area(STR)} = \frac{\sqrt{3}(\alpha-\beta)^2}{4(3+\beta)^2(3+\alpha)(1+\alpha)} \quad \text{if } \alpha > \beta$$

= 0 otherwise

So if we let p be the probability of monotonicity failure, we can find its value as follows:

$$\alpha > 0 \geq \beta \Rightarrow p = \frac{\alpha^2}{9(3+\alpha)(1+\alpha)}$$

$$\alpha > \beta > 0 \Rightarrow p = \frac{(\alpha-\beta)^2}{(3+\beta)^2(3+\alpha)(1+\alpha)}$$

else p=0

Or, by substituting,

$\gamma = \max\{\alpha, \beta, 0\}$

$\delta = \max\{\min\{\alpha, \beta\}, 0\}$

we obtain a single equation for p:

(P1) $$p = \frac{(\gamma-\delta)^2}{(3+\delta)^2(3+\gamma)(1+\gamma)}$$

3. Estimating the transfer patterns

Clearly we need to know the likely pattern of transfers between candidates from different parties, which requires access to the ballot papers of a typical British electorate voting by STV for real political parties. Last year an ERS/ MORI exit poll of 3,983 London voters was conducted during the European Parliament elections, in which they were asked to cast preferential votes in two multi-member constituencies. The results form by far the best available data on the likely behaviour of British voters in an election conducted by STV.

Details of the poll may be found in Reference 3, which includes tables of terminal transfers (transfers of votes from a candidate whose party has no further candidates left who are still eligible to receive votes). Unfortunately, there is no terminal data from Conservative candidates, since none occurred in the count of the mock vote, so this data cannot be used.

Instead I try to consider all the possible transfers of votes which could have taken place. For each of the two constituencies (London North and London South), and for every ordered triple of candidates (Conservative, Labour, Lib Dem), the following data extracted from the poll results is used.

The number of votes which would transfer to the Labour candidate (if the Conservative were to be eliminated leaving only the Labour and Lib Dem candidates); the number which would transfer to the Lib Dem candidate in such circumstances; and the number which would be non-transferable.

This data is repeated for the each of the Labour and Lib Dem candidates being eliminated, providing 840 data sets (sadly not independent!) on which to base the estimate of transfer patterns, and hence estimate *p*. The number of data sets arises from 216 ordered triples in London North (6-seater), 64 in London South (4-seater), and three data sets for each ordered triple.

4. Method

In outline, I employ the following method (using an Excel spreadsheet):

i) For each data set (representing the potential transfers from one candidate from one party to two candidates from the other parties), the proportions T_{ij} of votes transferred to each of the surviving candidates are calculated.

ii) These proportions are then adjusted using the following approximate shrinkage equation:

$$T'_{ij} = \frac{\bar{\bar{T}}_{ij}/t + nT_{ij}/s}{1/t + n/s}$$

where:

T'_{ij} represents the shrunken estimate of the proportion of *i*'s votes which transfer to *j* if *i* is eliminated.

$\bar{\bar{T}}_{ij}$ is the weighted sample mean of T_{ij} based on exclusions of candidates from the same party in a particular constituency.

s is the sample variance of T_{ij}.

n is the size of the data set (the number of first preferences credited to the excluded candidate).

$t = 0.0004$

Note that this is based on a two-stage hierarchical model, in which (for a given constituency and party) there is a party mean value of T_{ij}, with variance 0.0004, about which the candidates' T_{ij} values are distributed.

iii) Based on the values of T'_{ij}, γ, δ and p are calculated, using the above definitions and equation P1.

iv) For each ordered triple of candidates, the three values of p (one for each potential elimination) are summed to allow for all the possible ways in which monotonicity might fail, giving a total probability P.

v) For each constituency, a weighted mean of the probabilities is calculated.

vi) Finally a weighted mean of the probabilities for the two constituencies is taken to produce the result:

$$\varepsilon(P) = 2.5 \times 10^{-4}.$$

So, if the UK is divided into 138 multi-member constituencies, as proposed in Reference 4, and assuming an average of one General Election every four years, we would expect one instance of final-stage monotonicity failure affecting party standing under STV roughly every 115 years.

5. Justifying the approach

The problem of estimating the probability of monotonicity failure under STV is complicated, involving political considerations and statistical judgement as well as pure mathematics. So inevitably I have had to make a number of assumptions and simplifications. I will now attempt to identify all the potential objections to my approach and answer some of the possible criticisms.

5.1 Only monotonicity failure affecting parties is considered.

It is almost certainly true that the probability of affecting individual candidates within a party is much greater. For a start, far more voters are prepared to transfer within a party than between parties. This is supported if we look at ERS Council Elections (which are like elections between candidates of the same party since all support electoral reform), where potential instances have been observed.

Nevertheless, given that STV is the only system which even attempts to represent intra-party opinion, any minor ‘imperfection’ in this respect is irrelevant to the choice of an electoral system. And it is certainly the case that most of the opponents of STV are far more concerned with party representation.

Finally, it is a necessary simplification since intra-party transfer patterns are notoriously unpredictable and difficult to model.

5.2 The model only covers three parties and final-stage transfers.

Of course, earlier stages and a greater number of parties allow more opportunities for monotonicity failure. However, I claim that the probability of this making a difference to the final result is tiny compared to the figure I have calculated above.

To see this, consider the diagram in figure 1. The effect is only possible when there are three candidates with very similar votes (Q is the geometric centre). Thus, if there are four candidates competing for the final place, a candidate who ‘benefits’ in the penultimate stage is still very unlikely to benefit in terms of election (and one who ‘loses’ probably would not have been elected anyway).

If there are four candidates competing for two places, with three in danger of elimination, then the fourth may be discounted (as a certainty), and we are back to the original problem. Only in the case where there are four or more candidates all with similar votes might a relevant situation arise; it is reasonable to ignore such *n*th order terms.

5.3 The method assumes a Uniform (prior) distribution of votes between the three parties.

This assumes that the three parties each have the same marginal distribution. In a one-member constituency this is highly unlikely, but in a multi-member constituency the relevant distribution to consider is the remainder, once n-1 seats have been ‘allocated’, and the appropriate number of quotas deducted from each party's vote.

Therefore, in order for the assumption to be reasonable, all we need is to have across the country three parties capable of achieving proportions of votes over a range of at least one quota. This would typically be achieved by a party receiving 10% or more of the national (or regional) vote.

A similar principle is at work behind the Wichmann-Hill pseudo-random generator, where the sum of a number of variables is known to tend to normality, but the fractional part of the sum remains rectangular. There is room here for someone to conduct a proper analysis, which I am confident would uphold my assumption.

5.4 The results are based on an opinion poll conducted only in London.

This represents probably the biggest area of doubt about the result and, since this is the best data available so far, there is no way of avoiding it. The STV ballot paper was constructed by listing (nearly) all candidates in each of the Euro-constituencies represented. Since this was an election for MEPs, recognition of most individual candidates must have been relatively low.

However, we can only speculate on how voters would react in a General election conducted by STV, and it is by no means obvious that voting patterns would be substantially different. The same applies to the London factor. While the relative positions of the parties would vary across the country, there is no reason to suppose that the nature of voting patterns would be any different.

5.5 Why has shrinkage been applied in this way?

Shrinkage is one of the results of Bayesian analysis which has been accepted by non-Bayesian statisticians as representing a true effect which does not appear in more traditional models. I have judged that a hierarchical model is relevant to this situation, so we must take account of shrinkage. A reference will be given in the next issue of *Voting matters* to provide an explanation of shrinkage for non-statisticians.[Not produced?]

If the charge is that I have not defined a full Bayesian hierarchical model, with detailed multivariate prior distributions etc., then I plead guilty. This was done deliberately to avoid specifying prior distributions which might obscure the argument. The value of t is arbitrary but, I believe, reasonable. A little sensitivity analysis shows that it does not affect the final result by more than a tenth.

5.6 The weightings used in the final calculation do not allow for some votes having a greater effect.

Rather than try to work out what effect the voting patterns might have had in this particular election, I wanted to gain an estimate of overall voting patterns. This means considering both first and last place candidates, since in different constituencies each party will have somewhere between 0 and 5 ‘safe’ seats, so the candidate involved in a three-way battle could be anyone between the first and sixth most popular in that party.

The best way to cope with such uncertainty is to assign equal weightings to each elector.

6. Conclusions

Using the best data available and using reasonable assumptions I have estimated the probability that monotonicity failure would arise in a UK General Election conducted by STV. That probability turns out to be extremely small. In political terms it may as well be zero. Opponents of STV will need to come up with better reasons if they wish to reject it out of hand.

Acknowledgements

I am grateful to Professor Shaun Bowler of the University of California at Riverside for his help in supplying data from the ERS/MORI poll, and to Richard Wainwright and others for their encouragement and interest in this research.

References

1. Crispin Allard, ‘Lack of Monotonicity - Revisited’, *Representation* 33:2 (1995), pp48-50.

2. G. Doron and R. Kronick, *American Journal of Political Science* 21, pp303-311.

3. Shaun Bowler and David M. Farrell, 'A British PR Election: Testing STV with London's Voters', *Representation* 32:120 (1994/5), pp90-3.

4. Robert A. Newland, *Electing the United Kingdom Parliament*, 3rd Edition (ERS, 1992).

Appendix: Summary Statistics

Below is a table showing the transfer trends in North and South London. The transfers are weighted means, expressed as percentages of the respective first preference votes. The advantages (corresponding to α or β) are given after adjusting for shrinkage. See section 4 for a full explanation. Each party is shown with the number of first preference votes cast in the poll for candidates of that party.

Of the 3,983 voters polled, 3,013 expressed a valid first preference for a candidate from one of the three main parties, of whom 1,778 were from North London and 1,235 from South London. The overall probabilities of monotonicity failure were found to be 0.00013 in North London and 0.00043 in South London, giving a (weighted) mean of 0.00025 and a sample variance of 2×10^{-8}.

	%Transfers				Advantage		
Votes	Con	Lab	LD	NT	Mean	High	Low
North London							
Con:512	-	2.4	5.1	92.6			
Advantage(LD-Lab)					2.7	9.7	-9.8
Lab:1049	1.4	-	8.1	90.5			
Advantage(LD-Con)					6.7	22.5	-5.7
LD:217	4.8	11.3	-	83.9			
Advantage(Lab-Con)					6.5	20.6	-8.6
South London							
Con:400	-	3.0	11.6	85.4			
Advantage(LD-Lab)					8.6	23.2	-8.9
Lab:598	1.9	-	13.9	84.2			
Advantage(LD-Con)					12.0	21.2	3.8
LD:237	7.7	12.5	-	79.9			
Advantage(Lab-Con)					4.8	18.3	-13.0

An example showing that Condorcet infringes a precept of preferential voting systems

C H E Warren

Hugh Warren is a member of the ERS Council.

It is one of the precepts of preferential voting systems that a later preference should neither help nor harm an earlier preference. The purpose of this paper is to show that the Condorcet system of preferential voting infringes this precept.

Consider an election for one seat in which there are 3 candidates:

A is a Catholic Conservative White

B is a Protestant Labour White

C is a Catholic Labour Asian

There are 99 voters:

17 want Labour, they prefer a White to an Asian, and they are indifferent as to sect, so they vote BC.

16 want Labour, they prefer an Asian to a White, and they are indifferent as to sect, so they vote CB.

15 want a Catholic, they prefer Labour to Conservative and they are indifferent as to race, so they vote CA.

17 want a Catholic, they prefer Conservative to Labour, and they are indifferent as to race, so they vote AC.

16 want a White, they prefer Conservative to Labour, and they are indifferent as to sect, so they vote AB.

15 want a White, they prefer Labour to Conservative, and they are indifferent as to sect, so they vote BA.

1, whom we shall call Voter X, wants primarily a Conservative, and wants also an Asian and a Protestant, so is undecided whether to vote AC or AB, but settles for AC.

1, whom we shall call Voter Y, wants primarily a Protestant, and wants also a Conservative and an Asian, so is undecided whether to vote BA or BC, but settles for BA.

1, whom we shall call Voter Z, wants primarily an Asian, and wants also a Protestant and a Conservative, so is undecided whether to vote CB or CA, but settles for CB.

Accordingly the votes are as follows:

```
AB 16
AC 18
BA 16
BC 17
CA 15
CB 17
```

The Condorcet method for the election yields the following results:

C beats B by 50-49

A beats C by 50-49

B beats A by 50-49

Accordingly we see that Condorcet produces a paradox.

(Incidentally, the Single Transferable Vote, which amounts to the commonly called Alternative Vote in this case, would 'exclude the lowest', C, and hence would elect B.)

If the paradox is resolved by electing A, then, if instead of voting AC Voter X had voted AB, Candidate B would have beaten Candidate C, and accordingly by the Condorcet method Candidate B would have been elected. Therefore changing the second preference of Voter X from C to B works to the detriment of his first preference A.

If the paradox is resolved by electing B, then, if instead of voting BA Voter Y had voted BC, Candidate C would have beaten Candidate A, and accordingly by the Condorcet method Candidate C would have been elected. Therefore changing the second preference of Voter Y from A to C works to the detriment of his first preference B.

If the paradox is resolved by electing C, then, if instead of voting CB Voter Z had voted CA, Candidate A would have beaten Candidate B, and accordingly by the Condorcet method Candidate A would have been elected. Therefore changing the second preference of Voter Z from B to A works to the detriment of his first preference C.

Therefore, no matter how the paradox is resolved, the precept that a later preference should not harm an earlier preference is infringed.

Producing plausible party election data

B A Wichmann

The STV database lacks any data from public elections which involves political parties[1]. This is hardly surprising due to the legal constraints on public election data. However, from the point of view of election studies, this omission is very unfortunate. Statistical studies of real election data are important, since we know that desirable logical properties cannot be universally satisfied.

For public elections, the only information available is that of the result sheet. Unfortunately, this information is very much less than that contained in the ballot data itself. Only a few preferences expressed by votes are actually exercised in the counting process and therefore can be reconstructed from the result sheet. It is possible to produce minimal ballot papers which will give the same effect as the result sheet, but such ballot data is very unlike the (unknown) ballot data itself. In contrast, we are here attempting to produce ballot data which appears similar to the actual data, so that our constructed data can be used instead of the real data.

In this study, we are using the Irish election data for the years 1969 and 1973, since this is available in a convenient book format which is easy to process, see Knight and Baxter-Moore[3]. The first election in the book, is that for Carlow-Kilkenny. For this, we have:

	Information content
Result	9 bits
Result sheet	800 bits
Election data	800,000 bits

It might therefore appear that we have a hopeless task since the result sheet contains a thousand times less information than that of the (missing) election data.

However, we established[2] that if we can provide a matrix giving the probabilities of A being followed by B (for all candidates A, B), then election data can be constructed which appears to have the statistical properties one would expect, at least as far as the election results are concerned with the usual STV algorithms. Hence if we can produce an estimate for the A-B probabilities, we can construct plausible data.

Taking the result sheets for all the Irish elections for 1969, we can study just the first transfers made. These transfers are not restricted in the potential choice that can be made by the elector, and therefore can provide a basis for the probabilities we wish to estimate. To compare one constituency with another, we label the candidates FF1, FF2,.. for Fianna Fail in order of the first preferences, and similar for Fine Gael (FG1, etc), Labour (LA1, etc) and others (OT1, etc). (Fortunately, this is exactly the order listed in [3]) We only need to consider the three main parties since they account for around 97% of the first preference votes. However, the 'other' candidates must be taken into account with transfers, and hence appear as a notional party.

Table 1

All first transfers,

Irish elections 1969

Consistuency	FF1	FF2	FF3	FF4	FG1	FG2	FG3	FG4	LA1	LA2	LA3	LA4	OT1	OT2	OT3	NT
Carlow-Kilkenny	-411	0	313		43	7	2	1	21	22						0
Cavan	169	171	117		237	294	72		-1255	-495			458			232
Clare	0	64	23		19	42			346	-533						39
Clare-S Galway	15	51	20		82	10	18		348	-561						17
Cork NW	-4815	3828			114	120			128	75			550			0
Cork SE	-3679	3182			152	122			86	138						0
Mid Cork	-1165	490	391		91	41	15		41	96						0
NE Cork	-1159	450	454		69	3	44	4	127	8						0
SW Cork	141	87			1719	784	-3216		395							90
NE Donegal	-1539	1422			47	50			20							0
Dublin C	-935	662	168		20	6	3	4	14	6	4	8	21	18	1	0
Dublin NC	-3254	1743	676	630	48	64	28		0	42	23					0
Dublin NE	-4268	2054	1710		98	48	23	12	0	55	41	24	168	35		0
Dublin NW	89	46	27		57	99	35	21	-2305	535	719	677				0
Dublin SC	10	23	11		13	14	8	3	11	8	10	8	10	19	-149	0
Dublin SE	46	12	25		-1731	1469			132	21			19	7		0
Dublin SW	4	6	5		11	16	4	5	33	22	10	4	26	-154		6
NC Dublin	0	52	8		214	134	175	-688	31	8	19	36				11
SC Dublin	17	14	16		62	25	17		331	330	-830					18
Dun Laoghaire -	0	24	25		-3317	2030	956		102	53	35		59	33		0
NE Galway	19	26	9		-477	203	168		52							0
W Galway	-780	445	189		27	44	10		28	18			19			0
N Kerry	242	403	69		934	304			351				-2425			122
S Kerry	-1583	1243			57	122	18		143							0
Kildare	197	188	74		305	118	193		178	146			-1496			97
Laois-Offaly	0	55	34		-2075	444	688	487	91	68	65		44			0
E Limerick	12	8	7		19	18	5		112	131	-366		50			4
W Limerick	-3358	1098	1695		175	73	93		144	60			20			0
Longford-W	50	10	6		2	29	25	35	107	108	-420		33			15
Louth	18	89			-1048	614	244		63	20						0
E Mayo	74	58	39		226	145	233		-869							94
W Mayo	36	28	46		122	144	21		-445							51
Meath	99	82	49		107	25	32		981	-1408						33
Monaghan	64	30	22		68	76	33		372	-699						34
Roscommon -	28	36	4		-525	197	224		25	8	3					0
Sliogo-Leitrm	11	18	203		158	8	3		29	51	-506					25
N Tipperary	-1533	628	480		102	71			222	14	16					0
S Tipperary	-1942	1208	462		88	40	13		74	38			19			0
Waterford	1071	679	-2118		35	93			156	24						60
Wexford	51	23	21		39	101	13		343	29	24		112	-813		57
Wicklow	-1010	272	544		36	37			80	41						0

Table 2

Transfers from Fianna Fail

Consistuency	FF1	FF2	FF3	FF4	FG1	FG2	FG3	FG4	LA1	LA2	LA3	LA4	OT1	OT2	OT3	NT
Carlow-K *	-411	313			43	7	2	1	21	22						0
Cork NW	-4815	3828			114	120			128	75			550			0
Cork SE	-3679	3182			152	122			86	138						0
Mid Cork	-1165	490	391		91	41	15		41	96						0
NE Cork	-1159	450	454		69	3	44	4	127	8						0
NE Donegal	-1539	1422			47	50			20							0
Dublin C	-935	662	168		20	6	3	4	14	6	4	8	21	18	1	0
Dublin NC	-3254	1743	676	630	48	64	28		0	42	23					0
Dublin NE	-4268	2054	1710		98	48	23	12	0	55	41	24	168	35		0
W Galway	-780	445	189		27	44	10		28	18			19			0
S Kerry	-1583	1243			57	122	18		143							0
W Limerick	-3358	1098	1695		175	73	93		144	60			20			0
N Tipperary	-1533	628	480		102	71			222	14	16					0
S Tipperary	-1942	1208	462		88	40	13		74	38			19			0
Waterford *	-2118	1071	679		35	93			156	24						60
Wicklow	-1010	272	544		36	37			80	41						0

Table 3

Transfers of 1,000 preferences from Fianna Fail

FF1	FF2	FF3	FF4	FG1	FG2	FG3	FG4	LA1	LA2	LA3	LA4	OT1	OT2	OT3	NT
599	222	18		35	28	8	1	38	19	2	1	24	2	0	2

Table 4

Transfers from Fine Gael

FF1	FF2	FF3	FF4	FG1	FG2	FG3	FG4	LA1	LA2	LA3	LA4	OT1	OT2	OT3	NT
19	29	8	5	527	244	51		68	14	9	3	9	3	0	8

Table 5

Transfers from Labour

FF1	FF2	FF3	FF4	FG1	FG2	FG3	FG4	LA1	LA2	LA3	LA4	OT1	OT2	OT3	NT
59	54	54	0	108	86	46	5	296	125	63		51			53

Table 6

Transfers from other parties

FF1	FF2	FF3	FF4	FG1	FG2	FG3	FG4	LA1	LA2	LA3	LA4	OT1	OT2	OT3	NT
100	128	36		258	110	43	2	182	41	9	1	29	6		56

Table 1 gives the first transfers for all* the 1969 Irish elections. The candidates are labelled as above and NT (for Non-Transferable). A blank in the relevant columns indicates no such candidate. Others are listed in the order given in Knight and Baxter-Moore[3].

Table 2 shows the transfer from Fianna Fail alone. The star against the Waterford entry represents a change from the original. In this case alone, the FF transfer was by elimination; but we wish to put under FF1 the candidate from which transfers were made, which implies permuting the columns as shown. Again, the star against the Carlow-Kilkenny entry represents a change from the original. Here, the candidate FF2 already had the quota, and therefore was not eligible for transfers (or rather any such transfer would have been ignored) and hence the transfer to FF3 is regarded as being for FF2, being the next available FF candidate.

The columns can now be added up to see what the average transfers are. (The total transfers are 33,549, but we express this as votes transferred per thousand.) This result is shown in Table 3, where FF1 here represents the first Fianna Fail candidate to which transfers could be made. As expected, this indicates weak cross-party voting and that the most popular person within a party is that based on first-preference votes.

Tables 4, 5 and 6 give the corresponding transfers of 1,000 votes from Fine Gael, Labour and the other parties respectively.

Hence we now have estimates for our A-B probabilities, although these figures are very crude for the following reasons:

1. The tables show large variations between constituencies.

2. Comparing constituencies with different numbers of candidates for each party is dubious.

3. Grouping all other candidates into a notional party is clearly dubious also.

Nevertheless, we now have some estimates that are probably as good as we can get in the circumstances.

The next process is to use the above estimates for providing default transfer probabilities in those cases in which the result sheet does not provide this information.

For each of the Irish elections for 1969, we compute the transfer probabilities that can be found from the result sheet. For the other values, we use our estimates. This then allows for plausible ballot data to be computed by program.

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* Donegal-Leitrim is excluded since this has the Speaker of the Dail elected unopposed, so comparisons are difficult.

The computer program does need to reduce the ballot data to manageable proportions. For Carlow-Kilkenny in 1969, there were 46,073 ballot papers. If we constructed this number of ballot papers individually by program, we would have a 750K bytes data file — too big to process rapidly. We can reduce the data file to a more manageable size by having piles of identical papers, which all the computer algorithms can handle rapidly. The program uses piles of 500, 100, 50, 10, 5 and 1 paper(s), adjusted so that the correct number of total ballot papers is produced, and the first preference counts are the same as the result sheet. The data file is now reduced to about 11K bytes.

The program also attempts one further adjustment. The ballot papers match the first preferences and the total votes cast exactly, but the match to subsequent transfers is only similar in terms of the proportion of the occurrence of A-B's in the papers. To obtain a better, but not identical fit, the program computes many examples using different seeds for the random number generator, and selects the best example. Determining the fit between a ballot paper set and the result sheet is not straightforward. To undertake the comparison properly would require a computer version of the Irish STV rules which was not available. Instead, the ERS rules were used, which has a number of differences from the Irish version. The most obvious difference is rounding the votes to whole numbers (single ballot papers are transferred), rather than one hundredths; but this makes little difference in this case with over 10,000 votes cast in each election.

To summarise, the program takes as input:

1. The transfers between parties deduced from a set of elections.

2. The result sheet from a specific election from that set, giving the party affiliation of each candidate.

3. Seeds for the random number generator, and a number of trials from which to select the ballot set with the best fit.

From this, the program outputs a set of ballot papers giving a ‘good’ fit to the specified election. Note that by changing the seeds for the random number generator, slightly different sets of ballot papers will be produced.

This program was then used to construct plausible ballot sets for the 1969 and 1973 Irish elections. The elections in 1973 were regarded as distinct from 1969, so that the same process as illustrated above was used to construct another table of transfers per thousand votes between parties.

A summary of the results from analysing the election data appears below. The meaning of the entries in the table are as follows:

D*n* On my home computer, I have nine different STV-like algorithms. Listed here is the number of algorithms giving a different result from the actual Irish election. A result of D0 is not printed.

C*n* A Condorcet ranking is computed from the election data. From this, the lowest-ranked candidate is found who was elected. C*n* is the number of un-elected candidates ranked at least as high as that candidate.

P*n* From the Condorcet ranking, a Condorcet paradox is evident. P*n* indicates the number of candidates involved in the paradox. The plus sign indicates that the paradox involves both elected and un-elected candidates. (Note that a Condorcet paradox involving the ‘top’ candidate is undoubtedly a problem when electing a single candidate, but not necessarily in other cases.)

IEM Of the nine STV algorithms that were used to analyse the data, two are of special interest: Meek and the ERS hand-counting rules. Of the three when the Irish result is compared, the odd-one-out is noted (by a single letter). (Note that in the single case of Dublin SW for 1969, all three algorithms gave a different result, so there was not an ‘odd-one-out’.)

