Comments on the STV Rules Proposed by British Columbia

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1 Introduction

In May 2005, the Canadian province of British Columbia conducted a referendum to decide whether to adopt the single transferable vote (STV) to elect the members of its legislative assembly. Although 57% of the electorate voted in favor of adopting STV, the measure was not adopted as a super majority of 60% was required for adoption. A Citizens' Assembly drafted a proposed set of STV rules, which will henceforth be called BC-STV. These rules are set forth in pages 17-20 of a Technical Report drafted by the Citizens' Assembly [1] and are also included as an appendix to this article.

The purpose of this article is to clarify the details of the BC-STV implementation and provide some insight into the rationale underlying the rules. Much of the information presented in this article has been gleaned from email conversations with James Gilmour, Jonathan Lundell, Brian Wichmann, and Joe Wadsworth. I have implemented the BC-STV rules in the software package called OpenSTV.[6]

2 Unitary and Inclusive Philosophies

The primary difference between different STV rules is in how surplus votes are transfered. The different methods for transferring surplus votes can be grouped into two different categories, what I call the unitary and inclusive philosophies of transferring surplus votes.

Before describing these two categories, a distinction must be made between an initial surplus of votes and a secondary surplus of votes. An initial surplus arises when a candidate has more than a quota of first choices, i.e., a surplus after the first stage of counting. A secondary surplus occurs when a candidate does not have an initial surplus but later goes over the quota after receiving votes from other elected or excluded candidates.

Consider an election where the quota is 100. Suppose candidate A has 140 votes after the first stage and thus an initial surplus of 40 votes. Suppose candidate B has 90 votes after the first stage and 110 votes after the second stage, after receiving 20 votes of A's surplus. At the second stage, candidate B has a secondary surplus of 10 votes.

Under the unitary philosophy of surplus transfers, only whole votes are transferred. With candidate A, 40 of her votes transfered at full value, while the other votes remain with A at full value. Similarly with candidate B, 10 votes are transferred at full value. A common practice is to take these 10 votes from the 20 that B received during the second stage.

Under the inclusive philosophy of surplus transfers, a portion of each of a candidate's votes is transferred. With candidate A, each of her votes will be transferred to their second choices at a transfer value of 40/140. The total value of the votes transferred is 40. The transfer is inclusive because each of A's votes takes part. With candidate B, the idea is the same, except that one could (and should) account for the fact that some of the votes that B received in the second stage could already have a value of less than one.[1]

Some STV rules can be clearly classified as exemplifying one of these two philosophies, while others employ a hybrid of these two philosophies. I will now consider several STV rules in addition to BC-STV: Cambridge STV (Massachusetts, USA), Dail STV (Ireland), Northern Ireland STV, Malta STV, Tasmania STV (Australia), Australian Capital Territory or ACT STV, and Meek STV (New Zealand).

Cambridge and Dail STV are examples of the unitary philosophy. With Cambridge STV, the votes selected for transfer are chosen at random. With Dail

[1] Under a method used in Australia, all votes are treated the same even if some of them were received at less than full value. In contrast, BC-STV appropriately weights the votes received at less than full value [4].

STV, the votes selected for transfer are chosen in a manner that proportionally represents the following choices on the ballots but does not seek to proportionally represent later choices on the ballots. Both of these methods are ballot order dependent – the outcome is not guaranteed to be the same if the votes are recounted with the ballots in a different order – a fact that some people find highly undesirable. David Robinson has proposed an interesting unitary STV rule that is ballot order independent (or nearly so).[5]

Northern Ireland, Malta, Tasmania, and ACT STV employ a hybrid of the two philosophies and each is an example of the long-established Gregory method of STV counting. The idea underlying these methods appears to be to exemplify the unitary philosophy to the extent possible but to also ensure that the rules are ballot order independent. With these rules, the method of surplus transfer is different for an initial surplus and a secondary surplus. An initial surplus is transfered according to the inclusive philosophy. While not impossible, it is difficult to transfer an initial surplus in a unitary fashion that is also ballot order independent. The method for transferring secondary surpluses is still hybrid, but much closer to being unitary. For secondary surpluses, only the last batch of received votes is considered. This last batch could arrive from a previous transfer of surplus votes or from the exclusion of a candidate. For example, consider candidate B from above. The last batch of votes has a total value of 20 and the surplus is 10. Each of the votes in this last batch is transfered to the next candidate on the ballot with a transfer value of 10/20.[2] The transfer is thus inclusive among the last batch but much more unitary than a completely inclusive transfer.

BC-STV and Meek STV are examples of the inclusive philosophy. For both initial and secondary surpluses, a portion of each vote is transfered to its next choice. The primary difference between BC-STV and Meek STV is the following: with BC-STV votes are transfered only to unexcluded candidates with less than a quota while with Meek STV votes are transfered to all unexcluded candidates. Meek STV is clearly a better method than BC-STV, but Meek STV requires a computer program to count the votes while BC-STV can be counted by hand.

3 Provenance of the BC-STV Rules

Over the years, rules similar to the BC-STV rules have been considered in numerous places. The Proportional Representation Society of Australia urged Australia to replace an existing STV method with a method similar to BC-STV[4]; Douglas Amy's book includes a method similar to BC-STV[2]; and the model statute on the website of the Center for Voting and Democracy (a United States organization) is similar to BC-STV. Rules similar to BC-STV rules have likely been independently derived numerous times, and I present two possible derivations.

Among people familiar with the different STV rules, Meek STV is generally regarded as the "best" set of rules