Consistuency	Result 69				Result 73			
Carlow-Kilkenny		C1	P3+		D2		P5	
Cavan		C1			D2		P3	
Clare	D6	C2		M		C1	P4+	
Clare-S Galway		C1			D2	C1		
Cork NW	D3	C3	P4+		D2	C1		
Cork SE	D2				D2	C1		
Mid Cork	D2					C1		
NE Cork	D6	C1		M	D7	C1		M
SW Cork	D1	C2	P3+		D1	C2	P4+	
NE Donegal	D1							
Dublin C		C1						
Dublin NC	D1	C1				C2	P5+	
Dublin NE		C2			D2	C2	P3	
Dublin NW		C1	P5+			C1		
Dublin SC	D1	C4			D8	C1		M
Dublin SE		C1			D8	C1		M
Dublin SW	D9	C3	P6+	ME		C2	P4+	
NC Dublin						C1	P3+	
SC Dublin	D1				D1	C1		
Dun Laoghaire -					D2			
NE Galway						C1		
W Galway					D3	C1		M
N Kerry					D6	C2	P5+	I
S Kerry		C1	P4+			C1		
Kildare		C1						
Laois-Offaly	D6	C1	P4+	M	D3	C1		E
E Limerick		C1				C1		
W Limerick					D1			
Longford-W	D1	C4			D8	C4	P3,3+	M
Louth					D1		P3	
E Mayo		C1	P4+		D1	C1		
W Mayo		C1						
Meath	D1	C1				C1	P3+	
Monaghan	D2	C1			D8	C2		M
Roscommon -							P3	
Sliogo-Leitrm						C1		
N Tipperary		C1			D3	C1	P3+	
S Tipperary		C2						
Waterford	D7	C1	P3+	I	D2	C2	P5+	
Wexford		C3	P5+					
Wicklow	D9	C2		I				

The method of construction implies that it would be unwise to assume that there was an actual Condorcet paradox for South West Cork, since this property is dependent in part upon the data which has been added by statistical means. However, it would be reasonable to suppose that the fraction of elections in Ireland having a Condorcet paradox is about one third, and about a quarter have a paradox involving elected and unelected candidates.

In many cases, the election result is clearly marginal between two candidates, and hence differences between the STV algorithms is not surprising.

Two elections stand out as being very different. For Dublin South West for 1969, all three main algorithms gave a different result. After the top candidate, the next six were in a Condorcet paradox. It seems clear that this seat is a potential example of non-monotonicity. I have been unable to determine if this is so, since I do not know of any computationally feasible way of determining the property. As an exercise for the readers, I have reproduced the result sheet, together with the fit my program produces, to allow others to determine if non-monotonicity occurs. I have been able to simplify the data by reducing the number of piles substantially, and also reduced the number of votes by a factor of ten, but this still does not provide an easy way of determining this vital property. David Hill has commented on this by noting that perhaps the property is not so important if it is impractical to determine its validity for a specific election.

The other unusual result is that for Longford-Westmeath for 1973. This is the only case in which there were two sets of candidates involved in Condorcet paradoxes in one election.

There is only a weak correlation between those elections having C≠0 and those having D≠0. There is some correlation between the C's and P's, which is hardly surprising due to the underlying dependence upon Condorcet. A Condorcet paradox involving both elected and unelected candidates is no guarantee that any of the STV algorithms will produce a different result as can be seen from Dublin North Central for 1973.

All the computer data produced in this study is available from me on request.

Acknowledgement

This work would not have been possible without the excellent work of J Knight and N Baxter-Moore in tabulating and presenting the results of the 1969 and 1973 Irish elections.

References

1 B A Wichmann. An STV Database. *Voting matters*, issue 2, p9.

2 B A Wichmann. A simple model of voter behaviour. *Voting matters*, issue 4, pp3-5.

3 J Knight and N Baxter-Moore. *Republic of Ireland: The General Elections of 1969 and 1973*. The Arthur McDougall Fund. London. 1973.

Appendix

The table below is the Irish result sheet as from Knight and Baxter-Moore, except that additionally the results computed by the program from the plausible data are shown in italics.

The actual event elected FF1, LA1, LA2 and FF2. The ERS rules with the plausible data elected FF1, LA1, LA2 and FG1, while the Meek algorithm with the plausible data elected FF1, LA1, LA2 and FG2.

There is a single Condorcet winner in LA1, but the set of candidates FF1, FF2, FG1, FG2, LA2 and OT1 are in a Condorcet paradox with the plausible data

Candidate	Stage I	Stage II		Stage III		Stage IV		Stage V		Stage VI		Stage VII		Stage VIII		Stage IX		Stage X		Stage XI		Stage XII	
Dowling	5724	4	5728	12	5740	22	5762	15	5777	651	6428	-589	5839		5839		5839		5839		5839		5839
FF1	*5724*		*5724*		*5725*		*5726*		*5726*		*5726*		*6186*		*5839*		*5839*		*5839*		*5839*		*5839*
Lemass	2512	6	2518	11	2529	13	2542	3	2545	771	3256	520	3776	43	3819	28	3847	15	3862	73	3935	772	4707
FF2	*2512*		*2564*		*2564*		*2564*		*2564*		*3487*		*3757*		*3917*		*3983*		*4020*		*4210*		*5147*
Sherwin	1643	5	1648	12	1660	14	1674	5	1679	-1679													
FF3	*1643*		*1643*		*1643*		*1643*		*1643*														
O'Keeffe	1331	11	1342	341	1683	5	1688	242	1930	22	1952	2	1954	21	1975	88	2063	23	2086	-2086			
FG1	*1331*		*1343*		*1800*		*1800*		*2050*		*2050*		*2050*		*2050*		*2100*		*2174*				
McMahon	1203	16	1219	193	1412	18	1430	579	2009	43	2052	8	2060	26	2086	91	2177	22	2199	1539	3738	767	4505
FG2	*1203*		*1220*		*1320*		*1320*		*1933*		*1983*		*2021*		*2021*		*2021*		*2206*		*3689*		*5594*
Lowe	856	4	860	94	954	10	964	-964															
FG3	*856*		*862*		*963*		*963*																
Redmond	759	5	764	-764																			
FG4	*759*		*760*																				
O'Connell	5273	33	5306	38	5344	169	5513	31	5544	61	5605	10	5615	435	6050		6050	-211	5839		5839		5839
LA1	*5273*		*5298*		*5298*		*5509*		*5509*		*5609*		*5609*		*6359*		*6359*		*5839*		*5839*		*5839*
Dunne	5136	22	5158	23	5181	468	5649	20	5669	129	5798	23	5821	1065	6886	-1047	5839		5839		5839		5839
LA2	*5136*		*5149*		*5150*		*5771*		*5771*		*5781*		*5781*		*6459*		*5839*		*5839*		*5839*		*5839*
Butler	1643	10	1653	10	1663	136	1799	11	1810	10	1820	4	1824	-1824									
LA3	*1643*		*1649*		*1649*		*1659*		*1759*		*1809*		*1809*										
Farrell	893	4	897	1	898	-898																	
LA4	*893*		*894*		*894*																		
Corcoran	2066	28	2094	24	2118	29	2147	45	2192	38	2230	22	2252	186	2438	195	2633	90	2723	196	2919	-2919	
OT1	*2066*		*2086*		*2186*		*2186*		*2186*		*2236*		*2274*		*2444*		*2906*		*3128*		*3568*		
McKeown	154	-154																					
OT2	*154*																						
Non transferable		6	6	5	11	14	25	13	38	14	52		52	48	100	645	745	61	806	278	1084	1380	2464
NT			*1*		*1*		*52*		*52*		*52*		*54*		*105*		*147*		*149*		*210*		*936*
Total	29193		29193		29193		29193		29193		29193		29193		29193		29193		29193		29193		29193

Dublin South West, 1969

Issue 6, May 1996

Editorial

A survey has been conducted of the readership of *Voting matters* which has resulted in a number of changes; these changes are reported on page 9. I have written individually to all those that took the trouble to write to ERS. Please write again if you have further suggestions, and especially if you have material for potential inclusion.

This issue contains five articles. The first is a republication of a further article by Brian Meek. Readers should take note of the preface which points out the very different nature of this article from the other two that *Voting matters* has republished. The second article contains a description of mine of a two-tier form of STV. I am not advocating this, since it appears to be inferior to standard STV.

The third article is a very detailed analysis of the degree of representativity in Irish STV elections by Philip Kestelman. Please note the use of the term *magnitude* to mean the number of seats in a multi-seat election.

Douglas Woodall's article is a very detailed analysis of the rules that could be used for single-seat elections. The importance of this work in my view is that of questioning the desirability of the property that later preferences should not harm or help earlier ones. Whatever your own views are, I hope you will note the consequences of the various impossibility theorems which shows that, even with just one seat, conflicting properties abound. This article does define a large number of terms but I hope readers will find the explanation of those terms adequate.

The last article is by David Hill which analyses the results which have previously been reported in *Replaying the 1992 General Election*. This paper illustrates the difficulties in producing accurate predictions for an STV election when only 9,614 ballot papers are available for all of the UK.

Brian Wichmann

Note in this reprinting

Brian Meek died in 1997: see end of this issue, page 17.

A transferable voting system including intensity of preference

B L Meek

Brian Meek is now at the Computing Centre, King's College London, Strand, London WC2R 2LS. This article was originally published in *Mathématiques et Sciences Humaines*, 13, No 50 1975 pp23-29.

Preface to this republication

After I wrote the two papers describing what has since become known as ‘Meek's method’ — published (in French) in *Mathématiques et Sciences Humaines* in 1969 and 1970, and republished in English in *Voting matters* No.1 — I went on to write a third paper, which the same journal published (in English!) in 1975. Some people have been aware of the existence of this third paper, and this led to a request that it too be republished in *Voting matters*. I have no objection to this being done, but it is important to stress that its status is quite different from the other two.

The first two papers present my analysis of STV counting, and how it can be made as accurate as possible. The method totally accepts the basis of STV as it is, and does not alter or challenge its fundamental assumptions at all. (It does seem to challenge some people's own assumptions about STV, but that's not the same thing at all!) As such, ‘Meek's method’ was always intended as a practical method for conducting an STV count, albeit an expensive one at that time — far less so now, of course. Years later, David Hill, Brian Wichmann and Douglas Woodall demonstrated beyond question that it is a practical method, and earned my eternal gratitude for so doing.

This third paper does not have that status at all. It is in fact no more than an academic exercise, exploring an issue which arises from time to time in the literature on aggregation of individual preferences. It demonstrates that a method of taking account of intensity of preference is possible. This is very far from advancing it as a practical method for implementing an election.

I have never regarded it as a practical method. I do not advocate its adoption, and I shall be very annoyed if anyone attempts to present it as (say) ‘Meek's proposal’ or otherwise imply that I advocate its use. It should not even be linked to 'Meek's method' (e.g. by alleging it is an *extension* to my method), at least without very careful qualification. The reason is that 'Meek's method' is STV, whereas the process described in this paper is *not* STV. (It is certainly not a 'single' transferable vote, for a start.) The way that votes are cast and interpreted is quite different from STV.

To be sure, the vote *counting* shares some similarities, but that is only because the same logic that led to the invention of STV and to the Meek method has been applied to the aggregation process. The individual votes being aggregated are, however, not STV votes. The consequence is that the result can end up very far from STV, as the paper itself clearly shows.

So the paper should be read for what it is, a mathematical demonstration that individual preferences can be fairly aggregated while still taking intensity of preference into account, and not as a suggested practical method for conducting elections. If that is done, there should be no misunderstandings. A voting system, derived from the STV (Single Transferable Vote), is described which includes intensity of preference while avoiding difficulties due to inter-personal comparison of utilities. It is shown that this system allows the voters some control over the method used to aggregate their preferences.

Introduction

This paper describes a voting procedure with a number of interesting properties. Chief among these are the inclusion of intensity of preference in a non-controversial manner — i.e. in a way which avoids the difficulty of inter-personal comparison of utilities — and that in various limiting cases the procedure is equivalent to well-known voting systems such as simple majority, the single transferable vote, the single non-transferable vote, etc. The paper first describes the voting procedure, then looks at the properties mentioned, and finally shows that the procedure offers a partial solution to the problem of determining which voting procedure to use in some decision situation.

The procedure

Any voting procedure consists of two parts — that of vote casting, and that of vote counting. In this case the vote casting procedure for the elector is to assign weights to the different candidates to indicate the order and strength of his preferences between them. It is a basic assumption that strength of preference is transitive, e.g. that if a voter thinks that he prefers A twice as much as B, and B three times as much as C, then he prefers A six times as much as C and can express his preferences by assigning weights to A, B, and C in the ratio 6:3:1.

The vote counting procedure begins by normalising all the weights w_{ij} which the ith voter gives to the jth candidate, so that

$$\sum_{j=1}^{j=c} w_{ij} = 1, \text{ all } i$$

c being the number of candidates. This is the key, as we shall see later, to the avoidance of troubles due to inter-personal comparison of utilities, since it ensures that as far as possible each voter has an equal say in the voting procedure.

The count proceeds by summing all the weights for all the candidates, i.e. calculating

$$W_j = \sum_{i=1}^{i=v} w_{ij} \quad \text{for all } j,$$

v being the number of voters. Thereafter the count proceeds much in the same way as in the single transferable vote, as modified by the proposals in two earlier papers[1,2]. An STV-type quota is calculated according to the formula

$$\left[\, q = \frac{W}{s+1} + 1 \,\right]$$

where s is the number of seats to be filled and

$$W = \sum_{j=1}^{j=c} W_j$$

is the total vote, and the brackets indicate that the integral part is to be taken. q is the minimum number such that, if s candidates have that number, any other candidate must have less than that number.

(In practice it is likely that working will be to fractions of votes — say three decimal places, in which case the "+1" in the formula for q is replaced by "+0.001", or equivalently the weights w_{ij} are normalised to sum to 1000 for each voter and the formula for q is unaltered.)

The count may proceed by one of two steps. If no W_j exceeds q, i.e. no candidate has reached the quota, then the candidate with lowest W_j, say candidate x, is eliminated. All the w_{ij} are then renormalised with all w_{ix} made equal to zero. The principle adopted is that if a candidate is eliminated the count proceeds as if that candidate had never stood; the assumption is that the elimination of a candidate does not alter the voter's relative preferences between the remaining candidates. (It is of course quite possible to take issue with this assumption.)

If, however, a candidate, say y, has W_y greater than q, another renormalisation takes place so that W_y is reduced to q. This means that all w_{iy} are reduced by the factor q/W_y, and all w_{ij}, $j \neq y$, are increased by the factor $(1 + w_{iy}q/W_y)/(1 - w_{iy})$. By this means the weights allocated by each voter i are adjusted in a quite natural way, so that those supporting y give him no more support than is necessary to ensure his election.

Counting continues by the application of one or other of these rules until the requisite number s of candidates are elected. Once elected and allocated the quota q the weights for that candidate are of course not included in the recalculation. This makes the procedure somewhat simpler than in the modified form of STV described in [1]. However, if all of a voter's choices — i.e. those candidates he has allotted a positive weight — are eliminated, the quota q has to be recalculated as in [2] so that this undistributable vote is not included; similarly, when all a voter's choices have been elected and allotted recalculated weights, the residue is non-distributable and also must be subtracted from W. Recalculation of the quota does of course imply recalculation of the weights of elected candidates, and an iterative procedure as described in [2] can be used to obtain the new q and w_{iy} to any desired accuracy.

Intensity of preference

When expressed crudely in the form “It is of more benefit to me to have A rather than B than it is for you to have B rather than A”, inter-personal comparison of utilities is patently invidious. Nevertheless in actual voting situations intensity of preference is often taken into account. If A and B want to go to a museum when C wants to go to the funfair, the collective choice is frequently the funfair, without any sense of dictatorship or lack of democracy, simply because all know that C's preference is much the most intense.

Lest this be regarded as too trivial an example, it is often the case in committee that the collective choice for chairman is X, even though a majority prefer Y, simply because a substantial minority strongly object to Y. Any theory of voting which does not allow for intensity of preference is certainly incomplete, and any voting system which does not permit its expression cannot be wholly satisfactory.

The present system is based on two principles: that the only person who can gauge the intensity of his preferences is the voter himself; and that as far as possible each voter should contribute equally in the choice of those elected. In a multi-vacancy election ($s > 1$) there is more than just a single choice involved, and so it makes sense to allow a voter to express his preference intensities by contributing all his voting power to the choice of one candidate, or to share this power between the choices of different candidates. Of course, it is possible to regard an s-vacancy election as a single choice from the ${}^nC_s$ possible combinations of s candidates out of n elected, but this view invalidates the assumption that elimination of a candidate does not alter the voter's relative preferences. This is because each combination is independent; a voter may rank candidates individually A, B, C, D in that order, but rate them in pairs AB, BD, CD, BC, ... since he thinks A will only work satisfactorily in combination with B. This kind of multiple election is essentially the election of a ***team*** of s people, rather than s individuals. STV, and the present system, is concerned with choosing a set of s independent individuals from a larger set of c candidates. An STV vote is a vote for one individual (the first choice) and only subsidiarily and in special circumstances for lower choices. The present system enables the voter to have a say in all the s choices if he wishes, but his share in the whole decision process remains the same, up to the wastage involved in nontransferable votes or those given to unelected candidates who remain when the s winners have been chosen.

Equivalence to other voting systems

(a) STV

Let $1 > \varepsilon > 0$. Let the voters order their choices $1-\varepsilon$, $\varepsilon-\varepsilon^2$, $\varepsilon^2-\varepsilon^3$,..... $\varepsilon^{c-2}-\varepsilon^{c-1}$, ε^{c-1}. Then the closer ε is to 0 the closer the actual voting process becomes equivalent to STV. For example, suppose there are 5 candidates and $\varepsilon = 0.01$. A voter's choice will be in the proportions 0.99, 0.0099, 0.000099, 0.00000099, 0.00000001, counting 99% for his first choice. If his first choice is eliminated, the four lower votes remain, and total 0.01. These have to be renormalised to add up to 1, and so are multiplied by 100 to give 0.99, 0.0099, 0.000099, 0.000001. A similar argument applies to votes transferred from elected candidates.

(b) Single non-transferable vote

This, trivially, is when the voter gives 1 to his first choice and 0 to all the others.

(c) Simple majority with multiple vote

Here the voter gives $1/s$ to each of s candidates, or perhaps $1/k$ to each of k candidates, $k < s$. These are special cases of giving α to k candidates and β to $c-k$ candidates, where $\alpha k+\beta(c-k) = 1$ giving a weighting between a more preferred and a less preferred group.

(d) Cumulative vote

In this case the voter gives α_1, α_2, ... α_k, to k candidates respectively, such that

$$\sum_{i=1}^{i=k} \alpha_i = 1$$

For an exact analogy to the cumulative vote each α_i must be a multiple of $1/s$.

The choice of voting procedure

Such a voting system would require a more than usual sophistication on the part of the voter. This being so, one can consider a further sophistication. The choice of voting procedure is one of immense importance in the democratic process, and no system is wholly stable wherein a substantial minority is dissatisfied with the voting procedure in current use. The required consensus may either be achieved through ignorance or habit, or by general agreement that a system is fair even though another may be advantageous to many, perhaps even a majority. In situations where there is awareness of and controversy about the different properties of voting systems, the present system offers a possible way out of deadlock. For, if most voters use the system in one of the ways described in the last section, then the election will be largely determined according to that voting procedure. Looking at it from the point of view of parties, each party can urge the voters to use the method they favour of filling in the ballot forms. However, it is a weakness in this area that voting systems are so often argued about in terms of fairness to parties or candidates, seldom in terms of fairness to voters. The present system, whose main fault is its complexity, has the virtue of that fault in that each voter can specify as precisely as he wishes the way his vote is to be counted, without this being imposed by others on him or on others by him. Most voting systems allow some such flexibility; the virtue of this system is the much greater precision with which the sophisticated voter can specify his wishes, without his being able by strategic voting to exercise more influence on the final result than is implied by his actual possession of a vote.

Concluding remarks

Despite the scope for manipulation which the system offers, it is clearly derived from and shares the principles of the STV system, particularly with the concept of the quota and the transferability of votes above the quota. One of the chief objections to STV is that it does not guarantee the election of a Condorcet winner, e.g. when one candidate is everyone's *second* choice. While the present system does not guarantee the election of such a candidate (this is obvious, since as shown earlier the system can approach arbitrarily closely to STV), it does render it more likely, and will ensure it provided that the weights given to the candidate are large enough i.e. if the candidate is considered a *good enough* substitute for their first choice by a sufficient number of electors. The price that one has to pay for this improvement to STV is the greater complexity, particularly for the voter.

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A form of STV with single-member constituencies

B A Wichmann

Brian Wichmann is the editor of *Voting matters*

One over-riding concern that appears in the Plant report is the desire to retain single-member constituencies in any reform of the electoral system for the House of Commons. A natural question is if STV can be adapted in some way to retain single-member constituencies, but avoiding the non-proportionality of the Alternative Vote (AV). This paper presents such an adaptation.

The basic idea is to use a two-tier system in a similar manner to the German system by having single-member constituencies augmented by members elected in a more proportional manner. The second tier is a group of constituencies which, for convenience, we call a *county*. The electors provide two ‘votes’ by giving the usual preferences to the candidates in their constituency and then also providing a preference vote to *all* the candidates in their county.

The election proceeds in two stages, firstly each constituency is considered individually using STV (which degenerates to AV). However those votes which have *not* been used to elect a candidate here are forwarded to the county vote (or second stage). The county vote first eliminates those candidates already elected at the first stage, and then uses STV to fill the county seats.

The main parameters of this voting system are the number of single-member constituencies used to form a *county*, and the number of seats available at the county level. It appears that about 5 (or more) constituencies should be grouped into a county in order to provide reasonable proportionality and that the number of county seats should be not less than 2 for the same reason.

This system is quite different from conventional STV for a number of reasons:

1. This system, like FPTP has *safe* seats, whereas STV has no such equivalent. For instance, in the Irish elections, almost every constituency has a Fianna Fail or Fine Gael candidate who is *not* elected. My reason for concluding this is that I believe that the main parties, even for safe seats, would not propose more than one candidate since this would appear to present a divided party.

2. The elector's ability to select within a party is restricted. If you are a Conservative party supporter in a safe Conservative constituency with a male candidate, you could not select a woman candidate (given the restriction noted above of a single candidate). On the other hand, if you were in a Labour constituency, your vote would be *wasted*, allowing you to select a woman candidate from the county list as your first preference.

3. Minority interests would be represented at the county level. These interests would be accumulated as wasted votes and hence would have a good chance of representation, depending upon the number of county seats.

Of course, the advantage of this system is that there is no reliance upon the ordering of a party list which is outside direct voter control.

There are some technical details to resolve. I have based my proposal on the use of the Meek algorithm for STV, although this is not strictly essential. However, it is clearly important to compute the fraction of each vote which is wasted (from the first stage) in order to conduct the second stage. This is straightforward since for each voter who contributes to the elected candidate, the percentage wasted is simply the percentage of votes above the quota. This implies that about ½ of the votes would go forward to the second stage. This might imply that about half of the seats should be at county level, but a smaller number is probably satisfactory.

My belief is that this proposal would be quite easy to implement, at least using the Meek algorithm. However, since we have no similar system, it does not seem possible to construct realistic data with which to do any serious study of its suitability.

Is STV a form of PR?

P Kestelman

Philip Kestelman is keen on measuring electoral representativity, and works in the area of family planning

Introduction

In my view, Single Transferable Voting (STV) is the best electoral principle: whether electing one representative by Alternative Voting (AV), or several representatives by multi-member STV. The Collins English dictionary succinctly defines *proportional representation* (PR) as “representation by parties in an elective body in proportion to the votes they win”.

The 1937 Irish Constitution prescribes that both the President and parliamentary deputies (TDs) shall be elected “on the system of proportional representation by means of the single transferable vote”. Of course, PR is *not* an electoral system; but a principle, to which different elections approximate to widely varying degrees.

Accordingly, the basic question is whether STV achieves PR; and if so, how far? To answer this question, we need some overall measure of electoral representativity (‘pro-portionality’); of which the simplest is the *Rose Index*[12]. For reasons which will become apparent, I have renamed the Rose Index, *Party Total Representativity* (PTR).

Party

Table 1 demonstrates the calculation of PTR, for the 1994 European Parliamentary Election in the Irish Republic. Notice that the total over-representation of all over-represented party votes (+23.7% of first preferences) is equal and opposite to the total under-representation of all under-represented party votes (−23.7%). This overall deviation is the *Loosemore-Hanby Index* (LHI) of party disproportionality[10], — “the most widely used measure of disproportionality”[9].

Thus LHI measures the total under-representation of all under-represented party-voters. Complementing LHI is the Rose Index, PTR = 100.0 − 23.7 = 76.3% of first preference votes. For comparison, in the 1994 European Parliamentary Election in Britain (First-Past-the-Post), PTR = 70.4%. This low British PTR (definitely *not* PR) approximated the Irish PTR (76.3%); and the corresponding STV final count PTR (81.7%) was little higher.

Cole[3] over-estimated final count PTR by excluding non-transferable votes. Moreover, non-transferable votes are under-counted by conventional STV proportionating Droop Quota surplus votes among transferable next preferences (ie. continuing candidates only[11]). Besides, “using later-stage figures overstates the proportionality of STV”[6].

Table 1: Party Representativity analysis of the European Parliamentary Election, Irish Republic 1994.

Party	Votes (V%) first	Votes (V%) final	Seats (S%)	Deviation (S%-V%) first	Deviation (S%-V%) final
Total	100.0	100.0	100.0	0.0	0.0
Fianna Fáil	35.0	37.4	46.7	+11.7	+ 9.3
Fine Gael	24.3	30.8	26.7	+ 2.4	- 4.1
Labour	11.0	4.2	6.7	- 4.3	+ 2.5
Green	7.9	8.9	13.3	+ 5.4	+ 4.4
Cox (Munster)	2.5	4.6	6.7	+ 4.2	+ 2.1
Others/Non-transferable	19.4	14.2	0.0	-19.4	-14.2
Over-represented	69.6		93.3	+23.7	
		55.1	73.3		+18.3
Under-represented	30.4		6.7	-23.7	
		44.9	26.7		-18.3

Source: *Irish Times*, 14 June 1994.

In the first four European Parliamentary elections (1979-94), the Irish PTR ranged from 76.3% to 87.0% of STV first preferences; hardly more representative than the British PTR, ranging from 70.4% to 78.6%. In the 1990 Irish Presidential Election, PTR increased from 38.9% of first preferences to 51.9% of final preferences yet nobody regards AV as PR!

Indeed, none of the foregoing STV elections has achieved anything like PR. However, in the last six Irish general elections (1981-92), PTR has ranged from 90.1% to 96.9% of first preferences, as may be seen in Table 2.

Apparently, multi-member STV is only ‘semi-proportional’. More remarkably, three and five member STV constituencies mediated comparable representativity. This refutes the widespread belief that “political science research establishes conclusively that PR electoral districts must elect at least four MPs before they deliver proportional outcomes”[5]. Indeed, four member STV constituencies proved invariably *less* representative than either three or five member constituencies, although the differences were small.

Table 2: Party Total Representativity by district magnitude in Irish Republic general elections.

	District magnitude			
Date	All	3	4	5
1981	94.2	95.4	89.8	94.7
1982 (Feb)	96.6	97.4	95.6	95.8
1982 (Nov)	95.8	97.4	92.8	95.3
1987	90.1	89.5	89.1	89.9
1989	92.9	94.0	91.1	92.2
1992	91.8	90.2	89.5	91.5

Source: Dáil Éireann[4]

Cumbency

Bogdanor[1] observed that STV advocates prefer to secure “proportional representation of opinion ... which cuts across party lines. But since they do not give a clear operational definition enabling one to measure ‘proportionality of opinion’, it becomes difficult to offer any evaluation of their claim”. Nonetheless, published election results provide some usable, *non*-party data for each candidate, including *Cumbency*: that is, whether incumbent (immediately previous representative) or non-incumbent (‘excumbent’).

Analogously to party, consider the relationship between cumbency first preferences and seats. Instead of PTR, incumbent and excumbent candidates are treated as representing two different parties; and *Cumbency Total Representativity* (CTR) is calculated, as in Table 3.

Table 3: Cumbency Total Representativity by district magnitude for Irish Republic general elections

	District magnitude			
Date	All	3	4	5
1981	86.2	85.0	83.8	88.6
1982 (Feb)	85.0	77.9	92.6	83.0
1982 (Nov)	83.7	72.6	87.0	87.3
1987	81.6	80.2	79.9	83.8
1989	87.9	96.2	82.2	86.9
1992	85.4	76.7	90.4	86.3

Source: Dáil Éireann[4]

Such low CTRs arise from incumbents invariably over-representing their first preferences (high incumbent S%−V%). Notice the distinction between this finding and the unsurprisingly, greater electability of incumbent candidates (high incumbent S%−C%, where C% is the fraction of incumbent candidates).

Of course, incumbents are far more likely than excumbent candidates to be men. Hence the importance of disentangling cumbency from gender.

Gender

At the 1992 Irish General Election, 19% of candidates were women: 8% of incumbents and 24% of excumbent candidates. Among elected candidates (TDs), only 12% were women: 8% of incumbents, and 23% of excumbent TDs. Thus allowing for cumbency, TDs fairly represented candidates by gender.

What of the relationship between votes and seats, by gender (electoral representativity proper)? In 1992, voters cast 13% of their first preferences for women candidates: slightly under-represented by women TDs (12%).

Table 4: Gender Representativity Ratio by district magnitude in Irish Republic general elections 1981-89

	District magnitude			
Cumbency	All	3	4	5
All	0.94	1.20	1.04	0.80
Incumbent	1.10	1.01	1.19	1.02
Excumbent	1.07	2.26	0.98	0.89

Source: Dáil Éireann[4]

As with cumbency, we could calculate a *Gender Total Representativity* (GTR) for each election and district magnitude. However, because there are only two genders (non-transferable!), and so few women candidates (and hence votes for women), it seems more illuminating to aggregate the previous five general elections (1981-89); and to calculate *Gender Representativity Ratios* (GRRs).

GRR is the ratio of female seats per vote to male seats per vote (first preference). Table 4 gives GRR, by district magnitude and cumbency.

In 1981-89 overall, first preferences for women candidates were slightly under-represented (GRR = 0.94). However, allowing for cumbency, women TDs slightly over-represented their first preferences (excumbent GRR = 1.07).

Of particular interest, three member STV constituencies over-represented votes for women by 20% (GRR = 1.20); leaving them under-represented in five member constituencies by 20% (GRR = 0.80) overall. Among excumbent candidates in three member constituencies, first preferences for women were over-represented even more spectacularly; only 5% of votes electing 10% of the TDs (GRR = 2.26). By contrast, in five member constituencies, 15% of the voters for excumbent candidates preferred women, represented by 14% of the TDs (GRR = 0.89).

Alphabetic bias

It is widely believed that candidates appearing high on ballot-forms enjoy some electoral advantage. On Irish general election ballot-forms, candidates' names are listed in surname-alphabetic order. Voters' preferences for (less familiar) excumbent candidates may well be more vulnerable to 'Positional Voting Bias'[14].

Notice that we are interested here in three distinct relationships: between candidates and votes (first preferences): between candidates and seats; and between votes and seats (electoral representativity proper). Aggregating five Irish general elections (1981-89), Table 5 confirms that higher placed excumbent candidates attracted disproportionately more first preferences (V%/C% decreasing, from 1.18 for A-C surnames, to 0.89 for N-Z surnames).

Table 5: Excumbent Candidate Surname/Forename Representativity Index Irish Republic general elections 1981-89

Name	Initial letter	Vote/ Candidate =V%/C%	Seat/ Candidate =S%/C%	Seat/ Vote =S%/V%
Surname	A-C	1.18	1.20	1.02
	D-J	0.99	1.12	1.12
	K-M	0.96	0.80	0.84
	N-Z	0.89	0.91	1.01
Forename	A-E	0.95	0.95	1.01
	F-K	1.08	1.02	0.94
	L-P	1.06	1.32	1.24
	Q-Z	0.88	0.60	0.68

Source: Dáil Éireann[4]

Consequently, excumbent TDs over-represented candidates with A-C surnames by 20% (S%/C% = 1.20); under-representing those with K-M surnames by 20% (S%/C% = 0.80). However, notice something else: excumbent candidates with L-P forenames were even more over-represented (S%/C% = 1.32); leaving those with Q-Z forenames even more under-represented (S%/C% = 0.60). All the more remarkable, considering that forenames are *not* ordered alphabetically on ballot-forms; and perhaps voters' preferences for surnames were not positional, after all!

Relating excumbent first preferences to seats (electoral representativity proper), D-J surnames and L-P forenames were over-represented (by 12% and 24%, respectively); while K-M surnames and Q-Z forenames were under-represented (by 16% and 32%, respectively). How should we compare surname with forename representativities overall?

We could treat every single name-initial letter of the alphabet like a party (N=22), and calculate both *Surname Total Representativity* (STR) and *Forename Total Representativity* (FTR). Aggregating five Irish general elections again gives Table 6, comparing STR with FTR by district magnitude and cumbency.

Table 6: Excumbent Candidate Surname/Forename Total Representativity Index by district magnitude in Irish Republic general elections 1981-89

	District magnitude			
Cumbency	All	3	4	5
All	95.5/94.8	93.3/91.7	91.8/89.7	93.1/95.6
Incumbent	97.1/96.4	96.0/95.3	94.4/93.4	96.1/97.2
Excumbent	90.7/88.6	75.3/80.8	83.8/85.0	85.2/88.1

Source: Dáil Éireann[4]

Overall, first preferences for surnames and forenames were represented with comparable fidelity (STR = 95.5%: FTR = 94.8%); again, with little difference by district magnitude. Among excumbent candidates, TDs represented surnames slightly more faithfully than forenames (STR = 90.7%: FTR = 88.6%) overall; by district magnitude, somewhat less. Altogether a muddy picture, without obvious implications for ordering candidates' names on ballot forms (surname — alphabetical or random).

Conclusions

Considering the quantitative notion of PR, the measurement of electoral representativity remains curiously neglected. The simplest measure of overall party disproportionality, the Loosemore-Hanby Index (LHI), complements the Rose Index, or Party Total Representativity (PTR). Indeed, PTR may be construed as the degree to which any given election — from a national aggregate down to a single member constituency — achieves PR (rarely 100%).

Single member STV (Alternative Voting) hardly mediates PR, even at the national level (as in Australia[2]). In the four European Parliamentary elections in the Irish Republic, even multi-member STV has only achieved PTRs ranging from 76% to 87%: scarcely more representative than First-Past-the-Post in Britain: ranging from 70% to 79%.

However, the last six Irish general elections (1981-92) have proved considerably more representative, PTR ranging from 90% to 97%. Thus multi-member STV alone mediates quasi-PR[15]; requiring a few additional members to guarantee PR (eg. final count best losers: ‘STV-plus’, as in Malta[7]).

More remarkably, Irish three and five member STV constituencies have proved comparably representative. That is good news for voters, oppressed by the lengthy ballot-forms characterising larger STV constituencies (perhaps listing 20 names). It is equally good news for reformers, dismayed at the prospect of anonymously vast STV constituencies, electing as many as seven MPs (eg. representing all three London boroughs of Greenwich, Lewisham and Southwark[13]).

The concept of *Total Representativity* proves a most versatile tool, even beyond party considerations. In respect of cumbency, multi-member STV remains invariably non-PR; with Cumbency Total Representativity ranging from 82% to 88%. On the other hand, first preferences for women candidates have been represented near-proportionally; with an aggregate female-to-male S%/V% ratio of 0.94 overall. Nonetheless, three member STV constituencies over-represented votes for women, under-represented in five member constituencies.

Aggregating five Irish general elections also confirmed that excumbent candidates listed higher on ballot-forms tended to attract disproportionately more first preferences; thereby over-representing candidates with A-C surnames, and under-representing K-M surnames(S%/C%). Yet TDs over/under-represented candidates with L-P/Q-Z forenames even more steeply. Moreover, first preferences for both surnames and forenames were represented with comparable fidelity. It may not be so important to randomise ballot-forms after all: another relief for preferential voters accustomed to alphabetic order!

At best therefore, in mediating party first preferences (the main consideration), multi-member STV alone is not quite a form of PR. Nonetheless, in national parliamentary elections, Irish STV has proved far more representative than British FPP. That conclusion may be brought even closer to home, by calculating another measure (perhaps user-friendlier than PTR).

Under both AV and FPP, around half of all voters elect candidates; whereas under multi-member STV, nearly 90% of voters elect at least one representative of their preferred party. In Irish general elections, this *Constituency-Represented Party Vote-fraction* (CRPV) has also proved conspicuously invariant with district magnitude, as shown in Table 7.

Maximising each CRPV, multi-member STV minimises vote-wastage. Thus quantifying STV's principal virtue, CRPV should allay the concern over STV — apart from its complexity — expressed by the Plant Working Party on Electoral Systems[8]. Of course, STV enjoys other virtues!

Table 7: Constituency-Represented Party Vote-fraction by district size for Irish Republic General Elections 1981-89

Date	District size			
	All	3	4	5
1981	90.8	91.3	86.9	93.0
1982 (Feb)	92.6	94.0	91.3	92.8
1982 (Nov)	92.3	94.6	89.4	93.4
1987	83.6	85.3	81.0	84.8
1989	87.3	90.9	84.9	86.7
1992	86.7	85.7	84.9	88.7

Source: Dáil Éireann[4]

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Changes to *Voting matters*, as recommended by the Technical Committee of the ERS are as follows:

1. You can see that a subtitle now appears. The reason for this is that some readers did not appreciate the technical nature of the publication.

2. As Editor, I will try to avoid excessively technical jargon. I will attempt to ensure that terms like *monotonicity* are explained (even though that has been defined in a previous issue).

3. The main publication of ERS, *Representation*, is being asked to reproduce the contents list of our issues, so that those interested will be aware of *Voting matters*.

Monotonicity and Single-Seat Election Rules

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1. Introduction

This article investigates the monotonicity properties of preferential election rules for filling a single seat. Section 2 lists the properties of interest, which form a subset of those introduced in Woodall[4]. Section 3 describes several known election rules and two new ones (QLTD and DAC), whose properties are tabulated in Table 1. Section 4 describes a number of impossibility theorems, which are also represented symbolically in Table 1. These theorems say that certain combinations of properties cannot hold simultaneously, because the properties in question are mutually incompatible. In Section 5 I attempt to summarize the current state of knowledge and indicate what remains to be done.

Throughout this article I consider only the single-seat case. This does not reduce the force of the impossibility theorems in Section 4. We are interested in universal election rules, which will work for filling any number of seats. If certain properties are mutually incompatible even in the single-seat case — that is, there is not even a single-seat election rule with all these properties — then it is almost inconceivable that there will be an election rule with all these properties that works for any other number of seats, and there certainly cannot exist a universal election rule with them all. So, in practice, an impossibility theorem for single-seat election rules is as good as one that considers multi-seat elections as well. But in the case of the examples in Section 3, considering only single-seat elections is a real limitation, and I have resorted to it only because I have found the multi-seat case too hard to handle. There are many election rules that possess properties in the single-seat case that they do not possess in the multi-seat case, and there are many single-seat election rules that cannot apparently be extended to multi-seat elections in any sensible way, and so the multi-seat case is much harder to analyze.

I think the most important problems facing mathematicians who are interested in STV are, first, to discover which monotonicity properties are compatible with DPC (the Droop Proportionality Criterion)[4], or with majority (the property that DPC reduces to in single-seat elections—see Section 2 below); and then to find an election rule that satisfies DPC and as many monotonicity properties as possible. In the case of single-seat elections, I have found a rule (DAC) that satisfies majority and many monotonicity properties, which I would be prepared to recommend as preferable to the Alternative Vote (AV). Admittedly it fails to satisfy one important property of AV, that later preferences should not count against earlier preferences, but in return for this it gains five properties that AV does not possess. However, at the moment I have not been able to extend DAC in any sensible way to multi-seat elections, and I do not know whether this will prove to be possible, or whether it will be necessary to start afresh with a new idea.

2. The properties

These properties were all introduced in Woodall[4], where they were discussed in more detail, and so I shall only state them briefly here. Of the seven global or absolute properties mentioned there, three are of interest to us now:

Plurality. If some candidate *x* has strictly fewer votes in total than some other candidate *y* has first-preference votes, then *x* should not have greater probability than *y* of being elected.

Majority. If more than half the voters put the same set of candidates (not necessarily in the same order) at the top of their preference listings, then at least one of those candidates should be elected.

Condorcet. If there is a Condorcet winner (that is, a candidate who would beat every other candidate in pairwise comparisons), then the Condorcet winner should be elected.

Of these three properties, **majority** is by far and away the most important. **Plurality** is also important, but it is much less likely to be violated: every reasonable electoral system seems to satisfy it, whereas many systems proposed or actually used, such as first-past-the-post, point-scoring systems and approval voting, fail **majority**. **Condorcet** is a very attractive property, but, as we shall see in Section 4, it leads to problems with monotonicity. My aim is to find a system that satisfies **majority** and as many of the monotonicity properties as possible.

Among the local or relative properties introduced in Woodall[4] we shall consider seven of the nine versions of monotonicity, together with **participation**, **later-no-help** and **later-no-harm**. The remaining two versions of monotonicity, **mono-append** and **mono-add-plump**, are omitted because they hold for all the election rules discussed in Section 3 and do not feature in any of the impossibility theorems in Section 4.

Monotonicity. A candidate *x* should not be harmed if:

(**mono-raise**) *x* is raised on some ballots without changing the orders of the other candidates;

(**mono-raise-delete**) *x* is raised on some ballots and all candidates now below *x* on those ballots are deleted from them;

(**mono-raise-random**) *x* is raised on some ballots and the positions now below *x* on those ballots are filled (or left vacant) in any way that results in a valid ballot;

(**mono-sub-plump**) some ballots that do not have *x* top are replaced by ballots that have *x* top with no second choice;

(**mono-sub-top**) some ballots that do not have *x* top are replaced by ballots that have *x* top (and are otherwise arbitrary);

(**mono-add-top**) further ballots are added that have *x* top (and are otherwise arbitrary);

(**mono-remove-bottom**) some ballots are removed, all of which have *x* bottom, below all other candidates.

Participation. The addition of a further ballot should not, for any positive whole number *k*, reduce the probability that at least one candidate is elected out of the first *k* candidates listed on that ballot.

Later-no-help. Adding a later preference to a ballot should not help any candidate already listed.

Later-no-harm. Adding a later preference to a ballot should not harm any candidate already listed.

3. Examples of election rules

First-Preference Plurality (FPP), or First-Past-the-Post, elects the candidate with the largest number of first-preference votes. This rule behaves extremely well with regard to all the local properties (although it satisfies **later-no-harm** only if second and subsequent preferences are ignored totally, and are not used to separate ties). However, it does not satisfy **majority** or **Condorcet**: in Election 1 below, FPP elects *c*, but majority requires that *a* or *b* should be elected, and *a* is the Condorcet winner.

	ab	30
Election 1:	*ba*	25
	c	45

Point Scoring (PS) methods are those where each candidate is given a certain number of points for every voter who puts them first, a certain (smaller) number for every voter who puts them second, and so on, and the candidate with the largest total number of points is elected. These methods have very similar properties to FPP, although later preferences can now count against earlier preferences, so that **later-no-harm** fails, and **mono-raise-random** and **mono-sub-top** also fail in most cases. To see that PS systems do not satisfy **majority** or **Condorcet**, suppose that just over half the voters list three candidates in the order *abc*, and just under half list them in the order *bca*. Then both **majority** and **Condorcet** require that *a* should be elected, but any PS method will choose *b*.

Table 1

	Properties of specific election rules						Impossibility theorems		
	FPP	PS	AV	C-PS	QLTD	DAC	1	2	3
Plurality	√	√	√	√	√	√	•		
Majority	×	×	√	√	√	√			•
Condorcet	×	×	×	√	×	×	•	•	
Mono-raise	√	√	×	√	√	√			×
Mono-add-top	√	√	√	×	×	√	×		
Mono-remove-bottom	√	√	×	×	√	√			×
Participation	√	√	×	×	×	√		×	×
Mono-raise-random	√	×	×	×	×	×		×	×
Mono-sub-top	√	×	×	×	×	×		×	×
Mono-raise-delete	√	√	×	×	√	√		×	×
Mono-sub-plump	√	√	×	×	√	√		×	×
Later-no-help	√	√	√	×	√	√		×	•
Later-no-harm	√	×	√	×	×	×		×	•

The thick box delimits those properties that make sense even if truncated preference listings are not allowed. The top three properties are global while the others are local or relative.

The Alternative Vote (AV) was discussed at length in Woodall[4] and so I shall content myself here with tabulating its properties in Table 1. Unlike FPP and PS, it satisfies the all-important **majority** property, but it behaves rather badly with respect to monotonicity.

There are many known election rules that satisfy Condorcet's principle; for example, nine such rules are discussed by Fishburn[1]. In the present context (looking for a more monotonic substitute for AV) we are really only interested in rules that satisfy **majority**. Among such rules, the one with the largest number of other properties seems to be one that is not among the nine considered by Fishburn, namely to use a point scoring method to select a candidate from the Condorcet top tier. This method is described as C−PS in Table 1. It satisfies all three of the global properties that we are considering, but it behaves badly with respect to the local properties.

My first serious attempt to find a rule that would rival AV resulted in what I call Quota-Limited Trickle-Down (QLTD). Although this has now been superseded by DAC, I describe it here because it is simpler. One starts by crediting every candidate with all their first-preference votes. If no candidate exceeds the quota (of half the number of votes cast), then one gradually adds in the second-preference votes, then the third-preference votes, and so on, until some candidate reaches the quota. For example, it may be that if one credits every candidate with all their first-preference votes, all their second-preference votes and 0.53 times their number of third-preference votes, then exactly one candidate is brought up to the quota; that candidate is then declared elected.

	abcdef	12
	cabdef	11
Election 2:	*bcadef*	10
	def	27

It is easy to see that this rule satisfies **majority**. At first I thought it satisfied all the most important monotonicity properties as well. However, I now realize that it fails **mono-add-top**. This can be seen from Election 2 above. Here the quota is 30, and if one gives every candidate all their first and second-preference votes, plus 0.7 of their third-preference votes, then *a* gets 30 votes, *b* 29.7, *c* 29.4, *d* 27, *e* 27 and *f* 18.9; thus *a* is elected. However, if one adds six extra ballots marked *ad*, then the quota goes up to 33, but now *d* reaches the quota on first and second preferences alone: the count is *d* 33, *a* 29, *b* 22, *c* 21, *e* 27 and *f* zero. In Election 2 itself, *a* is behind *d* (by 23 to 27) on the basis of first and second-preference votes, but *a* overtakes *d* when the third-preference votes are added in. Adding six extra *ad* ballots increases *a*'s and *d*'s first and second-preference votes by the same amount, and this causes *d* to reach the quota: *a* would again overtake *d* if the third-preference votes were added in, but this does not happen because the election has already ended.

Election 3		Acquiescing Coalitions			
ab	11	{*a*, *b*, *c*}	30	{*c*}	12
b	7	{*b*, *c*}	19	{*a*}	11
c	12	{*a*, *b*}	18	{*b*}	7
		{*a*, *c*}	12		

My most recent attempt to find a substitute for AV has resulted in what I call the method of Descending Acquiescing Coalitions, or DAC, which is the first election rule that I am really happy with. The *coalition acquiescing* to any set of candidates comprises all voters who have not indicated that they prefer any candidate not in that set to any candidate in that set. For example, in Election 3 above, there are 19 voters who acquiesce to *b* and *c*, namely, the 7 who voted *b* and the 12 who voted *c*; none of them actually voted for both *b* and *c*, but none of them have said that they prefer *a* to either of these candidates, and so they are said to acquiesce to this set of two candidates. Similarly, the 18 voters who acquiesce to *a* and *b* comprise the 11 who voted *ab* and the 7 who voted *b*. The 12 voters who acquiesce to *a* and *c* are exactly the same as those who acquiesce to *c*, namely, the 12 *c* voters. And so on.

In DAC, one first lists the sizes of all the acquiescing coalitions in decreasing order, as I have done above, and then works down the list from the top, eliminating candidates until only one is left. The largest acquiescing coalition always contains every voter, since every voter acquiesces to the set of all candidates; this does not help towards deciding who should be eliminated. In the above example, the next largest acquiescing coalition comprises 19 voters, for {*b*, *c*}; the fact that *a* is not included in this set means that *a* is the first candidate to be eliminated. The next acquiescing coalition comprises 18 voters, for {*a*, *b*}. Since *c* is not included in this set, *c* is next to be eliminated. This leaves only one candidate not eliminated, namely *b*, and so *b* is declared elected. (Note that AV would exclude *b* first and then elect *c* in this example.)

Election 4		Largest Acquiescing Coalitions			
adcb	5	{*a*, *b*, *c*, *d*}	30	{*a*, *c*}	8
bcad	5	{*a*, *b*, *c*}	13	{*b*, *c*, *d*}	8
cabd	8	{*d*}	12	{*b*, *d*}	8
dabc	4	{*a*, *d*}	9	{*c*}	8
dbca	8				

Sometimes several candidates can be eliminated at once. For example, in Election 4, the largest acquiescing coalition not containing all voters comprises 13 voters, for {*a*, *b*, *c*}; thus *d* is the first candidate to be eliminated. The next largest acquiescing coalition is for {*d*}, and so it appears that *a*, *b* and *c* should all be eliminated at once, leaving no candidate remaining uneliminated. In this case one simply ignores this coalition: it does not help in distinguishing between the remaining three candidates. The next coalition is for {*a*, *d*}, and this causes *b* and *c* to be eliminated, so that *a* is elected.

Election 5		Largest Acquiescing Coalitions	
acbd	6	{*a*, *b*, *c*, *d*}	25
adbc	3	{*a*, *b*, *c*}	14
adcb	3	{*a*}	12
bcad	4	{*a*, *c*}	10
cabd	4	{*a*, *d*}	6
dbca	5		

It is not difficult to see that DAC satisfies **majority**, since if more than half the voters put the same set of candidates (in various orders) at the top of their preference listings, then every other candidate will be eliminated before any candidate in that set. With slightly more difficulty, it can be proved that DAC satisfies all the other properties ticked in Table 1. However, it does not satisfy **mono-raise-random** or **mono-sub-top**: if two of the four *dabc* ballots in Election 4 were replaced by *acbd* then *c* would be elected instead of *a*. Also, it does not satisfy **Condorcet**: in Election 5, DAC elects *a*, but *c* is the Condorcet winner. And it does not satisfy **later-no-harm**: if the seven *b* voters in Election 3 had voted *bc* instead, then *c* would have been elected instead of *b*. We shall see in the next section that there cannot exist any election rule satisfying **Condorcet** or **later-no-harm** as well as all the properties of DAC; but it is not clear whether there is any rule that satisfies **mono-raise-random** or **mono-sub-top** as well as everything that DAC does.

4. Impossibility theorems

Of the three theorems summarized symbolically in Table 1, the one of greatest interest in the present context is Theorem 3. However, it is also the most difficult to prove, and so I shall discuss the two simpler theorems first.

Theorem 1 says that if **plurality** and **Condorcet** hold then **mono-add-top** cannot hold; that is, there is no election rule that satisfies all three of these properties. This is easily seen by considering Election 3. Which candidate would such a rule elect? Since *c* has more first-preference votes than *a* has votes in total, *a* cannot be elected, by **plurality**. But adding two *ba* ballots would make *a* the Condorcet winner, and so *b* cannot be elected, by **Condorcet** and **mono-add-top**. And similarly *c* cannot be elected, because adding five *cb* ballots would make *b* the Condorcet winner. Thus, whichever candidate was elected, at least one of the three properties would be violated! (Of course, our rule could declare the result of Election 3 to be a tie; but this would lead to a contradiction in a similar way.)

It seems that most of the Condorcet-based properties discussed in the Social Choice literature would in fact elect *a* in Election 3, and so violate **plurality** (whereas AV elects *c* and DAC elects *b*). How seriously one regards the failure of plurality depends on how one interprets truncated preference listings, and that in turn may depend on the rubric on the ballot paper. If the 12 *c* voters are merely expressing indifference between *a* and *b* and not hostility to them, and so can be treated in exactly the same way as if half of them voted *cab* and half voted *cba*, then the violation is not too serious. But if, by plumping for *c*, these voters are not just saying that they prefer *c* to *a*, but that they want *c* and definitely do not want *a* (or *b*), and if the seven *b* voters also definitely do not want *a* (or *c*), then it is clear that *c* has more support than *a* and so *a* should not be elected.

	abc	3	*acb*	2
Election 6:	*bca*	3	*bac*	2
	cab	3	*cba*	2

Theorem 2 says that if an election rule satisfies Condorcet's principle, then it cannot possess any of the seven properties that are crossed in the column headed 2 in Table 1. This is a lot to prove. Fortunately most of it can be proved by considering variants of Election 6 above. The only bit that cannot is the incompatibility of **Condorcet** with **participation**; this is proved by Moulin[2], and I shall not attempt to reproduce his proof here. The following proof of the rest of Theorem 2 invokes the axioms of symmetry and discrimination, for a precise statement of which see Woodall[4].

So suppose we have an election rule that satisfies **Condorcet**. By symmetry, the result of this rule applied to Election 6 above must be a 3-way tie. But by the axiom of discrimination, there must be a profile P very close to the one in Election 6 (in terms of the *proportions* of ballots of each type) that does not yield a tie. So our election rule, applied to profile P, elects one candidate unambiguously; and there is no loss of generality in supposing that this candidate is a. However, there are ways of modifying the profile P so that c becomes the Condorcet winner, so that our election rule must then elect c instead of a. This happens, for example, if all the *bac* ballots are replaced by a; and the fact that this causes c to be elected instead of a means that our election rule does not satisfy **mono-raise-random**, **mono-raise-delete**, **mono-sub-top** or **mono-sub-plump**. It also happens if all the *abc* ballots are replaced by a, and this shows that our election rule does not satisfy **later-no-help**.

To prove that our election rule does not satisfy **later-no-harm**, it is necessary to consider a slight modification of the profile in Election 6, in which the second and third choices are deleted from all the *abc*, *bca* and *cab* ballots. Again, our election rule, applied to this profile, must result in a 3-way tie. But again, there must be a profile P' very close to this (in terms of the *proportions* of ballots of each type) that does not give rise to a tie, and we may suppose that our election rule elects a when applied to profile P'. But if we replace the a ballots in P' by *abc*, then b becomes the Condorcet winner, and so must be elected by Condorcet's principle; and this shows that our election rule does not satisfy **later-no-harm**.

Together with the result of Moulin[2] already mentioned, this completes the proof of Theorem 2, that an election rule that satisfies **Condorcet** cannot satisfy any of the seven properties crossed in the column headed 2 in Table 1.

Theorem 3 is a result that looks superficially similar to Theorem 2, and the proof is similar in character but much harder. The theorem says that if an election rule satisfies **majority**, **later-no-help** and **later-no-harm** then it cannot possess any of the seven properties crossed in the column headed 3 in Table 1. This is a substantial improvement on the result sometimes known as “Woodall's impossibility theorem”[3], which asserts that there is no election rule that satisfies **plurality**, **majority**, **later-no-help**, **later-no-harm** and **mono-sub-top**. In obtaining the improvement, I have needed to adopt an axiom of discrimination that is somewhat stronger than the one stated in Woodall[4], although one that must surely still hold for any real election rule. I am also grateful for help from my research student, Ben Tarlow.

A1	A2		A3	A4		A5	A6	
a	*ab*	0.34	*a*	*ab*	0.34	*ab*	*ab*	0.3
b	*b*	0.33	*b*	*b*	0.3	*b*	*ba*	0.3
c	*c*	0.33	*c*	*c*	0.36	*c*	*c*	0.4

Because the proofs of the different parts of Theorem 3 are quite complicated, I shall just sketch the proof of the easiest part, which is that there is no election rule that satisfies **majority**, **later-no-help**, **later-no-harm** and **mono-sub-plump** (or **mono-sub-top**). Suppose, on the contrary, that we have a rule that satisfies these four properties. The first part of the proof is to show that it must elect a in election A1 and c in election A3 in the above table. This is not too difficult to prove, using symmetry and **mono-sub-top**, provided that neither of these elections results in a tie. However, although it may seem highly implausible that either of them should yield a tie, I cannot see any way of proving that this is impossible. Instead, I have used the strong form of the axiom of discrimination in order to show that, if it does happen, then one can vary the proportions 0.34, 0.33, 0.3 and 0.36 in these profiles by very small amounts in a consistent way so as to obtain very similar profiles in which it does not happen.

The rest of the proof is much easier to explain. Let us write $X \to x$ to mean that x is definitely elected in Election X (that is, with probability 1), and $X \not\to x$ to mean that x is definitely not elected in Election X (that is, x does not even tie for election in Election X). Also, $\Rightarrow$ is used to mean "implies that". Therefore

$\mathrm{A1} \to a \Rightarrow \mathrm{A2} \to a$ by **later-no-harm**,

$\mathrm{A2} \to a \Rightarrow \mathrm{A2} \not\to b$ (clearly),

$\mathrm{A2} \not\to b \Rightarrow \mathrm{A4} \not\to b$ by **mono-sub-plump**,

$\mathrm{A3} \to c \Rightarrow \mathrm{A3} \not\to a$ (clearly),

$\mathrm{A3} \not\to a \Rightarrow \mathrm{A4} \not\to a$ by **later-no-help**,

$\mathrm{A4} \not\to a$ and $\mathrm{A4} \not\to b \Rightarrow \mathrm{A4} \to c$,

$\mathrm{A4} \to c \Rightarrow \mathrm{A5} \to c$ by **mono-sub-plump**,

$A5 \to c \Rightarrow A5 \not\to b$ (clearly),

$A5 \not\to b \Rightarrow A6 \not\to b$ by **later-no-help**.

However, majority requires that A6 should result in the election of either *a* or *b*, and the axiom of symmetry therefore requires that *a* and *b* should tie for election in A6, each with probability ½. This contradiction shows that there can be no election rule satisfying the four properties described.

The details of this proof, and the proof of the rest of Theorem 3, can be found in Woodall[5], which is not yet published but is available from the author at the Department of Mathematics, University Park, Nottingham, NG7 2RD, email drw@maths.nott.ac.uk .

5. Conclusions

In attempting to find a single-seat preferential election rule that satisfies majority and is generally monotonic, I have come up with only one rule, DAC, that I would be prepared to recommend as preferable to the Alternative Vote, and then only when the count is carried out by computer. DAC is much more complicated than AV, and I have not given great thought to how one would implement it on a computer, but I do not think there would be any great difficulty unless the number of candidates was unrealistically large. DAC admittedly fails to satisfy one important property of AV, that later preferences should not count against earlier preferences, but in return for this it gains five monotonicity properties that AV does not possess, including the very strong **participation** property, and so I would regard it as preferable.

However, DAC only works for filling a single seat, and I have not so far found any sensible way of extending it to multi-seat elections. The major remaining problem seems to me to be to find a multi-seat preferential election rule that satisfies the Droop Proportionality Criterion and is generally monotonic. It is not clear whether one can do this by modifying DAC, or whether it will be necessary to start afresh with a new idea.

From the mathematical point of view, there is still a great deal of work to be done on single-seat elections. The general problem is to determine which sets of the properties listed in Table 1 are mutually compatible. The examples discussed in Section 3 and the impossibility theorems in Section 4 give some answers. For example, Theorems 2 and 3 show that both FPP and AV possess maximal compatible sets of these properties, and that moreover these are the only two maximal compatible sets of properties that include both **later-no-help** and **later-no-harm**. Surprisingly, I have not been able to prove that the properties possessed by DAC form a maximal compatible set; Theorems 2 and 3 show that one cannot add either **Condorcet** or **later-no-harm** to these properties, but I cannot prove that one cannot add **mono-raise-random** or **mono-sub-top** (although this seems unlikely, since these last two are extremely strong properties, which hardly any election rules seem to possess). Another problem of this type is to determine whether there is any rule that satisfies **majority**, **Condorcet** and either **mono-add-top** or **mono-remove-bottom**. While problems of this type may seem to have little direct relevance to STV, the ideas generated by attempts to solve them may turn out to be more relevant than at first appears, and in any case we cannot afford to know less about such questions than our opponents do.

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Some comments on *Replaying the 1992 general election*

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At the time of the 1992 general election, Patrick Dunleavy, Helen Margetts and Stuart Weir conducted research designed to indicate how Britain would have voted under alternative forms of voting. Their report [1,2] states that the result "poses a problem for STV advocates" in that the allocation of seats is far from proportional by first preferences and severely disadvantages the Conservatives. They are very forthright in their claims that the study shows what would actually have happened. A subsequent letter[3] hoped that the Electoral Reform Society would "address the problems for STV that our ... study identified". It is, of course, not possible fully to address such problems without the data, and I am grateful to the authors for letting me have a copy.

In the comments that follow, I have concentrated entirely on the STV part of the document, ignoring the work that they also did on Alternative Vote, Additional Member, and List PR systems.

The data were obtained from a sample of 9614 people across 13 regions of the UK (excluding Northern Ireland). The sampling and interviewing was done by ICM, using their professional experience of getting a representative sample within each region. The interviewees were given a ballot paper of 17 candidates, in sections by party, their names being those of actual candidates in the general election in that region. They always consisted of 4 each from the 3 main parties, plus 5 others who included the nationalist parties in Scotland and Wales. Within this pattern, the aim was to give a mix of well-known and lesser-known, of men and women, etc.

The country was divided into 5-member constituencies so far as possible, but with some 4-member ones, by combining the actual single-member constituencies within each region. There were 133 such constituencies, consistently misquoted as 123 in their reports.

Much trouble was taken to get representative samples, but for analysis the regional results were reweighted for each multi-member constituency "to produce distinctive local profiles". I do not doubt that this was done with good intentions but, so far as I can see, the anomalous results that "pose a problem for STV advocates" result almost entirely from this reweighting.

My analysis has necessarily had to be slightly incomplete because of some missing files. I am told that some computer discs have become unreadable and these files are going to be difficult to retrieve, so it seems better to go ahead with reporting what I can without them. Those missing concern all four of their East Anglian constituencies, 5.4% of the total data, and three of the Greater London ones (those that they call Richmond and Kingston, Hillingdon, and Central London). We can derive the regional Greater London results, from the other constituencies in the region, but not the reweighting for each of these three missing constituencies.

Of the available files, there are some that show trouble in the data in that some spurious figure zeros appear, that lead much of the data to be ignored by my STV computer program that was used. Luckily only one of these instances leads to a different result by political party from what they found, but there are also four others not suffering from this particular trouble, where the results by party seem to have been incorrectly reported. For the reweighting of the data, the authors say that "the 1992 general election results provide a complete picture of people's first preferences" so they use those to reweight the voting patterns. Even if this actually gave an improvement, I completely disagree with the beliefs behind it. A very important reason for wanting electoral reform is that election results at present do *not* show people's true preferences. Common observation shows that vast numbers of people vote tactically, not for what they would most like to see but for candidates who, they think, have some chance of success, and trying to keep out the party they most dislike. The *squeeze* of the third candidate in by-elections is notorious and a similar effect in general elections, to a lesser extent, certainly exists. Whether a better electoral system would make much change in voters' stated preferences or not we simply do not know; until we try the real thing the evidence is not available.

Having done the reweighting, for better or for worse, they report (in their Table 11):

	Con	Lab	L/D	others
Pure proportionality	273	222	114	25
STV	256	250	102	26

and this is what they say that STV supporters have to ponder. If we do the analysis by what appear to be the original data for each region, without such reweighting, it means using the same voting pattern for every multi-member constituency within the region, which is a crude model and often unrealistic but is probably the best that can be done with the data available. The results (with some assumptions for missing files) are:

	Con	Lab	L/D	others
Pure proportionality	273	222	114	25
STV	274	230	108	22

I think that it is they who have some pondering to do.

I cannot see how the numerical values of their reweighting were derived, but my requests for clarification have not been successful. If, as I believe, it was intended to bring the first preferences, by party, closer to the general election votes, it does not seem to have done so. The results are in the large table.

If anything the results after reweighting seem further from the general election results than do the raw ones, and certainly the Conservatives have been marked down.

It might be claimed that it is the individual multi-member constituency figures that matter rather than these overall ones, so I have looked at one constituency in detail to see whether that improves the picture. I chose to do this for my home constituency which, in their scheme, would be the combination of the present constituencies of Herts SW, Herts W, Hertsmere, Watford and St Albans. As an example of their reweighting, in this constituency every vote in the raw data with a Conservative first preference has been treated as 98 identical votes, every vote with a Labour first preference as 121 identical votes, every vote with a Liberal Democrat first preference as 87 identical votes and every other vote as 94 identical votes.

For this constituency I find:

	Con	Lab	L/D	other
General election	53.3%	25.1%	20.3%	1.3%
STV (raw)	52.7%	23.0%	20.0%	4.3%
STV (reweighted)	51.2%	27.6%	17.3%	4.0%

	General election				STV (raw figures)				STV (after reweighting)			
Region	Con	Lab	L/D	other	Con	Lab	L/D	other	Con	Lab	L/D	other
East Anglia	48.7	37.1	12.7	1.5	-	-	-	-	-	-	-	-
East Midlands	48.9	35.2	14.8	1.1	46.5	39.2	12.0	2.3	46.3	39.5	12.0	2.3
Greater London	45.2	35.1	18.0	1.7	41.0	37.1	18.1	3.8	*40.3*	*39.5*	*16.0*	*4.2*
North West	39.4	45.1	14.2	1.3	37.6	46.3	13.9	2.1	36.7	47.3	13.8	2.2
Northern	29.6	55.1	15.2	0.2	26.0	56.8	15.2	2.0	26.1	56.8	15.1	2.0
South East	54.6	20.4	23.7	1.2	52.7	23.0	20.0	4.3	52.7	23.3	19.8	4.3
South West	48.1	17.2	32.8	1.9	49.3	21.7	24.8	4.1	49.2	21.9	24.8	4.1
West Midlands	49.4	34.1	15.7	0.7	45.2	40.5	10.7	3.6	44.3	41.6	10.4	3.7
Yorks & Humber	37.5	45.7	15.5	1.3	39.0	44.2	12.8	4.1	38.3	45.0	12.5	4.2
Highlands	38.0	11.0	20.1	30.9	23.2	28.0	17.4	31.4	22.0	28.5	18.9	30.5
Strathclyde	19.8	49.9	7.8	22.5	24.1	47.2	5.2	23.5	23.3	48.0	5.2	23.6
East Central Scotland	29.9	34.1	14.8	21.2	28.0	36.0	18.3	17.7	28.4	35.2	18.7	17.6
Wales	26.0	50.8	11.8	11.4	32.0	43.7	13.1	11.2	31.1	44.1	13.2	11.6

Percentage share of votes by region

East Anglia missing, italic figures approximate due to missing files

Apart from slightly reducing the *others* figure, which is far too big nevertheless, has the reweighting helped? I doubt it.

I am well aware that it is much easier to criticize such a study than to perform one, but it does seem to me that a better scheme would have been to take their 9614 interviews equally from each of their 133 multi-member constituencies, i.e. about 72 per constituency, and then use the results in raw form. It might be argued that 72 votes are not many for electing to 5 seats, but that is all you get with a total of 9614. You do not get any more actual information by using the same figures many times with reweighting.

The authors also comment on “some apparently extraordinary results — as with the election of 5 Green MPs in the south east region”, and that only 2 of the 5 would survive if Meek rules were used (I make it only 1 of the 5 actually). In interpreting this we need to remember that it is the same set of votes being analysed over and over again, and the identical person as Green candidate, merely with different reweighting for each constituency in the region. That may have been the best that could be done in the circumstances, but I wish they would not claim that this is what would actually happen in practice. Again it is the reweighting that has produced the odd effect — no Green is elected if the original, unmodified, observations are used.

They seem to think it a disadvantage of STV that it can react with different results when the votes change only slightly. I think it an advantage that most constituencies become marginal for their final seat. At present it is only the marginal constituencies that have any real effect on who wins a general election. Under STV nearly every voter can feel that it is worth voting as it could make a difference.

They also make a point of the fact that “STV is only contingently proportional” if comparing seats with first preferences by party. So it should be. It often helps to explain a point such as this by using an exaggerated example. To repeat one that I have used elsewhere, if we have 9 candidates for 3 seats, A1, A2 and A3 from party A; B1, B2 and B3 from party B; C1, C2 and C3 from party C and the votes are

```
20   A1   B1
20   A1   C1
```

a party-based proportional system would have to elect A1, A2 and A3 as all first preferences were for party A, whereas STV will elect A1, B1 and C1 and appear to do badly if one insists on comparing seats with first preferences by party, but it has done what the voters have asked for, and that should be the aim of an electoral system.

What is more their data show that, of all interviewees who selected at least two preferences with each of their first two from the three main parties, only 79% of them chose the same party for both choices. If this is nothing like what would happen in practice, then the exercise cannot be quoted as meaningful in this respect. Their report claims strongly that their figures do represent what would happen in practice, but they cannot have it both ways; if they are right in that, then the authors' wish to see party proportionality by first preferences is not shared by the electorate. I believe that the wishes of the electors are what matter.

My overall conclusion is that it has been claimed that STV advocates have some problems to deal with, but in fact it is the authors of the study who need to deal first with the problems that they have created.

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Added in this reprinting

Brian Lawrence Meek, M.Sc, FRAS, C.Eng, FBCS.
1934 - 1997.

Brian Meek died on 12 July 1997. He was a member of the Electoral Reform Society for many years, and in the annals of the Single Transferable Vote his name will surely be immortal. Alongside the three pioneers Hill, Andrae and Hare, the other great names are Droop, Gregory and Meek. Various others have made improvements from time to time. This is not intended as any disparagement of them — fine-tuning of the system is not to be despised; it all helps if well done. It was Meek though who re-thought the system from scratch for the age of the computer and put it upon a proper mathematical basis. It should be recorded that a major part of the Meek system was also devised, quite independently, by Douglas Woodall a little later.

It is a pity that, although Meek's system is simpler in principle and easier to understand than other versions of STV, it is too long-winded if tried by hand. A computer is necessary, and since not everyone is willing to use computers for counting all elections, it will be necessary for a number of years yet to keep the approximate methods, suitable for hand-counting, available too. However, for any organisation that is willing always to use computers for its elections it would be madness to continue with approximate methods when Meek's method is available.

We have lost a man who did something really great in this field. One day that fact will be common knowledge for all proponents of STV.

I.D. Hill

Issue 7, September 1996

Editorial

This issue contains five articles within the tradition that has now been established. This concentrates upon the properties of various STV algorithms as seen from examples or computer simulation.

In the first paper, Hugh Warren illustrates a counter-intuitive case of the application of STV where two halves are not the same as one whole.

My own article provides the results of a computer simulation of 'large' STV elections which casts doubt on the use of the hand-counting rules in that case.

David Hill provides a simple comparison between the hand-counting rules and the computer method due to Meek. In a separate article, he shows how one can compute with Meek how one's vote has contributed (or otherwise) to the elected candidates.

In another paper by Hugh Warren, an example is provided in which equality of preference does not have a property that one might reasonably expect. David Hill responds to this in the final paper of this issue.

On reviewing this material, I conclude that I should appeal for a broader spectrum of papers. STV is not just a minority interest. I am a member of the *John Muir Trust* which aims to preserve wild places in Scotland. The trustees are elected annually by the membership by STV using the Meek algorithm. (Nothing to do with me.) I have been given an impressive list by Eric Syddique of organisations known to ERS that use STV (107 in total, but omitting the *John Muir Trust*). Can I appeal to readers to send details of other organisations so that I can publish the list in a subsequent issue of *Voting matters*?

Brian Wichmann

On the lack of Convexity in STV

C H E Warren

Hugh Warren is a retired scientist

If the voters in a constituency are divided into two districts and the ballots are processed separately and the results in the two districts are the same, then there is said to be convexity if processing the ballots of all voters together gives the same result.

As Woodall[1] has pointed out, quoting an example of David Hill's, STV does not satisfy convexity. We give here a further example, in which the lack of convexity arises, not from the elimination of candidates as in David Hill's example, but from the transfer of surpluses. We assume that these transfers are made by the method currently recommended by the Electoral Reform Society, and which the Electoral Reform Society uses for its own elections — the Meek[2] method.

There are four candidates A, B, C, D, and three seats to be filled. The voting is as follows:

	District 1	District 2	Constituency
ABD	10	-	10
BAD	-	10	10
AC	-	8	8
AD	-	1	1
BC	8	-	8
BD	1	-	1
D	1	1	2
Totals	20	20	40
Elected	A,B,C	A,B,C	A,B,D

This further example reinforces Woodall's comment in the article quoted that ... *sadly, convexity is of no use to us*, as this seemingly ideal property conflicts with a more desirable property.

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1 D R Woodall, Properties of Preferential Election Rules, *Voting matters*, Issue 3, pp 8-15, December 1994.

2 B L Meek, A new approach to the Single Transferable Vote, reproduced in *Voting matters*, Issue 1, pp 1-10, March 1994.

Large elections by computer

B A Wichmann

Introduction

By a large election, in this article we mean elections in which there are a large number of candidates, say over 100. Such an election was reported in reference 1, in which the periodicals to be retained in a library were to be decided. In that case, the Meek algorithm was used[4], but on re-running the same data with the Newland-Britton (ERS) rules[5], a disturbing fact was noted. Towards the end of the count, none of the remaining candidates were credited with any votes at all, so that the last few ‘seats’ were filled at random from the remaining candidates. This was quite inappropriate, since the number of journals that received some support in the votes was more than enough to fill all the places. Hence Woodall has defined the property No-support in reference 2 to cover this issue.

In this paper, we are concerned not with the limiting case of the ERS rules electing candidates without support, but with other large elections in which some candidates are elected despite having less than half the quota. In such situations, it might appear that the ERS rules might elect the ‘wrong’ person. Unfortunately, it is not easy to devise a means of determining the ‘right’ choice. Here we use random ballot papers with some characteristics of a real election.

UKCC

The United Kingdom Central Council for Nursing, Mid-wifery and Health Visiting (UKCC) election is the largest one conducted by Electoral Reform Ballot Services Ltd (at least, using STV). It is possible that other such elections could arise of this type if multi-national organisations undertake employee-council elections to satisfy the ‘Social Chapter’.

The data from the last UKCC election is impressive: 129 candidates for 7 seats with 62,216 ballot papers. The election is conducted to the ERS rules assisted by Rosenstiel's program.

Mr Wadsworth of ERBS has kindly given me the information above and also a print-out from the Rosenstiel program which gives for the seven elected candidates:

Candidate	First Preference Votes/ Quota	Stage when elected	Votes when elected/ Quota
A	112.8%	1	100.0%
B	17.3%	122	47.7%
C	19.8%	121	53.2%
D	21.0%	121	50.5%
E	16.0%	123	34.9%
F	11.1%	123	36.4%
G	11.0%	123	42.6%

The concern here is that since one candidate was elected on only about one third of the votes that had to be retained by the most popular candidate, can one be sure the correct choice was made? The result of that particular election is not being questioned, but the choice of algorithm for elections of this type.

Computer processing

Since the computer programs to conduct elections are not used for the large public elections, there is no experience in using these programs for very large elections. As noted above, Rosenstiel's program was used for UKCC, but this program is for assisting a manual count, and could not be used for the Meek algorithm (for instance). Although the programs for the ERS rules (by I D Hill) or that for Meek do not have hard limits, it is not immediately obvious that they could be used for elections as large as that for UKCC.

To determine the feasibility of using these programs on a PC (personal computer) for elections like UKCC, a program was written to construct a large number of random ballot papers. Of course, real ballot papers were not available, and even if they were, the data preparation problem would be formidable.

At this point, a major problem arose. Both Meek and the ERS computer programs allow for the storage of the complete set of preferences. If this information is written to temporary disc storage, then the programs will run quite slowly. However, the total storage for UKCC-like elections is around 8Mbytes, which is only just within the reach of current PCs. The obvious solution was to undertake modifications to both programs to take advantage of the fact that only a small fraction of the total possible number of preferences would be specified. In fact, the modification to Meek was very easy and undertaken, but that for ERS (which is much more complex in computer terms) was too difficult. In any case, both programs were successfully run with random data on my home computer.

The conclusion from this study was that running the Meek or ERS rules on a modern PC would be possible for large elections. However, program modifications would be desirable to ensure that the programs kept within system limits. It was also observed that both programs produced result files which were excessive in size (and too big to print with convenience). With the preferences kept in main store, the time both programs took to execute was limited by the speed of processing; moreover, it was linear in the number of ballot papers. The time taken for the programs on my home computer was about 500 seconds per 10,000 papers for Meek and about ten times faster than that for the ERS rules. These times are clearly minor compared with the data preparation overheads in undertaking such counts.

Random UKCC-like data

Having determined that it is feasible to undertake UKCC-like elections on a computer with either Meek or the ERS rules, we now wish to see if there is a significant risk of either algorithm producing the 'wrong' result.

For this part of the study we use simulated data with only 1,000 papers, rather than the 62,000 that were actually recorded for UKCC. The reason for this reduction is to save on the computer time required, since many elections must be analysed (in fact, 100 elections were used). However, to have a realistic chance of determining the effect of using either algorithm, it is clear that the ballot papers must adhere to some of the characteristics of the real data.

The method used to construct the papers was to use a random number generator, but to use some of the characteristics of the UKCC election to determine the distribution functions used. The two major parameters are the popularity of each candidate and the length of each ballot paper. We can estimate the popularity of each candidate in the real election by means of their (known) number of first preference votes. Hence the popularities of the candidates in the simulated elected were adjusted so that the leading candidate had more than the quota of first-preference votes, candidates numbered 2 to 20 had reducing popularity of 95% of the previous candidate, and the remaining candidates had a constant popularity of 95% of the 20th candidate. The reason for this constant tail is that if the 95% rule was carried on, it was observed that the lower candidates had virtually no votes at all.

The distribution of the length of preferences chosen was as follows: For those expressing a single preference: 8.0% of the papers; for two preferences, 8.7%; for 3: 9.4%; for 4: 10.1%; for 5: 10.9%; for 6: 11.6%, for 7: 12.3%, and for 8 to 11 preferences: 7.2%. This distribution increases linearly to 7, the number of candidates then drops to a constant amount.

We can now compare a randomly produced set of papers with those above from UKCC. In this case with random ballots, the quota becomes 1000/8=125, instead of 62216/8=7777 for the real election. The table entries below and for the comparative table for UKCC are expressed in proportion to the quota to give directly comparable data.

Candidate	First Preference Votes/ Quota	Stage when elected	Votes when elected/ Quota
A	129.6%	1	100.0%
B	16.8%	119	53.3%
C	10.4%	121	46.1%
D	13.6%	121	46.9%
E	12.0%	121	45.3%
F	8.8%	121	44.1%
G	9.6%	121	42.2%

The pattern is clearly similar. We need not be concerned about minor differences, since the study is of elections of this general type. To generate each set of ballot papers merely requires as input the three integer seeds for the random number generator. In consequence, all the data presented here which is based upon a set of 100 elections can be recomputed from 300 integers. The seeds for the election in the above table were 1, 1 and 18.

Comparative tests: Meek versus ERS

We now have the ability to generate large election data and process the results with two algorithms: Meek and the ERS rules. The remaining problem is to determine characteristics of the results which would decide between the two. In fact, four different tests were applied as follows:

Non-transferables: In this test, the number of non-transferable votes of each algorithm are compared. The 'better' algorithm is the one which gives the lower figure.

Condorcet: In this test, we take those elections produced in which the two algorithms elected different candidates. We then compare the first candidate elected by Meek who was not elected by ERS with the first candidate elected by ERS who was not elected by Meek. The comparison is by Condorcet. Since there is no correlation between the votes for different candidates, the winning algorithm for this test is the one which has the higher number of Condorcet winners.

No-hopers: In this test, we eliminate the candidates with no realistic hope of being elected, namely the candidates numbered 21-129, so there are 20 candidates. Again, since there is no correlation between the votes of different candidates, the winning algorithm for this test is the one for whom this change makes the least difference. In other words, one is expecting the removal of the no-hope candidates to make no difference.

Steadiness: This test is that specified by I D Hill in reference 3. The test is applied when there is only one pair of candidates elected differently by Meek and ERS. The election is then re-run with only 8 candidates. The winner is the algorithm for which this makes the least difference to the result.

At this point, the author thinks that readers should reflect upon the tests above. If the results are against your favourite algorithm, will you be convinced that your algorithm should not be used for such elections?

We now consider the results of each of these tests:

Non-transferables: There is a consistent pattern with the number of non-transferable votes with each algorithm which can be summarised as follows:

Meek 559.0 (±18.8); ERS 482.6 (±13.7); Meek/ERS 1.159 (±0.031); where the range represents two standard deviations. Hence Meek consistently gives 16% more non-transferable votes.

Condorcet: Out of the 100 elections constructed with the random ballot papers, 30 produced a different result. Hence for these 30, the Condorcet test could be applied. The results were that for 24 cases, Meek elected the Condorcet winner, and for 6 cases, ERS elected the Condorcet winner.

No-hopers: In this test, we wish to know if the elimination of the no-hope candidates changed those that were elected. For Meek no change occurred for any of the 29 cases examined, but there were changes for all but three cases with ERS. Hence Meek is a clear winner here.

Steadiness: This test is applied to the 29 cases in which there was one difference between the two algorithms. To pass the test, the result of the election with just eight candidates must be the same as for the full election. Meek passes the test for all of the 29 cases, and ERS 6 times (and failed 23 times). Again, Meek is the clear winner of this test.

The above analysis understates the differences between the two algorithms. Of the 29 cases that can be compared for steadiness, the following table indicates how the results compared in 20 cases:

	Meek Elects	ERS Elects
129 candidates	[S]+A	[S]+B
20 candidates	[S]+A	[S]+A
8 candidates	[S]+A	[S]+A

Here `[S]` represents a set of six candidates and `A` and `B` are different candidates not in the set `[S]`. In other words, ERS reverts to the Meek result when the no-hope candidates are removed, and this reversion is retained when only 8 candidates are considered. This is clearly strong evidence that the full election using the ERS rules produces the ‘wrong’ result.

Conclusions

The study indicates that it is feasible to use computer algorithms such as Meek on a PC for elections as large as that for UKCC (although the data preparation problem has not been considered). Furthermore, a comparison between Meek and ERS shows that Meek is superior except for the number of non-transferable votes. The increased number of non-transferable votes is clearly secondary to producing the ‘correct’ result, and from that perspective, the Meek algorithm appears to be superior. The fact that random papers from no-hope candidates can change the result is strong evidence against the ERS rules.

Of course, this study only relates to elections with a large number of candidates. It can hardly be considered a criticism of Newland and Britton, since it is doubtful whether they ever conducted an election of the size considered here. [Added in this printing: See Issue 8, page 3].

Acknowledgements

I should like to thank Joe Wadsworth of ERBS for providing me with a summary of the UKCC election result ‘sheet’. Also, David Hill and Douglas Woodall provided many comments on a draft of this paper which I hope has resulted in a clearer presentation here.

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4. B L Meek. A New Approach to the Single Transferable Vote. Reprinted in *Voting matters*, Issue 1, pp1-10, March 1994.

5. R A Newland and F S Britton. How to conduct an election by the Single Transferable Vote, second edition, ERS, 1976.

Meek style STV − a simple introduction

I D Hill

Until recently, David Hill was Chairman of the ERS Technical Committee

For its 1996 Council election, ERS used the Meek counting rules, instead of the Newland and Britton rules that are suitable for counting by hand. Now that there is sufficient availability of computers, I believe that ERS owes it to itself and to its members to use the best rules of which we are aware.

However many people seem to be muddled as to what this involves and some seem to be sadly misinformed. It is therefore desirable to have available a simple listing of what is the same and what is different in these systems.

It needs to be said clearly that there is no intention of abandoning STV. The system adopted (taking its name from B L Meek who first proposed it) retains all the essential features and aims of STV, but uses the power of modern computers to get a closer realisation of the voters' wishes, better meeting all the traditional STV virtues.

Some of the main changes were mentioned by Robert Newland in *Comparative Electoral Systems*, section 7.8(c). He wrote that these further refinements ‘which would be likely rarely to change the result of an election but which greatly lengthen the count, are not recommended’. At the time, that was probably a reasonable judgement but information gained since then has shown it to be untrue that the result would rarely change, whereas lengthening the count is unimportant when counting is by computer where, either way, the counting time is trivial compared with the effort needed to input the data.

Meek style STV - what is the same?

1. Each voter votes by listing some or all of the candidates in order of preference.

2. Each voter is treated as having one vote, which is assigned initially to that voter's first-preference candidate.

3. A quota is calculated, as the minimum number of votes needed by a candidate to secure election.

4. If a candidate receives a quota of votes or more, then that candidate is elected, and any surplus votes (over the quota) are transferred to other candidates in accordance with the later preferences expressed by the relevant voters.

5. If, at any stage of the count, no surplus remains to be transferred, but not all seats are yet filled, then the candidate who currently has fewest votes is excluded. Votes assigned to that candidate are then transferred to other candidates in accordance with the later preferences of the relevant voters.

Meek style STV - what is different?

6. All surpluses are transferred simultaneously instead of in a particular order.

7. Surpluses are taken, in due proportion, from all relevant votes, not only from those most recently received.

8. To make that work properly it is necessary to give votes to already-elected candidates and not "leap frog" over them. This does not waste votes as the same number are transferred away again, but now in due proportion to all relevant votes.

9. Whenever a candidate is excluded, the count behaves as if that candidate had never existed (except that anyone previously excluded cannot be reinstated).

10. Whenever any votes become non-transferable, the quota is re-calculated, based on active votes only. This lower quota then applies not only for future election of candidates, but also to already-elected candidates giving them all new surpluses.

11. No candidate is ever elected without reaching the current quota.

12. For surpluses, every relevant vote goes to the voter's next choice, at fractional value. If there is no next choice, the fraction becomes non-transferable.

13. At an exclusion all the relevant votes are dealt with at once. There is no doing one little bit at a time.

14. The only disadvantage is that it is too tedious to do by hand, but has to be by computer.

Examples

1. A very simple, though artificial, example of the superiority of the Meek method is seen in 4 candidates for 3 seats. If there are only 5 voters and the votes are: 2 ABC, 2 ABD, 1 BC it is obvious to anyone, whether knowing anything of STV or not, that the right solution must be to elect A, B and C, as the Meek method does, yet traditional hand-counting rules elect A and B but declare the third seat to be a tie between C and D.

2. In a real election held recently, I shall call 4 of the

candidates A, B, C and D of whom at the last stage, A and B had each been elected with a surplus, C had been excluded and D was still continuing, to be either the last elected or the runner-up. Four of the votes gave preferences as ABCD, ACBD, CABD and ABD. As C had been excluded, these became identical votes, each now having A as first preference, B as second and D as third. The Meek method would have treated them identically, but the rules actually in use gave D wildly different portions of these votes, as follows:

Vote	Rules as used				Meek rules			
	Portion of vote assigned to				Portion of vote assigned to			
	A	B	C	D	A	B	C	D
ABCD	0.72	0.28	-	-	0.471	0.285	-	0.244
ACBD	0.72	-	-	0.28	0.471	0.285	-	0.244
CABD	-	-	-	1.00	0.471	0.285	-	0.244
ABD	0.72	0.28	-	-	0.471	0.285	-	0.244

The variation between all of the vote going to D, and none of it doing so, is really startling.

How was my vote used?

I D Hill

If an election has been conducted by STV using Meek counting, and the final keep values have been published (as I think that they should be), any voters who remember their preference orders can work out how their votes were used, as follows.

Suppose you voted for Bodkins as first preference, for Edkins as second preference, etc., where their final keep values were published as 0.310, 0.772, etc., as shown in the table below. The first thing to do is to make such a table with the order of preference that you actually used for the real candidates and fill in their published final keep values in column (3).

Always start with 1.000 as the first item, one line above your first candidate, in column (6), and then in each row in turn, fill in columns (4), (5) and (6) using the rules shown.

Preference	Candidate	Final keep value	Previous vote remaining	Vote kept	Vote remaining
(1)	(2)	(3)	(4)	(5)	(6)
			previous (6)	(3) × (4)	(4)−(5)
					1.000
1	Bodkins	0.310	1.000	0.310	0.690
2	Edkins	0.772	0.690	0.533	0.157
3	Atkins	0.000	0.157	0.000	0.157
4	Dawkins	0.702	0.157	0.110	0.047
5	Firkins	1.000	0.047	0.047	0.000
6	Gaskins	0.570			
7	Catkins	0.978			

When an excluded candidate appears, such as Atkins above, the keep value is 0.000, so no part of the vote is kept. When a candidate was either the runner-up or the last to be elected, such as Firkins, the keep value is 1.000, so that candidate keeps everything received and later preferences get nothing.

Column (5) tells how the vote was used. 0.310 of it went to help elect Bodkins, 0.533 of it went to help elect Edkins, 0.110 of it went to help elect Dawkins and the remaining 0.047 went to Firkins and, if Firkins was runner-up, was unused.

I have been asked by someone who has seen the above to produce something similar for traditional-style STV (and, in particular, for Newland and Britton rules, second edition). Having had a look at the problem, I have concluded that, for anyone who really understands what is going on, the information can be derived from the result sheet in an *ad hoc* way, but that it is not possible to do anything as general, or as simple, as the above.

This should be offered as an exercise for those who think the traditional rules simpler than the Meek rules. Let them do it. I do not deny, of course, that the traditional rules are less long-winded for making a hand-count, but in every other way, in principle and in practice, the Meek rules are much the simpler.

STV and Equality of Preference

C H E Warren

The Single Transferable Vote is a preferential voting system, in which the voter has to list the candidates in the order in which he prefers them.

One of the questions which is asked is whether a voter should be permitted to express an equality of preference between two candidates whom the voter assesses as equal in his judgement. My view is that the expression of equality of preference should be permitted in principle, although of course it would complicate both the voting and the subsequent count.

If a voter does express an equality of preference between two candidates A and B, then it is assumed that this is tantamount to his expressing two half-votes with non-equal preferences, one half-vote for A followed by B, and the other half-vote for B followed by A, but the half-votes otherwise identical.

However, Bernard Black is concerned that, if equality of preference is permitted, a voter may see neither of his equal preferences elected, whereas if the voter had given one of his two a clear preference then at least he would have got that one elected.

The following example of an election for 3 seats from 6 candidates by 30 voters, for which the quota is 7.5, exemplifies Black's concern. 29 of the voters vote as follows:

```
1 AB
1 BA
9 CAB
1 CEF
9 DBA
1 DEF
3 EF
4 F
```

The thirtieth voter is undecided between A and B. If this thirtieth voter votes AB, or votes BA or expresses an equality of preference between A and B, then the votes after the surpluses of C and D have been transferred are:

AB		BA		½AB + ½BA	
A	4.25	A	3.25	A	3.75
B	3.25	B	4.25	B	3.75
C	7.5	C	7.5	C	7.5
D	7.5	D	7.5	D	7.5
E	3.5	E	3.5	E	3.5
F	4	F	4	F	4

We see that if the voter gives a clear preference for either A or B, then that one gets elected, because the other one is now eliminated and his votes then transferred to the preferred one. However, if the voter expresses equality of preference, then E is now eliminated, and E's votes then transferred to F who is elected, so that neither A nor B is elected. Hence Black's concern is justified.

The main benefit that is likely to arise from permitting equality of preference, as Douglas Woodall has said, is not for voters who are undecided between their top preferences, but for voters who want to put certain candidates as their bottom preferences, below a whole lot of candidates whom they do not know much about, but for whom being able to give equality of preference would be ideal.

David Hill has shown, in an unpublished paper, that, in a real election, this middle group of candidates whom the voter does not know much about is more likely to be of relevance with Meek[1] counting than with Warren[2] counting, because with Warren counting the count does not extend down to this middle group of candidates.

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Equality of preference − an alternative view

I D Hill

In the preceding paper[1], Hugh Warren states 'Hence Black's concern is justified', but the example from which he derives this opinion is not convincing. It really concerns the question of how a tie is to be resolved, since in each of his three cases the AB supporters have 7.5 votes and the EF supporters have 7.5 votes. This makes it critically dependent on using a version of STV in which the quota is defined to give precisely 7.5 as in Newland and Britton, second edition[2] and not 7.5 plus a minimal amount as in most versions of STV, such as Newland and Britton, first edition[3], for example. It also depends on the rule that anyone reaching the quota is to be deemed elected at once even though some other candidate could catch up if the process were continued.

I am not objecting to those features, but if we are prepared to base conclusions on examples that depend critically on them, it is easy enough to construct one that points to the opposite conclusion. Consider 4 candidates for 3 seats with an odd number, n, of voters who support A and B, and an equal number, n, who support C and D. The quota will be $n/2$ and if the AB party do not use equality, no matter how they arrange their votes between saying AB and saying BA, one of their candidates will have more than a quota, and the other less than a quota, on the first count. If the CD party all put C and D as equal first, each of their candidates will have exactly a quota on the first count and consequently either ACD or BCD will be elected.

It follows that Black's concern is *not* justified. In these extreme cases use of equality could either harm or help and it is not possible to know which. In reality such extreme cases rarely, if ever, occur. What would normally happen if equality were used would be for one of the two candidates to go out (either as excluded or elected) at some stage and then the relevant part of the vote would be transferred to the other candidate, so nothing would be lost.

References

1. C H E Warren. STV and Equality of Preference. *Voting matters*, Issue 7, p6, September 1996.

2. R A Newland and F S Britton. How to conduct an election by the Single Transferable Vote, second edition, ERS, 1976.

3. R A Newland and F S Britton. How to conduct an election by the Single Transferable Vote, first edition, ERS, 1973.

Issue 8, May 1997

Editorial

In this issue, a new format is being used, but without any change to the content or type of material being published.

It is hoped that future issues of *Voting matters* will be made available via the Internet. However, printed copies will continue to be made which can be ordered from ERS. Due to some limitations of the most straightforward means of producing material on the World Wide Web, the printed copies will be the master ones, and presentation on the Web may have some defects.

The first article which lists those organisations known to use STV is an example of material which should be available on the Web anyway. Given this, then updating the list can more easily be undertaken.

As before, I am concerned about the lack of variety in the authors of material. Electronic publication could easily encourage contributions from other countries.

Brian Wichmann.

Organisations using STV

The following is an alphabetical list of organisations known to use STV in the UK or the Republic of Ireland. In the interests of brevity, local organisations are not always included.

3M plc
Aberdeen University SRC
Adlerian Society of the UK
Allied Dunbar
Amnesty International
Association for Jewish Youth
Association of Municipal Engineers
Association of University Teachers (AUT)
Association of Teachers & Lecturers (ATL)
Association of Logic Programming
Automobile Association
Avon Cosmetics
Bar Council
Bardsey Island Trust
Bass plc
Beechlawn School
Birmingham Labour Group
Birmingham University
Bow Group
British Airports Authority
British Dental Association
British Psychological Society
British Association of Colliery Management
British Association of Dermatologists
British Association of Counselling
British Computer Society
British Council
British Humanist Association
British Medical Association
British Mensa Ltd
British Union of Anti-Vivisection
Brittle Bone Society
BUPA plc
Cambridge University Student Union
Campaign for Homosexual Equality
Cardiff Union Services
Celtic Film and TV Association
Church of England
Church of Wales
City Literary Institute
Committee of Vice Chancellors & Principals
Consumers' Association
Coopers & Lybrand
Crosslinks
Derbyshire E. R. Group
Drake & Scull Engineering Ltd
Du Pont UK Ltd
Eastern Electricity
East Midlands Electricity
Engineering Council
Electronic Data Systems
Express Newspapers Pension Ltd
Faculty of Public Health Medicine
Family Law Bar Association
Gateshead & South Tyneside LMC
General Dental Council
General Medical Council
Gilberd School
Glasgow Caledonian University
Gouldens
Greater Manchester Police
Greater London Unison
Guild of Hospital Pharmacists
Headmasters' Conference
Hoechst UK Ltd
ICL plc
Imperial Tobacco
Institute of the Motor Industry
Institute of Chartered Accountants
Institute of Civil Engineers
Institute of Electrical Engineers
Institute of Linguists
Institute of Management Services
Institute of Mechanical Engineers
Institute of Public Relations
International Association of Teachers of English as a Foreign Language
John Muir Trust
King's College London Students' Union
Leeds University Union
Lewisham & Kent Islamic Centre
London Borough of Sutton
London Electricity plc
London School of Economics Students' Union
Liberal Democratic Party
Liberty
Logica plc
Manweb plc
Mercury Communications
Methodist Conference
Midland Bank plc
Mountain Bothies Association
National Association of Teachers in Further & Higher Education (NATHE)
National Citizens' Advice Bureaux
National Federation of Housing Associations
National Freight Consortium
National Grid plc
National Westminster Group
National Power
National Union of Journalists
National Union of Mineworkers
National Union of Rail, Maritime & Transport Workers (RMT)
National Union of Students
National Union of Teachers
Neural Computing Applications Forum
News International Newspapers Ltd
Northeast Fife D.C.
Northern Electric plc
Northern Sinfonia
Norweb plc
Pensions Management Institute
Pensions Trust
Pharmaceutical Society of Great Britain
Powergen
Price Waterhouse
Professional Association of Teachers
Prudential Assurance
Royal College of General Practitioners
Royal College of Midwives
Royal College of Nursing
Royal College of Pathologists
Royal Statistical Society
Royal Town Planning Institute
Scottish Nuclear
Secondary Heads Association
Shantiniketan Centre, Southall
Shell UK
SeeBoard plc
Smith & Nephew plc
Solicitors Family Law Association

South Oxfordshire D.C.
Stoneham Housing Association
Southern Electricity plc
South East Electricity plc
South Wales Electricity plc
South West Electricity plc
Telegraph Newspapers
Total Oil Ltd
Theatrical Management Association
UK Central Council for Nursing, Midwifery and Health Visiting (UKCC)
UK Council for Graduate Education
University of Bristol
University of Wales Swansea Students' Union
University of Ulster Students' Union
Union of Democratic Mineworkers
Union of UEA Students
Yorkshire Housing Association
Yorkshire Water
Zionist Federation of Great Britain

The various companies named above will not be using STV to elect their Boards of Directors which are usually Yes/No ballots, but to elect Pension Fund Trustees. The accountancy partnerships of Coopers & Lybrand and Price Waterhouse use it to elect their Executive partners. These particular elections are unique in that, apart from partners retiring during the year, all partners are automatically candidates.

Quotation Marks

Dear Sir,

There are one or two matters I would like to comment upon.

In his article *Large Elections by Computer*, Dr Wichmann says there is strong evidence that the traditional method of STV counting produces the 'wrong' result. I would suggest that even with the use of quotation marks this is an unfortunate comment. The result is surely correct within the rules which have been used, and to suggest otherwise is to imply that there is something inaccurate, or wrong, with the count. It might lead to defeated candidates thinking they were defeated as a result of some procedural error by the Returning Officer, which would not be the case. It would be wiser to say that the election result might be different. I do not think we would wish to appear to cast doubts upon our own ballot organisation to count an election by STV.

The real problem with elections of this kind is the proportion of candidates to the number of places to be filled. In the UKCC example there were over 18 times more candidates than the 7 places to be filled. 129 candidates appears to offer those voting the widest possible choice, but the choice is unreal. Unfortunately few of the voters have sufficient knowledge about the candidates to be able to put more that a small number of candidates in preferential order. The candidates are allowed to provide information about themselves but there is still a great deal of information to read. One answer might be to re-examine the nomination process, with a view to there being more assentors to the nominations. The organisation may, of course, not wish to do this because it might create an unreasonable hurdle to nomination.

Dr Wichmann is under a misapprehension when he says *It can hardly be considered a criticism of Newland and Britton, since it is doubtful whether they ever conducted an election of the size considered here*. Major Britton and Mr Newland were closely involved with drafting the electoral arrangements for the UKCC and took a very close interest in the first two elections at Chancel Street to see how the counts went. The report of the first UKCC election records that 441 candidates were nominated and that 61,715 people voted. Therefore it would appear that there has been a decline in the number of candidates nominated. I can recall both Major Britton and Mr Newland being somewhat concerned at the number of candidates nominated for the first election, but thought the number would decline when it was realised that most of those nominated had no real hope of being elected. At the first two elections no candidate achieved the quota, the whole election consisting of exclusions, candidates being elected with a reduced quotient as votes became non-transferable. It was not until the third or fourth UKCC election that I recall being told that for the very first time a candidate had attained the quota during the count. My recollection is that Major Britton and Mr Newland would probably have recommended that the nomination procedure be amended if the number of candidates had not declined to a more manageable number for the voters.

E M Syddique, ERS

A reply

I owe readers an apology if they were under any misapprehension on the use of the quotation marks. Of course, there was no implication that the rules were not correctly applied; indeed the simulations I made assumed that. It is also worth noting that since I used artificial ballot papers, the implications for any specific election (like the last UKCC one) are unclear.

I cannot apologise to Major Britton and Mr Newland for not realising their involvement in the early UKCC elections which were even bigger than the one I analysed.

I believe that a major contributory factor to the results I obtained was that not only was one candidate elected with the quota, but that the others were elected with very much less than the quota. The re-computation of the quota undertaken by the Meek method therefore makes a bigger difference than would typically be the case.

Lastly, it seems to me that an advantage of STV should be that many candidates can compete. Hence introducing barriers to nominations seems against the spirit of STV.

B A Wichmann.

Are non-transferables bad?

I D Hill

Brian Wichmann[1] put forward four different tests of whether one vote-counting algorithm had done better than another and invited readers, before reading on, to consider whether they would regard failure on each test as a serious matter.

I did not cheat, but made the requested consideration before reading on. I concluded that I accepted his tests called Condorcet, No-hopers and Steadiness but I totally rejected his test called Non-transferables. I then found, not much to my surprise, that he had found Meek's method to be a clear winner (on his particular data) on the three tests that I accepted as valid, while Newland and Britton (2nd edition) rules had done ‘better’ on the Non-transferables test which I had rejected, so I think it important to explain just why I had rejected it.

My view is that everything should always be in accordance with what the votes say, in proportion to their numbers and, if some votes, in whole or in part, are entitled to transfer and do not indicate a wish to be transferred anywhere, then it is morally wrong not to make them non-transferable, in whole or in part as the case may be.

That being so we cannot say which of two methods is better on the basis of the number of non-transferables, until we know the cause of the difference. If method 1 shows more than method 2, we must ask whether this is due to method 1 making some votes non-transferable unnecessarily, or to method 2 failing to make votes non-transferable when they should be. With methods of which we know nothing except the outcome of this particular test, we can really say no more than that.

In the actual case, however, we do know the methods in detail and are aware that Meek's method never makes anything non-transferable except when it is right to do so. It follows that, if the Newland and Britton rules get a smaller number, it is they that are failing to do the right thing.

Reference

1. B A Wichmann. Large elections by computer. *Voting matters*, 1996, issue 7, 2-4.

Some Council Elections

B A Wichmann

Introduction

This paper is an analysis of some Council elections based upon computer simulation in a similar manner to two previous papers [1,2]. The analysis starts with (five) result sheets, since they are the publicly available record of the elections. The first stage consists of using a computer program to produce a set of ballot papers which reproduces the result sheet (or gets very close to that). The second stage consists of running a number of experiments based upon elections which select a random subset of the ballot papers. The third stage is a further analysis of the results.

This paper is concerned with STV elections in which there are no ‘party’ affiliations. Hence the voting patterns are different from those which applied in the Irish elections analysed in the first reference. The identity of the actual council elections used for this study is not stated here, since this is irrelevant and could detract from the conclusions which are thought to be relevant for all elections for several seats in which there are no parties involved.

Constructing ballot papers

Given a result sheet, then it is possible to construct a set of papers which would produce the same results. In producing such a set by hand, the obvious method is to work forward stage by stage. However if no transfers occur from candidate A (say), such a method will give preferences that, if A appears, stop at that point. In other words, preferences that are not required to produce the results as given in the result sheet are not given. Clearly, the voter will not necessarily do this, and more significantly, other algorithms may use subsequent preferences. Hence a more general means is required of producing ballot papers.

The program used in this study works as follows. The program computes transfer rates from A to B if candidate A was eliminated or had a surplus to transfer (and B was available for transfers). If no such transfer occurred, then an estimate is used based upon the first preferences for B.

Ballot papers are now constructed using a random number generator with an exact match for the first preferences. This set is then used as the starting point of an iterative process, working stage by stage, to obtain a very close fit to the actual result sheet. The program cannot necessarily obtain a perfect match when transfers of surpluses are involved. Experiments showed that the starting position which was dependent upon the seeds for the random number generator did not have a large effect on the accuracy of the final fit to the actual election.

An example

To give a fuller explanation of the method of constructing ballot papers from a result sheet, we give a simple example. Consider an election in which the votes for electing one candidate from 4 was:

```
10 AB
5 BCD
6 BAD
6 CDA
1 C
8 DAB
```

The result sheet from these ballot papers using Newland-Britton is:

	Stage 1	Stage 2	Stage 3
A	10	10	0
B	11	11	21
C	7	0	0
D	8	14	14
Non-T	0	1	1

Since we are concerned with a council election without parties, we consider each candidate in the same way. We can judge the overall popularity of each candidate from the first preference votes. We now construct a matrix to represent the probability of X being followed by Y in any preference (X could clearly be the last preference given, so Y is allowed to be the Non-Transferable option). For instance, given candidate D, then the preference specified after D is assumed to be A, B or C in the ratio 10:11:7 (since these are the ratios of the votes on the first preferences).

We can make a better estimate of the transfer probabilities, since we do have a limited amount of information from the result sheet. In this case, for stage 2 in which C is eliminated, we know that the next preferences were either D or non-transferable in the ratio 6:1, respectively. Hence, we can adjust our matrix accordingly. For stage 3, in which A is eliminated, the transfers were entirely to B, but the papers could have had a preference to C which would have been ignored. This clearly reflects the adjustments made to the matrix. The final matrix, based upon one hundredths of a vote, in this case becomes:

			TO		
FROM	NT	A	B	C	D
STRT	-	278	306	194	222
A	0	-	1000	194	222
B	0	278	-	194	222
C	143	278	306	-	857
D	0	278	306	194	-

The program now computes a trial set of ballot papers with an exact match on the first preferences, but using a pseudo-random number generator and the above matrix to produce the remaining preferences. Finally, adjustments are made to the papers to obtain a better match to the result sheet. The root mean square error is computed over the entries in the result sheet, which gives 0 in this case for the 3×5 entries, since we have a perfect match.

The ballot papers produced in this case (which depends upon the seeds used for the random-number generator) were:

```
2 ABCD
7 ABDC
1 ACBD
3 BADC
4 BCDA
2 BDAC
1 BDC
1 BDCA
1 C
2 CDAB
4 CDBA
1 DABC
4 DBAC
1 DBCA
1 DCAB
1 DCBA
```

There are clearly many differences between the initial ballot papers and the above. However, since there are 64 ways of voting, it is quite unlikely that 10 ballot papers would be identical as with the initial papers (and in this sense, the final set must be regarded as more likely than the starting set). The construction method in this case gives very few papers with incomplete preferences, since the result sheet had few non-transferables.

Five real elections

The results of running the program for the five elections are given in Table 1. The result sheets were from the application of Newland-Britton[3]. A very close fit was obtained in all cases. The entry *Next* gives the difference in the number of votes between the last candidate elected and the next highest. This figure is also divided by the number of votes to give a numeric indication of how close the choice of the last elected candidate is. For election B, the result was very close since this difference was a mere 14 votes (from 8739, ie 0.16%). In performing both Newland-Britton and Meek upon the ballot papers constructed by the program, only one result was obtained which was different from the actual result. For election B, Meek produced a result different from the actual election, but this is hardly surprising, due to the closeness of the final candidate elected.

The experiment

The experiment concerns the influence of candidates with no realistic hope of being elected upon the result. With the UKCC analysis[2], it was observed that such no-hopers had a bigger influence with Newland-Britton rules than with

Table 1: Five Council Elections

Election	A	B	C	D	E	Total
Candidates	17	17	16	13	12	75
Seats	4	4	4	4	6	22
Votes	5764	8739	9364	8486	1669	34022
Stages	13	15	12	10	10	
RMS error(votes)	0.05	0.04	0.41	0.06	0.28	
Next	128	14	221	75	7.46	
Next/Votes	0.0222	0.0016	0.0236	0.0088	0.0045	
Actual=New-Br	yes	yes	yes	yes	yes	
Actual=Meek	yes	no	yes	yes	yes	

Meek. In this case, 100 elections were run by selecting 200 ballot papers at random (repeated five times for each actual election). For these 100 elections, both Newland-Britton and Meek were run. The second row in Table 2 gives the number of times out of the 100 that the results from Newland-Britton and Meek were different. For the 500 elections the result was different for 88 cases, which implies that 4% of the candidates were treated differently.

The first row in Table 2 gives the number of candidates which were never elected in any of the 100 elections, called *no-hopers*. It would seem that this is not an unreasonable definition of those that have no chance of election, since we know that the number of first-preference votes is not always a good indication.

The 88 elections in which Newland-Britton/Meek gave a different result were now re-run with the no-hopers eliminated. The results of this are recorded in Table 2 in the rows with indented titles. In all but one case, the difference between the two algorithms was just one candidate. However, the result of the re-runs is somewhat confusing except for the simple case in which the elimination of the no-hopers makes no difference. The results in the table are classified as follows:

No change. In this case, the elimination makes no difference and hence these cases are not supportive of either Newland-Britton or Meek.

Revert to Meek. In this case, the result from Meek does not change, but that for Newland-Britton changes to that of Meek. Such a case is taken as supporting the use of Meek.

Revert to Newland-Britton. This is the exact opposite of the previous case and is taken as supporting the use of Newland-Britton.

Meek unchanged. In this case, the result for Meek does not change, but that for Newland-Britton does (but not to that of Meek). This case is regarded as supporting the use of Meek.

Table 2: Results of simulations

Election	A	B	C	D	E	Total
No. No-hopers	7	7	6	3	3	26
New-Br = Meek	78	84	83	86	81	412
New-Br≠Meek	22	16	17	14	19	88
No change	13	8	11	10	9	51
Revert to Meek	6	5	3	2	7	23
Revert to New-Br	1	2	1	1	0	5
Meek unchanged	2	1	0	1	0	4
Both change	0	0	2	0	1	3
Invert both	0	0	0	0	1	1
Other	0	0	0	0	1	1

Both change. In this case, both change to a different result. This is obviously not supportive of either algorithm.

Invert both. In this case, the results of both algorithms change to the previous result of the other one! Clearly not indicative of either Newland-Britton or Meek.

Other. None of the above, and again not supportive of either algorithm.

The overall count from the above classification is that 56 cases are neutral, 27 support Meek and 5 support Newland-Britton.

Conclusions

In appears that realistic ballot papers can be computed from the result sheets. However, it is difficult to validate this process, since at the moment, actual ballot papers are not available from real elections of any size. I would like to appeal for such ballot papers, perhaps in computer format, since such papers could be made available without revealing the source which surely would be satisfactory once the period of elected candidates had finished. All the election data obtained so far is for small elections for which the study above could not be applied.

The first result from this study is that Newland-Britton and Meek produce a different result for about 4% of the seats. The observed rate for the Irish elections in 1969 was 2.8% (3 out of 143) and for 1973 was 4.9% (7 out of 143). The difference between 1969 and 1973 is due to a decline in the party voting and hence is consistent with a figure of 4% given in this study.

Does a difference of 4% matter between two STV algorithms? Obviously, it is reasonable to say this is insignificant against a difference of around 30% when STV is compared to First Past The Post. On the other hand, for the Electoral Reform Society, it is surely unsatisfactory to have such differences. Unfortunately, resolving this issue, as we are all aware, is not easy.

The remaining result is that Meek has more indicative cases in its support than Newland-Britton by about 5 to 1 in the above experiment. Does this matter? Surely, a key advantage of STV is that candidates can enter without upsetting the result if they have no realistic chance of being elected. Providing other hurdles for candidates seems against the spirit of democracy.

References

1. B A Wichmann. Producing plausible party election data. *Voting matters*, Issue 5. pp6-9. January 1996.

2. B A Wichmann. Large elections by computer. *Voting matters*, Issue 7. pp2-4. September 1996.

3. R A Newland and F S Britton, How to conduct an election by the Single Transferable Vote, second edition, ERS 1976.

4. B L Meek, A new approach to the Single Transferable Vote, reproduced in *Voting matters*, Issue 1, pp1-10, March 1994.

Measuring proportionality

I D Hill

When you can measure what you are speaking about and express it in numbers, you know something about it, but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind. Lord Kelvin.

It is important to consider what the problem actually is, and solve it as well as you can, even if only approximately, rather than invent a substitute problem that can be solved exactly but is irrelevant. Anon.

I agree with the first of those quotations but I agree much more strongly with the second one. As Philip Kestelman points out in a recent article[1], if we are to talk of proportional representation, and to claim that one aim of STV is to achieve it, it is desirable that we should have some idea of how to measure it and thus be able to detect the extent to which one system or another is able to achieve it.

Many indices have been proposed for the purpose, of which Kestelman prefers the Rose index, or Party Total Representativity (PTR) as he renames it. While differently formulated, the various indices all seem to have similar effects, usually placing different elections in the same order of merit even if the numbers that they assign are very different. They mostly depend, in one way or another, on the differences between percentages of votes by party and percentages of seats by party. It seems a little odd when considering a multiplicative type of thing, like proportionality, to use an additive type of measure, but this does overcome some difficulties that might otherwise arise when parties get zero seats.

A correlation measure

There is an additional measure that is rather different from all these, mentioned by Douglas Woodall[2] as having been proposed by Dr J E G Farina and depending on the cosine of an angle in multi-dimensional space. This is not a concept with which the general public would feel easily at home, but the measure does turn out to be closely associated with the statistical measure known as the correlation coefficient, and many people seem to feel happy that they know what correlation means (even if, in fact, they do not). However the ordinary correlation will not do, because it measures whether points tend to be grouped around a straight line, but not all straight lines give proportionality.

For example with votes of 200, 400 and 600 and the proportional 2, 4 and 6 seats we get a correlation of 1.0, but the non-proportional 3, 4 and 5 seats equally get 1.0 as those points also fall on a straight line. To get a suitable measure we also need to include the same numbers over again, but negated. Thus 200, 400, 600, −200, −400, −600 with 2, 4, 6, −2, −4, −6 gives a correlation of 1.0 as before, but 200, 400, 600, −200, −400, −600 with 3, 4, 5, −3, −4, −5 gives only 0.983 demonstrating a less good fit.

The fatal flaw

If going for any of these measures, I like the last one best, but they all have one fatal flaw — they depend only upon party representation and only upon first preference votes. It is possible to use them upon features other than formal political parties if there is enough information available on those other features, which usually there is not. Kestelman does so, but this is rarely done, while how to extend them to deal with anything other than first preferences does not even seem to be discussed. They therefore, to my mind, fall within the terms of the second quotation in my heading, as the substitute problem that is irrelevant.

It is true that, in many elections, voting is mainly in terms of party, and that most people's party allegiances will be detectable in terms of their first-preference votes, but I object to those who say that all we need to know about an electorate is to be found in those things. I much more strongly object to any suggestion that voters ought not to vote cross-party if they wish, or even should not be allowed to do so.

It often helps discussion to look at an exaggerated case, even though it is far removed from what normally happens in practice. An example that I have used before concerns 9 candidates: A1, A2 and A3 from party A; B1, B2 and B3 from party B; C1, C2 and C3 from party C. The election is for 3 seats and the votes are, say,

```
50% A1 B1
50% A1 C1
```

If a system elects A1, A2 and A3 the above measures will all say that it has done well — with 100% of the votes for party A and 100% of the seats for party A. Yet nobody actually voted for A2 or A3 at any level of preference. From that election STV would elect A1, B1 and C1, the candidates whom the voters mentioned, yet such measures will all say that it has done badly. While I believe that a measure of proportionality, if we can find a suitable one, would be a good thing I am not prepared to accept as useful any measure that cannot deal sensibly with that case.

Minor parties and independents

A further difficulty with all these measures occurs if there are a number of minor parties (and/or independent candidates), none of which get enough votes to be entitled to a seat. If each of them is put into the formula as a separate entity, you get one answer, but if you put them together as “others” you may get a very different answer because that number of votes for a single party would have been worth a seat (or more). Such minor parties are likely to be so divergent that to elect any one of their candidates to represent all their voters would be quite unsatisfactory.

STV's proportionality

STV's proportionality comes from what Woodall [3] calls DPC for “Droop proportionality criterion”. This says that if, for some whole numbers k and m (where k is greater than 0 and m is greater than or equal to k), more than k Droop quotas of voters put the same m candidates (not necessarily in the same order) as their top m preferences, then at least k of those m candidates will be elected. In particular this must lead to proportionality by party (except for one Droop quota necessarily unrepresented) if voters decide to vote solely by party. Anti-STVites may argue that this is not altogether relevant because people may not vote like that, but they cannot have it both ways — if voters are not concerned solely with party, and do not vote solely by party, then measures that assume that only party matters must be wrong.

The STV argument is that it will behave proportionately, as defined above, so long as voters do vote solely by one thing, whether that is party or not, but if (as is usual) voters have a mixture of aims and motives it will adjust itself to match what they do want to a reasonable degree. Looking at how it works suggests that it must do so, but I know of no way of proving it. What I find obnoxious is to find those who oppose it saying that it cannot be guaranteed to do so, and therefore wanting instead some system that does not even attempt it.

Furthermore STV gives the voters freedom to show their true wishes, major party, minor party, independents, sole party or cross-party, by sex or race or religion or colour of socks, or whatever they wish, whether others think that a sensible way of choosing or not. Even if it did not give a reasonable degree of proportionality as well, it would be worth it for that freedom and choice. Party proportionality is a bonus, not the be-all-and-end-all. It may be that “when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind” but can we measure love, or aesthetic pleasure, or scientific curiosity? Perhaps there would be some advantages if we could measure them, but our inability to do so does not in the least affect our conviction that they are things worth having. Let us continue to seek a useful measure, but not be bound by imperfect ones.

First-preference measures unsatisfactory

Even within strictly party voting, the first-preference measures are unsatisfactory. Consider a 5-seater constituency and several candidates from each of Right, Left and Far-left parties. Suppose that all voters vote first for all the candidates of their favoured parties, but Left and Far-left then put the other of those on the ends of their lists. If the first preferences are 48% Right, 43% Left, 9% Far-left, all the measures will say that 3, 2, 0 is a more proportional result than 2, 3, 0. Yet STV will elect 2, 3, 0 and that is the genuinely best result, because there were more left-wing than right-wing voters. There is no escape by comparing with final preferences, after redistribution, instead of first preferences. That is merely to claim that STV has done well by comparing it with itself. Our opponents may sometimes be dim, but I doubt whether they are dim enough to fall for that one.

Conclusion

I remain of the opinion that a measure of proportionality is very much desired if we can find a suitable one, but we know of none, and an unsuitable one may be worse than useless. What do others think?

References

1. P Kestelman. Is STV a form of PR? *Voting matters*, 1996, issue 6, 5-9.

2. D R Woodall. How proportional is proportional representation? *Mathematical intelligencer*, 1986, 8, 36-46.

3. D R Woodall. Properties of preferential election rules. *Voting matters*, 1994, issue 3, 8-15.

Issue 9, May 1998

Editorial

I must apologise for the absence of an issue since May1997, but this has been due to a lack of material. There is no doubt that the primary reason for this lack has been the May 1997 elections and the consequences in terms of the political debate on voting reform which has engaged many potential contributors.

The first article by David Hill considers the vexed question of constraints. After producing an elegant possible solution to the problem, he advocates that constraints should not be used. It seems to me that constraints can be used, but only modestly. For instance, if a Council is to be elected having a treasurer who must be a qualified accountant, then a constraint is better than having a separate election. Also, for national bodies, it is difficult to get young people elected since they are not as well-known which again seems to me to be reasonable grounds for a constraint. What do others think?

The second paper was prepared to submit to the Scottish Office as a result of the paper giving the proposed electoral system for the Scottish Parliament. This has obviously been partly overtaken by events.

The third paper on voter choice and proportionality was prepared as a result of the ERS AGM, and has been submitted to the Electoral Commission.

The last two papers consider a topical issue: how to prepare an ordered list of candidates given preferential voting. In this case, the Liberal Democrats and the Green Party have decided upon different methods which are specified in these two papers. The one point of agreement, which is also supported by others, is that the method of ordering a list given by Newland and Britton should *not* be used for this purpose! In both the second and third edition of Newland and Britton's book, they suggest that the order of election within the STV stages should be used to order the candidates (see section 2.5).

For the tenth issue, I plan to produce a combined index of all the issues to that date. I also hope to produce a volume containing all these issues in one binding, hopefully with good reproductive quality. The intent is to provide a more convenient permanent record.

I also plan to provide Internet access to *Voting matters* via the UK Citizens On-Line Democracy, which has been agreed by ERS. This Internet site provides discussion groups and other informal material on democratic issues. UKCOD has also collected comments on the Government proposals for a Freedom of Information Bill (`http://foi.democracy.org.uk/`). I believe that providing access to *Voting matters* by this means will encourage further international contributions. The printed version will be the authoritative one, since it is not possible to control layout precisely on the Internet, nor can references be guaranteed to remain valid over a long time-frame. The Internet version may also be delayed by the conversion effort required.

Brian Wichmann

STV with constraints

I D Hill

David Hill is the author of the computer program certified for use in the Church of England elections.

Introduction

Elections sometimes include constraints such as, for example, saying that those elected must include at least a given number of each sex. How is it to be done?

The traditional way is set out, for example, in the ERS booklet by Grey and Fitzgerald, that preceded the later rules by Newland and Britton. I have sometimes heard their method referred to rather rudely as “the naïve rules”. Basically they are the same as those in the Church of England's 1981 regulations and say: (1) that if a point is reached in the count where a specified maximum number of candidates of the constrained type has been elected, then any other candidate of that type must be excluded as soon as possible; (2) that if a point is reached where the number unexcluded of the constrained type equals a specified minimum, then any such candidate not yet elected must be guarded, such that when choosing a candidate for exclusion at any later point, the lowest non-guarded one must be chosen.

Multiple constraints

Grey and Fitzgerald make no mention of the possibility of more than one constraint or how such is to be handled. The Church of England's 1981 regulations, however, specified that the same rules should be applied to each constraint independently. It was pointed out that this could lead to trouble because two constraints may interfere with each other. The example used was: suppose there are 3 seats to be filled, and one constraint requires at least 2 women, while another requires at least 2 black people. If the available candidates are 2 black men, 2 white women and 1 black woman, where no-one has a quota and the last-named has fewest votes, she would be excluded by looking at each constraint separately, whereas that exclusion makes it impossible for the constraints to be met. It might be objected that such requirements are unlikely but: (a) regulations must allow for all possibilities; (b) however unlikely for a complete election, such a thing could easily arise at a late stage of something larger.

In consequence, the Church of England's 1990 regulations gave no specific rule for handling multiple constraints but left it to the presiding officer to do as seemed right at the time.

An alternative for a single constraint

An alternative approach has been devised by Colin Rosenstiel and colleagues for use by the Liberal Democrats in their internal elections, where STV is to be used with a constraint on the minimum number of each sex to be elected. Their method is (a) to run STV with the correct number of seats and no constraints. If more of one sex are found to be required, then (b) to rerun with more and more seats until it is found which extra candidates of that sex to elect, and (c) to rerun with fewer and fewer seats until it is found which candidates of the other sex to exclude. There are some difficulties, but on the whole this seems at first glance to be an elegant solution for a single constraint, though it is not feasible unless the count is to be by computer and it is not easy to see how it could cope with more than one constraint. It should be noted, however, that it is incompatible with any promise to voters that their later preferences cannot upset their earlier ones.

Although attractive at first sight, I have now come to the opinion that this method is wrong in principle. Indeed this opinion relates to any scheme that starts with ordinary STV and says that, if that produces a result that meets the constraints, it should be accepted. Such a method is always wrong. This opinion may seem odd; does it mean that there is something wrong with ordinary STV? Yes, of course there is. We know well that a perfect electoral method is impossible. The main fault with ordinary STV lies in its “exclude the lowest” rule, which can lead to unjustified exclusion on occasions. The justification of the rule is that it seems to be impossible to find a better one without violating the promise that a voter's later choices cannot upset that voter's earlier choices. It is generally thought to be better to accept the fault than to violate that.

Excluding the lowest is on the grounds that we must exclude someone and that candidate looks, on current evidence, the least likely to succeed. But if we have a constraint that makes it totally impossible for some other candidate to succeed, it is plain daft not to exclude that candidate first.

A simple example can explain the point clearly. Suppose 4 candidates for 2 seats. A and B are men, C and D are women. The votes are:

```
19 ABD.
 8 CD..
 3 DC..
```

giving a quota of 10. A is elected at once and passes his surplus to B, but with no further surplus someone must be excluded and, without constraints, it is most sensible to exclude D, who looks the least likely to succeed, and make a fair fight between B and C for the second seat, which C wins.

Suppose, however, that there is a constraint to say that 1 man and 1 woman must be elected. A and C, as by plain STV, are 1 man and 1 woman, but the reasoning by which they were chosen is now quite inadmissible. It cannot be said that D "looks the least likely to succeed" because, no matter what happens, it is absolutely certain that B cannot succeed and a fair fight between B and C is impossible. The remaining seat must go to a woman and a fair fight between C and D is what is necessary. Excluding B, D beats C.

So, by plain STV, A and C are elected, 1 man and 1 woman. Yet, with the constraint of 1 man and 1 woman, it is right to elect A and D instead. This may seem remarkable, but if there is any flaw in the logic I should like to hear of it. The conclusion must be that the title "naïve method" has been wrongly ascribed.

Tackling multiple constraints

How then should multiple constraints be tackled? I believe that the traditional way for a single constraint is right but it needs to be extended to deal with multiple constraints as such, not with each single constraint independently. This is not easy, but if people will introduce multiple constraints, the difficulties are their fault.

The Grey and Fitzgerald rules must be extended to say that whenever a situation is reached such that certain candidates must be elected, or must fail to be elected, if all constraints are to be met then the appropriate action is required. It should be noted that such situations can be met even before vote counting starts, and it may even be that no solution is possible. Regulations need to deal with such cases.

It is superficially attractive to look at each possible set of candidates that could be elected and enquire of each set whether it meets all the constraints, classifying each set as positive or negative. At every stage, each set ruled out as inconsistent with those now elected or excluded would be reclassified as negative, any candidate appearing in every positive set would be marked as "guarded" (i.e. not to be excluded, but still to receive votes until reaching a quota), while any candidate appearing in no positive set would be at once excluded. However, if thoughtlessly implemented, this scheme could easily lead to a combinatorial explosion. For example to elect 20 candidates out of 40, there would be over 10,000,000,000 sets and if a computer could examine 1000 sets per second to classify them, it would take over 4 years merely to go through them once.

A more practicable scheme is to note that the candidates can be grouped according to which constraint features they possess. Usually there are many identical in such respects and looking at them individually is not necessary but only at the number in each group. With that simplification it has been found possible to implement a solution, but it remains sufficiently complicated that to try to do it without computer help is inadvisable. By hand and eye it is all too easy to miss the vital moment when constraints need to be applied, and if missed, disaster can ensue later.

A (disguised) real election

An example of this can be seen in an election that actually occurred though, for obvious reasons, I shall disguise it. I shall also simplify it a little.

Suppose an election in which there are 28 candidates for 14 seats. The candidates, with two-letter code-names for the groups are

```
 4 English men    (EM)
 7 English women  (EW)
11 Scottish men   (SM)
 3 Scottish women (SW)
 2 Welsh men      (WM)
 1 Welsh woman    (WW)
```

Constraints say that those elected have to be 7 English, 6 Scottish, 1 Welsh and additionally 7 men, 7 women.

Suppose that the first to be elected is a Welsh man. Anyone would at once see that the other Welsh man and the Welsh woman cannot now succeed so it is right to exclude them at once to let their supporters move elsewhere.

Suppose that the next to be elected are 2 English men and 2 English women, and that the next step after that is to exclude a Scottish woman. How many people would notice that this is a critical point, where everything will go wrong later unless constraints are applied? I think that few people would; without careful analysis it is hard to notice.

The point is that only 2 Scottish women remain, we have to elect 6 Scottish altogether, and have elected none as yet. Therefore we must elect at least 4 Scottish men. But we are restricted to 7 men in total and we have already elected 3. It follows that we must elect exactly 4 Scottish men, and that means that the 2 remaining Scottish women must now be guarded, and that the 2 remaining English men must be excluded as soon as possible, as they cannot now succeed without upsetting the constraints.

If such an election has to be carried out by hand, the best way is to prepare in advance, preferably with a computer to help, a list of all the possible ways, by groups not by individuals, in which a conformant result could be obtained. This can be done as soon as the candidates are known, when there is time to devote to it before the count. In the present instance, there are 8 possibilities:

EM	EW	SM	SW	WM	WW
0	7	6	0	1	0
1	6	5	1	1	0
1	6	6	0	0	1
2	5	4	2	1	0
2	5	5	1	0	1
3	4	3	3	1	0
3	4	4	2	0	1
4	3	3	3	0	1

With such a list at hand during the count, its lines can be deleted as soon as they become impossible. Thus as soon as the first to be elected is found to be a Welsh man, any line with WM set to 0 goes out, leaving just

EM	EW	SM	SW	WM	WW
0	7	6	0	1	0
1	6	5	1	1	0
2	5	4	2	1	0
3	4	3	3	1	0

The election of 2 English men and 2 English women leaves just

EM	EW	SM	SW	WM	WW
2	5	4	2	1	0
3	4	3	3	1	0

and the second of these becomes untenable when only 2 Scottish women remain. Knowing that the first line is now the only way to meet the constraints shows the steps necessary much more clearly than could be seen without it. With a bit of practice, to follow such a list, as an indication of the interaction of the constraints with the STV count, becomes a little easier. However, it can never be really easy.

In case anyone should suggest that such a complicated example is implausible, I should repeat that it did actually occur except that I have disguised it and *simplified* it.

Conclusions

I believe that the approach given above is the best way, within STV, to implement constraints but that they should not be employed unless it cannot be avoided.

The mechanisms of STV are already designed to give voters what they want, so far as possible, in proportion to their numbers. It should be for the voters to decide what they want, not for anyone else to tell them what they ought to want.

The magazine *Punch* in 1845 included “Advice to persons about to marry — Don't”. My advice on constraints is similar.

Comments on the Scottish Electoral proposals

I D Hill, R F Maddock and B A Wichmann

Co-incidentally, all three authors have been members of the British Standards Institution programming language standards committee at various times.

It is clear than the proposal (made in July by the Government[1] in advance of the September 1997 Referendum) is an incomplete draft. Nevertheless, it seems appropriate to list the logical problems which are in this draft, since it is unclear how a complete proposal would rectify the flaws. In some cases, aspects which are undefined could be resolved by taking the proposals made at the Scottish Constitutional Convention, but this is something to be submitted to a referendum to authorise a constitutional change. No matter how worthy that body, it would be absurd to regard its proposals as being in any way definitive for such a purpose.

Why admit the existence of parties?

Although the existence of parties is a key aspect of the proposals, we feel bound to query this for the reasons below.

1. To formally acknowledge the existence of political parties is not currently part of the UK electoral framework. Surely such a significant step should be justified by showing that the general objectives can only be satisfied by this step.

2. Who is to be entitled to register a party? How are the names of such parties to be resolved to avoid confusion? Several names could cause confusion: The New Labour Party or The Tory Party or even just Liberal.

3. The proposal appears to suggest that the stated objective is to attain proportionality of party representation within the Scottish Parliament. However, the UK already accepts that proportionality can be attained without formal recognition of parties by means of the Single Transferable Voting system used for the Northern Ireland European Elections.

4. The proposal has several logical flaws, most of which arise from the party identification (see below).

The whole process appears to have been designed to give as much power as possible to party organisations and as little as possible to the electorate, making a mockery of what democracy should be.

On the other hand, the recent case of the Literal Democrats indicates that standardized party labels have some benefits.

Can one have independent MSPs?

It is clear that independent MSPs could not be elected from the Party lists, but for the constituency MSPs this appears to be possible. After all, we have such an MP for Westminster and therefore the question is not academic. The basic right for anyone to seek election should not be unreasonably restricted and therefore one must assume that those seeking election as a constituency MSP need not have a party affiliation.

Can a 'rejected' MSP be elected?

This can happen under the German system and results in the electorate being very sceptical about elections. This happens as follows:

A candidate who is seeking re-election is both a constituency candidate and is on a party list. If the candidate fails to obtain election for the constituency, the person can nevertheless be elected via the party list. If the person concerned was overtly unpopular and lost by a significant swing, then to be subsequently elected is perverse.

No electoral system should give rise to anomalies as gross as the above, since it can seriously damage the electoral process in the eyes of the electorate. (However, we know that 'perfection' is not possible for electoral systems which implies that minor anomalies cannot be avoided.)

One party list or many?

It is not clear if there is a single party list for each party, or one for each European Constituency. Note that the rules appear to allow for a party which is already over-represented to obtain additional seats due to being under-represented within one European Constituency (thus increasing the lack of proportionality).

Better proportionality would be obtained for a single list allocated on the basis of the entire Scottish vote. If the aim is to elect on the basis of European Constituencies, then why not STV for each such constituency?

Some problems

A list is made here of the main flaws that we have noticed. We cannot guarantee that the list is complete.

1. Who specifies the party lists? In practice, a good fraction of the MSPs are not determined by the electorate but by those who draw up the lists. In consequence, it is most important that the mechanism for producing these lists should be well-defined (or even an explicit statement that the party organisations determine the list by means of their own choosing). If the list is specified by the party organisations without any electoral process, then it is clear that this aspect is less democratic than any other mechanism currently in use within the UK.

2. When are the party lists published, and by whom? Is the list on the ballot paper? Surely the lists have to be published by the returning officers, but what restriction, if any, is placed upon the lists? (One could allow 'cross-benchers' to appear, as in the Lords. We assume that the lists are published before polling day!) The Scottish Constitutional Convention proposals appear to suggest that the list is just that, with no 'party' as such, which leaves open how parties are linked to constituency MSP's to determine the number of additional members.

3. Can a (previously) sitting MSP also be on a party list? If this is allowed, then the German problem arises, as noted above. In consequence, it seems best to exclude this. Obviously, if an MSP is elected as a constituency member, then one must assume that his/her name is deleted from the party list. This might present a practical problem if the MSP appeared on a different list from his/her own European Constituency.

4. What duplicates can appear on the party lists? If a person could appear on the party list for more than one European constituency, then logical problems arise due to the coupling of the voting between the European Constituencies. In particular, the result would depend upon the order in which the European Constituencies were considered.

5. The dependence of the proposals on the European Constituencies seems odd since the government has indicated its intention that the next European election, which will occur before the elections to the Scottish parliament, will use a regional list system, and thus the current European constituencies will no longer exist. The white paper does say that if the European constituencies are changed the boundary commission will make "appropriate arrangements for the Scottish Parliament".

6. A popular MSP could stand as an 'independent' so that his/her seat would not count for his/her party, thus increasing their additional members by one.

7. In a somewhat similar position to the last problem, a party could have a different label for its constituency candidates than for its party list. This would make the party list label appear under-represented (no seats), thus being eligible for additional members.

8. Apart from the voting system, we regard it as quite wrong that Scottish MPs will apparently be allowed to continue to vote at Westminster for what is to happen in England on the devolved issues.

9. The statement that the number of Scottish seats [in Westminster] will be reviewed begs more questions than it answers. The number of seats could even be increased! (However, Donald Dewar, introducing the white paper in the Commons, indicated that the number of Westminster constituencies was likely to be reduced at the next boundary review, and the white paper says that such changes would lead to corresponding changes in the number of both constituency and additional members in the Scottish parliament.)

10. It has been noted in New Zealand that a result of a mixed system of constituency members and party lists is a potential conflict between local party workers (who want to get their constituency member elected) and the party organisation (who might prefer the next person on the list instead).

11. The proposals call for 129 members which appears to be a consequence of the constituency numbers with the need for 56 additional members to obtain proportionality. Contrast this with STV for each of the 7 European Constituencies which could obtain the same degree of proportionality with around half the number of MSPs. (The cost saving would be very significant, and the body might well be more effective.)

12. Candidates must be *resident in the UK*, including therefore resident outside Scotland, which is different from most local elections in Britain, where the candidate must reside in the area administered by the assembly in question.

13. Can a Westminster MP simultaneously be an MSP? Nothing is mentioned about this, so one assumes the answer is yes, as it is for MEP, MP, county councillor, district councillor, parish councillor,... However, the proposals made by the Scottish Constitutional Convention state that being an MSP is *a full-time appointment* and thus excluding such roles (except perhaps being a Peer).

14. The arrangements for by-elections are not stated, although proposals were made by the Scottish Constitutional Convention, which we assume apply (namely, a conventional by-election for constituency MSPs, and the next on the party list for the additional members).

15. It is not specified what happens if a party list is exhausted.

16. If an MSP, elected from the party list, resigns from the party or is expelled from it, is resignation as an MSP to be required?

Reference

1. The Internet Scottish Office pages, and those from the Scottish Constitutional Convention.

The above paper records our comments at the time that it was written. We recognise that some of its queries have now been answered.

Voter Choice and Proportionality

B A Wichmann and R F Maddock

At the Electoral Reform Society 1997 AGM, Hugh Warren produced an eye-catching diagram in which several electoral systems were plotted on a diagram in which the two axes were voter-choice and proportionality. The diagram was not intended to give precise measures of the characteristics of each electoral system, but merely their relationship. However, for (party) proportionality, the Rose Index is a reasonable approximate measure. For voter-choice, no existing measure appears to be available which would be necessary to provide a more accurate representation of the diagram.

A possible measure of voter choice is the information-theoretic value of the result of an election, which appears to be new. For instance, in a dictatorship which has mock elections, the result is known beforehand, and therefore the information-theoretic value is zero. On the other hand, if the electorate is given a choice between three candidates then, assuming that each outcome is equally likely, the information-theoretic value is $\log_2(3)=1.58$. As the number of possible outcomes increases, so does this measure of voter choice.

For values of the Rose Index, Kestelman[1] gives values for the major electoral systems. It must be acknowledged that the Rose Index as a measure of party proportionality, may not be appropriate for STV elections, as pointed out by David Hill[2].

We compute the values for a hypothetical election for a 600 seat assembly in which there are three parties. For the use of STV, we take 120 constituencies each electing 5 members. For the regional list, we take 10 regions electing 60 candidates each. For the additional member system, we assume 300 seats elected directly and 300 added by proportionality. Note that if n seats are to be filled with 3 parties, then the number of ways to do this is $n^2/2+3n/2+1$. We assume that all possible outcomes are equally likely. The entries in the diagram are as follows:

First Past The Post (**FPTP)**: Rose Index 70% (UK), voter choice is $600 \log_2(3)=951$.

Alternative Vote (**AV**): Rose Index 72% (Australia), voter choice is $600 \log_2(3)=951$.

Single Transferable Vote (**STV**): Rose Index 92% (Ireland), voter choice is $120 \log_2({}^5C_{15})=1386$. (We are assuming each party has five candidates and therefore could theoretically obtain all five seats; hence the number of possibilities is the number of ways of selecting 5 from 15.)

Additional Member System (**AMS**): Rose Index 98% (estimated), voter choice

$300 \log_2(3)+\log_2(300^2/2 +3\times300/2+1)=491$.

Party List (**PL**): Rose Index 98% (estimated), voter choice is $\log_2(600^2/2+3\times600/2+1)=17.5$.

Regional party Lists (**RL**): Rose Index 98% (estimated), voter choice is $10 \log_2(60^2/2+3\times60/2+1)=109$.

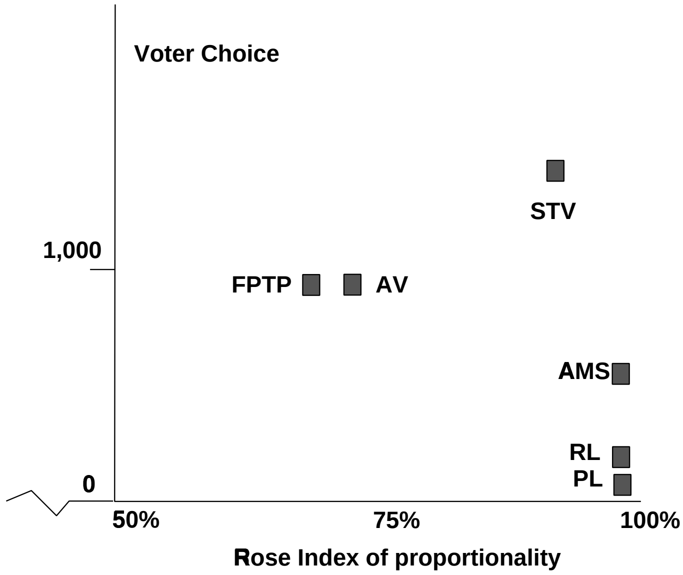
It is important to note that this diagram will change if the underlying assumptions are changed, for instance, if the number of parties was increased from 3 to 4. An alternative way to compute voter choice values would be to take into account the probability of the various outcomes, based upon appropriate statistical data. This was considered initially but rejected due to the difficulty of the calculation and the problems in finding appropriate statistical data. If the voting system was changed, then one can only guess at the future statistical data. (The diagram here has the *x*-axis reflected from Hugh Warren's version so that the Rose Index is increasing.)

The conclusion from this diagram is hardly unexpected: party lists do not give voter choice, and FPTP/AV do not give party proportionality, while STV can claim, to a reasonable degree, to provide both.

References

1. P Kestelman. Is STV a form of PR? *Voting matters*. Issue 6. p5-9.

2. I D Hill. Measuring proportionality. *Voting matters*. Issue 8. p7-8.



Producing a Party List using STV

C Rosenstiel

Colin Rosenstiel is a member of the Council of ERS and the author of a computer program for STV.

With some of the current proposals for electoral reform, parties will be required to produce a list from whom candidates will be elected in order from the top. STV can be used to construct the ordered list, given a preferential ballot of all party members.

The conventional use of STV to elect n members gives members of equal status, since the order in which STV elects does not necessarily determine the strength of their support. Repeated use of STV elections can be used to determine an order as follows:

Given a total list of 10 (say), then the first step is to determine those on the list (without an ordering) by running an STV election with all the candidates and 10 seats to fill. The next step is to run an STV election for 9 seats with 10 candidates being those previously elected (using the same ballot papers). The eliminated candidate is then placed last on the list. Next, an STV election if run with the remaining 9 candidates with 8 seats to determine the next lowest candidate, and so on.

This process might sound tedious, since so many STV elections are run, but if a computer is used, it is straightforward. Note that the above process will not work in reverse, i.e. selecting the top candidate first. The reason for this is that when electing two candidates, it can happen that neither of those elected is the previously selected 'top' candidate.

Two elections were taken in which there was more than ten candidates to which we have applied the algorithm above to order the top 10 candidates. The results obtained were as follows:

```
                     Election 1     Election 2
This algorithm    :  ABCDEFGHIJ     ABCDEFGHIJ
Order of election :  CABDEFGHJI     CBAFEHDGIJ
```

As expected, it can be seen that the order of election does not give the same result as successive elimination. Hence this algorithm is recommended in producing party lists.

Editorial comment

It has been suggested to me that if the Meek method is used, then just one election would suffice (to determine the order of the 10 candidates). Their order can be found from the retention factor in the final table of the election results — the smallest retention factor implying the strongest candidate since that candidate required the smallest proportion of the votes retained to get the quota. These values do give a measure of their relative support, unlike the order of election. In the elections above the Meek results were:

```
                     Election 1     Election 2
Meek 'keep' factor:  ABCDEFGJHI     ABCEFDHGIJ
```

This would appear to indicate that the methods of ordering of the candidates produce a similar result. In practice, both methods would need to use a computer and hence there seems to be little to choose between them.

Ordered List Selection

J Otten

Joseph Otten is the Green Party Policy Co-ordinator.

Rationale

The electoral system to be used for the next European Elections requires ordered lists of candidates from each party. It was felt that the advice in the ERS booklet [1] that *If an order is desired, this is provided by the order of election* (2.5) was inadequate — it would effectively lead to a First Past the Post contest for the top place on the list.

Were we to know in advance that we would win, say, *n* seats in a region, then it would be straightforward to use STV to select *n* candidates from the potential candidates and put them in the top *n* places in our list. If we don't know *n* in advance (which we don't!) then we can perform this operation for every possible *n*, i.e. from 1 up to the number of seats available in the region, and attempt to construct a list whose top *n* candidates are those victorious in the *n*th selection ballot. (There is really only 1 ballot — the division into *n* ballots is notional.)

This ideal solution fails when a candidate elected for one value of *n* is not elected for a larger *n*. In such cases either the STV result for a smaller *n* must constrain that for the larger (top-down) or vice versa (bottom up). Reasoning that the Green Party would be unlikely to win large numbers of seats in any region, we opted for top-down.

Algorithm

Each count is conducted by ERS rules [1] with the following alterations. We start with the count for the first place (*n*=1) and work down.

5.1.6 Calculate the quota

Divide the total valid vote by one more than the ordinal number of the count. Eg for the third count, divide the total valid vote by 4. If the result is not exact, round up to the nearest 0.01.

5.2.5 Excluding Candidates

Do not exclude any candidate in one count if they have already been elected to the list in an earlier count. This may introduce distortions to the results of later counts, but is necessary to preserve the integrity of the earlier counts.

If a count is proceeding identically to an earlier count, and an exclusion by lot is required, then the result of the earlier lot should be taken as read. Otherwise the lot must be recast. (cf 5.6.3)

5.3.3 and 5.4.2

For the purpose of these rules (i.e. receiving transfers), a candidate elected in a previous count (not stage) should be treated as a continuing candidate for purpose of receiving transfers during the count, until they are deemed elected again.

5.5.2 Completion of the Count

For the purpose of this rule, any candidate elected to the list in a previous count shall be deemed elected. Therefore the count may stop as soon as a single candidate is deemed elected, who was not elected in a previous count. In exceptional circumstances it is possible that two candidates, not previously elected may exceed the quota in the same stage. Only one may be elected. Resolve as follows (in order of priority):

1. If more than one value of papers is transferred during that stage, and only one candidate is elected as a result of the transfer of an earlier (i.e. higher valued) batch, then that candidate is deemed elected.

2. If both exceed the quota during the transfer of the same batch, then elect the one with the higher vote.

3. According to 5.6.2

4. By lot.

Other deviations

My apologies to the Electoral Reform Society for these, but they do seem to be popular in some quarters.

Where regional parties have agreed to adopt gender balance constraints, then the usual constraint rules shall be used. This usually means excluding all the candidates of a particular sex at the beginning of an even-numbered count.

Each region was free to determine its own gender balance formula. For example one region might require a list of half men and half women with no constraints on position, and another region might require that the top two candidates were a man and a woman with no constraints on the other candidates. Whatever formula was chosen, this was applied within the system by excluding any ineligible candidates at the beginning of a round. Hence the top place on each list would be open to both sexes, and subsequent places would only be constrained in the event of an imbalance. Notably the London region decided not to impose a gender balance formula, and the top three candidates are all women.

On each ballot form there is a notional candidate called "Re-Open Nominations" (who is of indeterminate sex). If Re-Open Nominations is elected to the list, then there must be a fresh election for that place and lower places on the list. This is a distortion of STV which could be used by a majority to deny minority representation, although there is no evidence of this happening. STV, rightly in my view, omits this sort of negative voting, but it is popular in the real world outside public elections, such as in student unions.

Conclusions

Although the justification for starting at the top of the list and working down, as opposed to starting at the bottom or even in the middle, is not particularly strong, this system is a reasonable solution to the question of seeking an ordered list. In particular it ensures that however MEPs are elected in any region from the party, they are as proportionally representative of the range of opinion in the party as their number allows.

Reference

1. R A Newland and F S Britton. How to conduct an election by the Single Transferable Vote. ERS 3rd Edition. 1997.

Issue 10, March 1999

Editorial

The publication of the Jenkins Commission report has presented ERS with a dilemma. The role of STV is minimal and on any reasonable measure, the degree of voter choice is not on the same scale that STV would provide. However, the report in my mind raises a technical challenge. If one accepts as a political imperative that 80% of the seats must be from single-member constituencies, can one devise a scheme with an increased voter choice which is simple to understand?

I certainly believe that this is possible. Moreover, I think that one should accept the significant support for First Past The Post (FPTP).

The Jenkins proposal of having two votes seems to me to be basically flawed since it then requires a mathematical adjustment to correct the mis-representation from the single member constituencies. Why not have just one vote, which either elects your chosen candidate for the single-member constituency or is transferred to the 'county' vote?

With an additional 20% to be elected at the 'county' level, and STV being the electoral mechanism, one needs about 15 single member constituencies to be merged into counties. These counties would therefore elect three members, giving useful voter-choice and good proportionality.

What would such a scheme look like from the point of view of the voter? The single-member voting would retain FPTP and hence would correspond to the existing system apart from a 20% increase in the size of these constituencies.

This implies that votes which would undeniably be 'wasted' under the present system would now be transferred to the county vote. Here the voter has a bigger choice, but more difficult decisions to make. With perhaps 12-20 candidates to rank in order to elect 3 people, the situation would be very similar to that of the voter in the Irish Republic. The key difference is that this STV vote would only apply to those voters who did not select a winning candidate at the single-member constituency level. Surely this scheme would end the need for strategic voting. The use of the Alternative Vote, as proposed by Jenkins, would therefore be unnecessary.

If voter choice is to mean anything at all, surely the voter must be able to choose between candidates of the same party. By having STV with three seats, such a choice would be effective. Increasing the size of the STV areas would have some advantages in terms of proportionality, but would probably give a ballot paper that was too cumbersome (compared with current practice).

Brian Wichmann.

The Handsomely Supported Candidate Ploy

C H E Warren

Hugh Warren is a retired mathematician

There is an electorate of 1400, who have to elect candidates to fill 6 seats, so clearly the quota is 200. The electorate is made up of 418 members of the Labour Party and 982 members of the Conservative Party. Labour should, therefore, get 2 seats, and the Conservatives 4.

The Conservatives put up 5 candidates — L, A, B, Z and Y. Candidate L is the Party Leader, and is handsomely supported because of his ability to hold the party together, despite its Europhile and Europhobe wings. Candidates A, B are on the Europhile wing, and Candidates Z, Y on the Europhobe wing. If all the Conservatives voted sincerely their voting pattern would be as follows:

```
503 LAB
479 LZY
```

Whether the count is done by Newland & Britton[1], Meek[2] or Warren[3], 4 Conservatives would be elected — L, A, Z and B. Not surprisingly the Europhiles get one more seat than the Europhobes because they are the slightly larger faction. 182 Conservative votes would be ‘wasted’, as would 18 Labour votes, thereby making up a quota of 200 votes in total which are perforce ‘wasted’ in any STV election.

However, the Europhobe Conservatives adopt the Handsomely Supported Candidate Ploy. Above everything else they want to see their leader, Candidate L, elected. But they argue that their support of 479 voters should be enough to ensure that Candidate L is elected if they insincerely give him their second preference only, even if those Europhiles are even more insincere and don't give Candidate L a preference at all!

In practice the Europhiles vote sincerely, so the voting pattern turns out to be:

```
503 LAB
479 ZLY
```

If the count is done by Newland & Britton or Meek, the Europhobes' ploy pays off, because the 4 Conservatives elected are L, A, Z, Y. So, by their ploy, the Europhobes have ‘captured’ the fourth Conservative seat for the Europhobes.

Of course one can not guarantee that one will always gain an advantage by adopting the ploy, but it is always worth trying on, for one can not lose provided it is done prudently, as in the example here, by not relegating a handsomely supported candidate to a preference where one has not the support to get him elected no matter what other voters do.

The Handsomely Supported Candidate Ploy, if practised by a group, can lead to a discernible gain, as just demonstrated. However, the principle, that one can gain an advantage by not giving one's first preference to a handsomely supported candidate, holds even for voters voting individually.

Consider an election for nine seats by 100 voters, so the quota is 10, in which the voting pattern is as follows:

```
39 H...
19 M...
41 ....
 1 HM...
```

H is clearly a handsomely supported candidate, and M a moderately supported candidate. These two candidates do not figure in the voting pattern other than in the places shown.

If the count is done by Meek the individual voter HM... will have 0.37025 of a vote to pass on to his third preference after H and M have retained just the votes necessary to attain the quota.

However, if the individual voter decides not to give his first preference to the handsomely supported candidate H, who would be his sincere first preference, but instead to vote MH..., then he finds that he has 0.37342 of a vote to pass on to his third preference.

It is the principle that is salient from this example — that one can get more out of one's single vote by not giving one's first preference to a handsomely supported candidate.

References

1. R A Newland and F S Britton. How to conduct an election by the Single Transferable Vote. ERS 3rd Edition. 1997.

2. B L Meek, A new approach to the Single Transferable Vote, reproduced in *Voting matters*, Issue 1, pp1-11, March 1994.

3 C H E Warren. Counting in STV Elections. *Voting matters*, Issue 1, pp12-13. March 1994.

An example of ordering elected candidates

C H E Warren

Colin Rosenstiel has proposed that elected candidates can be ordered by successive elimination[1]. In an unpublished note of the same date (May 1998), Eric Syddique has proposed essentially the same method. However, in Newland & Britton[2], the method proposed is to take the order of election. The purpose of this paper is to show that these two methods can produce very different results.

Consider the following election of 4 candidates from 7 contenders by 600 voters, for which the voting profile is:

```
 50 AC
 70 AD
115 BED
100 CD
115 D
 65 ED
 50 FCD
 35 GBED
```

Since the quota is 120, we obtain the following result sheet from the ascription of the Newland & Britton principles, avoiding the rounding errors which the practical application of their method as given by them introduces.

A	120		120		120		120		120
B	115	+35	150	-30	120		120		120
C	100		100		100	+50	150	-30	120
D	115		115		115		115	+30	145
E	65		65	+30	95		95		95
F	50		50		50	-50	0		0
G	35	-35	0		0		0		0

Hence the order of election is A, B, C and then D.

With the Rosenstiel/Syddique method of successive elimination, with E, F and G eliminated the votes are:

```
120 A
150 B
150 C
180 D
```

B, C and D are selected and A is placed fourth and eliminated henceforth. The votes are then:

```
150 B
200 C
250 D
```

C, D are selected, and B is placed third and eliminated henceforth. The votes are then:

```
200 C
400 D
```

C is now placed second and D first. To summarise, the order is D, C, B and then A.

Hence the two methods produce ordering which is exactly the opposite of each other.

References

1. C Rosenstiel. Producing a Party List using STV. *Voting matters*, Issue 9, pp7-8, May 1998.

2. R A Newland and F S Britton. How to conduct an election by the Single Transferable Vote. ERS 3rd Edition. 1997.

STV with constraints

Earl Kitchener

Hill[1] describes a useful way of dealing with constraints, but then says that “It should be for the voters to decide what they want, not for anyone else to tell them what they ought to want”. If, as is normally the case, the rules for elections have been set by the voters, there *is* no-one else, because it is the voters themselves who have decided in advance that they want, say, at least one new member and at least one sitting member. I feel that the ERS should encourage the use of constraints, so that we can learn whether they turn out to be helpful.

When voting is on party lines, it may be desirable to ‘entrench’ the rules by only allowing them to be changed by more than a simple majority. This is because a party in power can often find alterations whose only merit is that they would favour it. In some cases this would force constraints on unwilling voters.

Reference

1. I D Hill. STV with constraints, *Voting matters*, Issue 9, pp2-4, May 1998.

Response by I D Hill

I am grateful to Lord Kitchener for his courtesy in letting me see his paper in advance and for having no objection to my putting a reply in the same issue.

Although it is true that constraints would sometimes have been set by the voters themselves, it is by no means always so. For example, some Church of England elections are subject to constraints that have been set by Act of Parliament. Even where the voters have set them, it will usually be an earlier set of voters who have done so, constraints being set in the bye-laws of the organisation and continuing to exist for many years; the actual voters have no opportunity to alter them at the time of an election.

There is much to be said in favour of rules specifying that at least a certain number of people of particular types must be among those nominated as candidates, but it should be for the voters to decide whether they wish to elect them or not. As soon as they are forced to elect some, the whole election can become distorted by that fact. So I stick to my point of view that, in general, constraints within STV are a bad thing and should be avoided if at all possible. If there is no avoiding their use, the method employed should be as in my article.

A problem for Andrae and Hare

I D Hill

David Hill's great-great-great-grandfather is the Thomas Wright Hill mentioned in this article.

With any form of STV there is a question about the best way to transfer surpluses when they arise. Some people seem to think that provided the right number are taken, and no vote is specifically misused, it does not much matter how it is done. Others claim that such conditions are not sufficient, and that methods should be used that correctly interpret the wishes of the relevant voters as a whole.

The argument turns up interestingly in a fascinating book, to be found in the McDougall Library *Andrae and Proportional Representation* by Poul Andrae, son of Carl Andrae who introduced STV to Danish elections in 1855. The book is partly a complaint that Thomas Hare gets all the credit for the invention of STV and his father very little. Hare first suggested STV in 1857, whereas Andrae actually introduced it in 1855. The complaint appears to be justified and it looks as though perhaps Hare himself did not really want to know about Andrae, but it is always dangerous to judge something like this after hearing argument from one side only. The author of the book is evidently totally unaware of what Thomas Wright Hill did in 1819.

Andrae's system was simply to shuffle the voting papers and then count them just once, allocating each to its earliest preference who had not already attained a quota, and finally elect all those with a quota, plus the highest of those with less, to give the right number to fill the seats. There was no system of exclusions, with redistribution of those votes. Hare's earliest versions were somewhat similar to this.

On the question of how to redistribute a surplus, there is in the book a problem that was put to Andrae, of a case where it was said that his system could give an absurd answer. Andrae, in reply, points out that one of the rules of his system is that the voting papers are to be thoroughly shuffled before counting and, if that rule is obeyed, the probability that they are counted in the particular order on which the absurd result depends is so small that it can be ignored. In this he is correct (and he calculates the probability correctly too).

However the problem was also put to Hare, and Hare's reply is to try to justify the absurd answer as reasonable. I wonder whether any STV supporter nowadays would agree with Hare.

The problem concerns 5 candidates for 3 seats, and votes:

```
299 ABD
200 ACB
101 ACE
```

Hare and Andrae agree that the quota should be 600/3 = 200 and for present purposes let us not dispute that, even though we think that Droop's quota is preferable. The problem says: suppose the votes are counted in the order as given, using Andrae's system. Then, of the first 299, 200 go to A and 99 to B, the next 200 all go to C (leap-frogging A) and the final 101 to E (leap-frogging both A and C). As the system does not use exclusions, the final seat is awarded to E, because 101 exceeds 99 even though nowhere near a quota.

Andrae's correct reply is that, even in the unlikely event of such votes being made, the probability merely that all the 299 come out before any of the 200 is $1/q$ where q is a number of 117 digits, without even taking account of the fact that all those have to come before the final 101. This is certainly a remote enough probability to be ignored.

He does not mention that a similarly silly answer could result from

```
2 ABD
2 ACB
2 ACE
```

where the probability is as high as 1/90, but I feel sure that he would have said that his system was designed for big elections, not such tiny ones, though to my mind a good system ought to work sensibly for any size of election.

Hare, however, according to the book, wrote

> I am willing ... to adopt the result, which I believe is perfectly reconcilable with the principle that is at the foundation of this method of voting, and also reconcilable with justice. The object is to give the elector the means of voting for the candidate who most perfectly attains his ideal of what a legislator should be, but it does not contemplate giving him the choice of more than one ...

> The primary purpose of giving the voter the opportunity of adding to his paper the second, third, or other names for one of whom his vote is to be taken on the contingency of the name at the head not requiring it, is not to add greater weight to his vote, but to prevent it from being thrown away or lost owing to a greater number of voters than is necessary placing the same popular candidate at the head of their papers ...

> Thus the first 200 voters, whose voting papers are appropriated to A, have no ground of complaint (because of the non-election of B), for their votes have been attended with entire success ... Still less have the second 200 voters, whose votes were appropriated to C, any reason to complain, for they also have not only elected a favourite candidate of their own, but, equally with the first 200, they are gratified by the triumphant success of A. The 99 voters for B have also the latter satisfaction, and if they failed to return their next favourite candidate, it is simply because 101 are more than 99.

I should have to change my mind about supporting STV, if that were good STV reasoning, but I do not accept that it is. I agree that it is right to allow each voter just the one vote, but if 299 say AB whereas 301 say AC, to pass A's surplus as 301 to C and only 99 to B, instead of dividing it out in proportion to the voters' wishes, is grotesque.

It is extraordinary that Hare thinks it just and reasonable to elect E even though the total number of voters mentioning E at any level of preference is far less than a Droop quota. Any modern STV system would take the quota as 600/4 = 150, elect A with a surplus of 450 to be divided almost equally between B and C, who then each have more than a quota and all seats are filled.

Even if the votes had been merely

```
200 ACB
101 ACE
```

to elect ACE rather than ACB would be obviously absurd. With the additional 299 ABD votes it becomes even more so. Does any reader think that Hare was talking sense?

A review of the ERS97 rules

B A Wichmann

Recently, I was asked to interpret the Newland and Britton 3rd edition rules[1] (referenced as ERS97) with some specific examples and therefore read the rules carefully for the first time. I think I was largely successful in interpreting the rules correctly, but was surprised at a number of features of their presentation.

Over the last 20 years, I have been involved with the specification of programming languages for the International Standardization Organization (ISO). The requirements here are again for precision and clarity. ISO have adopted drafting rules for standards which I think are very helpful and are not far removed from the style of the presentation of section 5 of ERS97. There are a number of detailed differences in which I prefer the ISO approach. These differences are as follows:

1. Separation of normative (requirements of the standard) and non-normative text. In ISO, the model election would be a non-normative annex. In fact only sections 5 and 6 are normative.

2. In ISO, defined terms would appear before the main text. In ERS97, the Glossary in section 6 appears after their usage in section 5.

3. In ISO, notes are non-normative and laid out in a manner to make this clear. The note in 5.6.2 in ERS97 is clearly normative (and uses *shall*, as in ISO standards).

It seems to me that the adoption of the ISO drafting rules would be a worthwhile undertaking if any revision of the rules was contemplated. Indeed, I see no reason why a suitable revision should not be proposed to ISO as a standard, since it would allow other organisations (in any country) to use it by reference. Currently, many organisations contain rules for STV in their constitution which is unsatisfactory when the rules themselves are very old — a method of reference would be useful in such contexts.

My major and perhaps controversial comments on the rules arise from my desire to see it formulated more closely as an algorithm, rather than as a description. In trying to interpret the rules, one is necessarily performing a function like that of executing a computer program. Since the main purpose of the rules is surely to aid Returning Officers, then the computer program approach is helpful. Of course, I am not suggesting that computer terminology should be used, but merely that the style should allow for conversion into a program in an obvious manner.

My specific points arising from the above computer perspective, and from other analysis are as follows:

1. There is no provision for conducting a count with the aid of a computer or by an entirely automatic process. Since computer programs of both types are routinely used for this purpose, this is a major fault. Note that the Church of England rules[2] make specific provision for this, including the certification of appropriate programs by ERS. Breaking ties by lot needs a different wording allowing for the use of a pseudo-random number generator.

2. I think the wording associated with checking and records should be separated by being in a paragraph after the corresponding actions. (This is not straightforward as some paragraphs are a mixture.)

3. As I see it, only those paragraphs which are needed for reference purposes need be numbered. This would reduce the apparent complexity of the rules. Currently, the whole of section 5 needs to be read to determine what use is made of each part of the rules.

4. Section 5.5 (completion of the count) is not referenced at all, since it is invoked when appropriate conditions are satisfied. This is not algorithmic in the conventional sense, indeed, in computer terms could be seen as ‘interrupt-driven’. I think this section should be used explicitly.

5. The calculation of the quota and the recording of transfers appears to give the impression of undertaking computations to one hundredth of the vote. However, this is not achieved, since that accuracy requires that the transfer values are computed to a greater accuracy. Indeed, if p votes are transferred, then there is a truncation error of at most $p/100$, which implies that transfer values should be computed to about *(number of digits in total votes)*+1 digits. I do not believe that an arithmetic approximation which can lose a whole vote is acceptable since the voter could reasonably equate the loss to his/her vote. Unfortunately, the rules depend upon (*number of papers*)×(*transfer value*) in hundredths of a vote, so it is difficult to increase the accuracy without complexities elsewhere. Hence I conclude that this problem is inherent in this type of rule and could be seen as a defect in ERS97.

6. The rules mention coloured forms, but the colouring is not apparent in the copy of the forms in the example — the solution is to print the ‘beige/blue/green/white/pink/yellow’ on the forms, so that photocopying them retains the information (or so they can be photocopied onto the correct colour paper).

7. Not all uses of the defined terms appear in bold in the rules. I would suggest that the uses of a defined term uses a different font (say, italic).

8. Paragraphs used in more than one place should be given a name and referenced by name (as with the sections 5.3 and 5.4).

9. A batch is a set of bundles each having the same transfer value, not a type of bundle as given in 6.1.

10. The definition of stage of the count is ambiguous, or perhaps depends upon the layout to parse.

11. The definition of transfer value should have ‘deemed elected’ rather than ‘elected’.

12. The statement that for small elections counting slips are not required should be made once at the start, rather than each time slips are mentioned.

13. The second sentence of 5.6.4 is confusing. Surely the point is that an auditable record of the count should be kept? If all recording forms are optional, then why are counting slips specifically mentioned in 5.1.3, 5.3.12 and 5.4.3?

14. The term ‘formally excluded’ (in 5.5.2) clearly means exclusion without the application of the rules associated with exclusion, although this is not explicitly stated.

I have attempted to reformulate the rules along the lines that ISO would use, but I do not regard the result as at all satisfactory. My attempts were based upon a minimal change to the wording, but it appears that a more radical approach is needed.

A few issues have been noted by others that I should also add for completeness:

a) Conventional practice appears to be that the transfer values are not included in the result sheet. I do not like this, since the values are hard to reconstruct and are available.

b) The handling of withdrawn candidates is not mentioned in section 5 of ERS97, although it is surely a possibility with all elections (and is noted in section 2.2).

c) A minor ambiguity has been noted in the rules. (I hope to report fully on this in the next issue of *Voting matters*.)

Conclusions

Is any 'improvement' to the wording needed? I think the rules should be readily usable just from the booklet. In this regard, the model election and examples given are very helpful. However, they do not cover all the situations that can arise. Moreover, for the model election, the actual papers are not included (not unreasonable for 785 voters, but this means that this single long example cannot be re-worked completely by the reader). Also, the explanations given are not always adequate. For instance, in Section 8.2 it is said that, because the surplus could change the order of the last two, it 'must be transferred', without any hint being given that it is required to look at whether the next two or more to go out are definite, in which case it must not be transferred. In the particular instance the action taken is correct, but that is not the point. How to decide that it is correct is not fully stated as it should be[3].

Of course, the fact that ERS runs courses in conducting an STV election is very helpful as is the large number of people that have had such training and can pass on their skills to others.

Hence I conclude that improving the wording is not that vital, but it would be a shame not to consider the ISO approach if a revision was produced in the future.

References

1. R A Newland and F S Britton. How to conduct an election by the Single Transferable Vote. ERS 3rd Edition. 1997.

2. Church of England, General Synod. Regulations for the conduct of elections by the method of the single transferable vote. GS930. 1990.

3. I D Hill. Private communication.

Quantifying Representativity

P Kestelman

Philip Kestelman is keen on measuring electoral representativity, and works in family planning.

Introduction

What is a Proportional Representation (PR) electoral system? Seriously begging that question, Gallagher (1991: 49) argued that "Each method of PR minimizes disproportionality according to the way it defines disproportionality, and thus each in effect generates its own measure of disproportionality".

However, Gallagher overlooked Single Transferable Voting (STV); an omission repaired by Hill (1997), invoking a 'Droop proportionality criterion' (DPC: Woodall, 1994: 10): "If, for some whole numbers *k* and *m* (where *k* is greater than 0 and *m* is greater than or equal to *k*), more than *k* Droop quotas of voters put the same *m* candidates (not necessarily in the same order) as their top *m* preferences, then at least *k* of those *m* candidates will be elected. In particular this must lead to proportionality by party (except for one Droop quota necessarily unrepresented) if voters decide to vote solely by party".

Thus defined, PR systems include Alternative Voting (AV: *k*=1); though over half the voters may be unrepresented! According to the 1937 Irish Constitution, not only parliamentary deputies (multi-member STV), but also the President (AV), shall be elected "on the system of proportional representation by means of the single transferable vote".

Yet nobody regards AV as a PR electoral system. In fairness to Woodall (1994: 10), "Any system that satisfies DPC deserves ... to be regarded as a system of proportional representation (within each constituency)". At that level, Hill's "exaggerated case" (three-member STV) is persuasive; however disproportional to Party First Preferences. Nonetheless, *constituency* level 'PR' (including AV) is not enough for PR as normally construed.

Hill (1992) reasoned that, if voters vote solely by party, each nominating sufficient candidates, "then STV will produce splendid proportionality, ... , while any discrepancy due to fractions of quotas can be expected to even out over a number of multi-member constituencies". Indeed, the main political question is how faithfully total seats reflect Party First Preferences overall (regionally and/or nationally).

Party Total Representativity

In parliamentary elections, the simplest measure of total disproportionality is the overall deviation between over-represented Party Seat-fractions and Vote-fractions: the *Loosemore-Hanby Index* (LHI),

$$\text{LHI} = \tfrac{1}{2} \Sigma \text{ ABS (S\% - V\%)},$$

where S% = Party Seat-fraction (percent);
V% = Party Vote-fraction (percent); and
Σ ABS = Sum of magnitudes (over all parties).

LHI complements the *Rose Index of Proportionality* (RIP); for which I prefer the more explicit term, *Party Total Representativity* (PTR).

Table 1 below demonstrates the calculation of PTR = 100%-LHI for the 1997 Irish General Election, which proved unprecedentedly disproportional to Party First Preferences.

Table 1: STV Party (First Preference) Votes and Seats: Numbers, Fractions and Deviations: General Election, Irish Republic, 1997

Party	Number		Fraction (%)		Deviation
(Constituency)	Votes (V)	Seats (S)	Votes (V%)	Seats (S%)	(S%-V%)
Total	1,788,985	166	100.0	100.0	0.0
Fianna Fáil	703,682	77	39.3	46.4	+7.1
Fine Gael	499,936	54	27.9	32.5	+4.6
Labour	186,044	17	10.4	10.2	-0.2
Progressive Democrats	83,765	4	4.7	2.4	-2.3
Green	49,323	2	2.8	1.2	-1.6
Sinn Féin	45,614	1	2.5	0.6	-1.9
Democratic Left	44,901	4	2.5	2.4	-0.1
Socialist	12,445	1	0.7	0.6	-0.1
Lowry (Tipperary N)	11,638	1	0.7	0.6	-0.0
Blaney (Donegal NE)	7,484	1	0.4	0.6	+0.2
Healy-Rae (Kerry S)	7,220	1	0.4	0.6	+0.2
Gildea (Donegal SW)	5,592	1	0.3	0.6	+0.3
Fox (Wicklow)	5,590	1	0.3	0.6	+0.3
Gregory (Dublin C)	5,261	1	0.3	0.6	+0.3
Unrepresented	120,490	0	6.7	0.0	-6.7
Over-represented	1,234,765	136	69.0	81.9	+12.9*
Under-represented	554,220	30	31.0	18.1	-12.9

* *Loosemore-Hanby Index* (LHI) = 12.9 percent = Overall deviation between over-represented Party Seat-fractions and Vote-fractions: complementing *Party Total Representativity* (PTR) = 87.1 percent.
Source: Dáil Éireann (1998).

The Independent Commission on the Voting System (Jenkins, 1998: 47) gave a 1997 Irish General Election LHI of only 9.8 percent (their DV or 'deviation from proportionality': Dunleavy *et al*, 1997: 10). However, the two main parties (Fianna Fáil and Fine Gael) alone received 11.6 percent more Seats than Votes (First Preferences); and exact LHI=12.9 per cent (Table 1). LHI (and hence PTR) are often miscalculated.

Other Measures

McBride (1997: 9) invoked "O'Leary's index of proportionality": the ratio of each party's Seat-fraction to its First Preference Vote-fraction (S%/V%). However, the problem is how to combine such party-specific ratios (or deviations, S% - V%: see Table 1 above) into some measure of overall disproportionality. O'Leary (1979: 100) favoured the *Rae Index of Disproportionality* (RID), measuring party average disproportionality (contrast LHI above):

$$\text{RID} = 1/\text{N}\ \Sigma \text{ ABS (S\% - V\%)},$$

where N = Number of parties exceeding 0.5 percent of votes.

The palpable arbitrariness of this average disproportionality per party (why not a cutoff-point of 0.1 percent, or 5.0 percent of votes, for that matter?) may be redeemed somewhat by defining N as the 'effective number of parties' (Laakso and Taagepera, 1979):

$$N_1 = 1 / \Pi\ P^P \qquad \text{or} \qquad N_2 = 1 / \Sigma\ P^2,$$

where P = Party Vote-fraction or Seat-fraction;
and Π = Product (over all parties).

Taagepera and Shugart (1989: 260) preferred N_2 on practical grounds; though (entropy-based) N_1 enjoyed "equally good conceptual credentials".

Gallagher (1991) argued that RID was "too sensitive to the number of parties"; to which LHI was "much too insensitive". Accordingly, he proposed a "least squares index": the *Gallagher Index of Disproportionality*,

$$\text{GID} = \left(\tfrac{1}{2}\Sigma \text{ (S\% - V\%)}^2\right)^{1/2}.$$

Nevertheless, Gallagher (1991: 47) considered "probably the soundest of all the measures" the *Sainte-Laguë Index*,

$$\text{SLI} = \Sigma \text{ (S\% - V\%)}^2 / \text{V\%} = (\Sigma\ \text{S\%}^2 / \text{V\%}) - 100\ \%.$$

Unfortunately, SLI ranges theoretically from zero to infinity; which Gallagher acknowledged was "less easily interpreted" than LHI or GID (ranging 0 - 100 percent). Thus in the 1997 Irish Presidential Election, AV First Count LHI = 55 percent (complementing PTR = 45 percent: President McAleese's First Preference Vote-fraction: Table 2 below); whereas SLI = 121 percent!

Table 2: AV Party Vote-fractions, Seat-fractions and Deviations, by Count: Presidential Election, Irish Republic, 1997

Candidate (Party)	Vote-fraction (V%) First	Vote-fraction (V%) Final	Seat-fraction (S%)	Deviation (S%-V%) First	Deviation (S%-V%) Final
Total	100.0	100.0	100.0	0.0	0.0
McAleese (FF)	45.2	55.6	100.0	+54.8	+44.4
Banotti (FG)	29.3	39.2	0.0	-29.3	-39.2
Scallon (Ind)	13.8	0.0	0.0	-13.8	0.0
Roche (Labour)	7.0	0.0	0.0	-7.0	0.0
Nally (Ind)	4.7	0.0	0.0	-4.7	0.0
Non-transferable	0.0	5.2	0.0	0.0	-5.2
Over-represented	45.2	55.6	100.0	+54.8*	+44.4†
Under-represented	54.8	44.4	0.0	-54.8	-44.4

* First Count LHI = 54.8 percent: PTR = 45.2 percent.

† Final Count LHI = 44.4 percent: PTR = 55.6 percent.
Source: Irish Times, 1 November 1997.

Lijphart (1994: 60) preferred GID as steering “A middle course between the Rae and Loosemore-Hanby indices. Its key feature is that it registers a few large deviations much more strongly than a lot of small ones”; and contrasted two hypothetical elections (abstracted in Table 3 below).

Without defining any ‘Lijphart Proportionality Criterion’, he maintained that Election 1 was “highly disproportional” (GID = LHI = 5.0 percent); whereas Election 2 was “highly proportional” (GID = 2.2 percent; but LHI = 5.0 percent). Ironically, his intuitively “much more proportional” Election 2 yielded the higher SLI, considered by Gallagher (1991: 49) “the standard measure of disproportionality” !

Woodall (1986: 45) preferred the *Farina Index,*

$$FI = \cos^{-1} (\Sigma S\% V\% / [\Sigma S\%^2 \Sigma V\%^2]^{1/2}).$$

FI is the angle between two multi-dimensional vectors, whose coordinates are Party Seat-fractions and Vote-fractions: theoretically ranging between 90° (cos FI=0) and zero degrees (parallel vectors: exact PR). As a fraction of a right angle, FID = FI/90°; so ranging 0 - 100 percent (instead of 0 - 90°).

In Table 3, FID (like RID and GID) evaluates Election 1 as more disproportional than Election 2. However, as Hill (1997) recognised, FID also poses problems of interpretation; remaining a far cry from the pristine simplicity of LHI.

Hill (1997) reproached PTR and other measures (their “fatal flaw”) as confined to Party First Preferences. Nonetheless, he acknowledged that the concept of Total Representativity may be generalised (e.g. from Party to ‘Cumbency’, Gender and Name: Kestelman, 1996); and extended beyond the STV First Count. Yet he regarded Final Count PTR as merely comparing STV with itself!

Table 3: Five Measures of Overall Disproportionality: Two Hypothetical Elections

Party	Election 1 Votes (V%)	Election 1 Seats (S%)	Election 2 Votes (V%)	Election 2 Seats (S%)
Total	100	100	100	100
A	55	60	15	16
B	-	-	15	16
C	-	-	15	16
D	-	-	15	16
E	-	-	15	16
F	45	40	5	4
G	-	-	5	4
H	-	-	5	4
J	-	-	5	4
K	-	-	5	4
Disproportionality Index (percent)*				
LHI	5.0		5.0	
RID	5.0		1.0	
GID	5.0		2.2	
SLI	1.0		1.3	
FID	6.2		4.9	

*As defined in the text above.

Minor / Micro-Parties

As Hill (1997) implied, minor parties and independents (‘microparties’ — representing nobody but themselves) may need disaggregating before calculating overall measures of disproportionality. Exact LHI necessitates disaggregating the votes for every represented party (and elected independent) from unrepresented parties; as in Table 1 above. SLI may also be calculated without disaggregating unrepresented parties.

On the other hand, exact GID requires disaggregating even unrepresented party votes. Moreover, in evaluating a few large deviations (S% - V%) as more disproportional than many small deviations, with the same total deviation (and hence LHI), GID implies that, the more fissiparously people vote, the more they *deserve* to be under-represented. In contrast, LHI consistently measures the total under-representation (Σ S%-V%) of all under-represented voters.

Conclusions

Gallagher (1991: 33-34) lamented that “There is remarkably little discussion of what exactly we mean by proportionality and how we should measure it ... how do we decide which is closer to perfect proportionality?” — when comparing different elections. Notice already two different senses of the term ‘proportionality’ here! Hence my preference for the term ‘representativity’ for measures admitting matters of degree to the relationship between votes and seats.

Gallagher (1991: 46) reported that, at 82 national elections in 23 countries (1979-89), LHI, GID and SLI (but not RID) proved impressively correlated with each other: so why complicate matters? Besides, measuring average disproportionality (RID) necessitates counting parties — a rather moveable feast — and there seems little virtue in quantifying some hybrid between the distinct concepts of total and average disproportionality.

There remains legitimate scope for debating the relative merits of STV first or final preference representativity, in national aggregate or constituency average, respecting party or other considerations. In evaluating the representativity mediated by different electoral systems, no measure is perfect.

A generation after its introduction (Loosemore and Hanby, 1971), LHI survives relatively unscathed. I remain peculiarly susceptible to the complement (PTR) of that simplest LHI; doubting whether more complex measures of overall disproportionality would materially affect electoral comparisons (for example, STV representativity by District Magnitude: Kestelman, 1996).

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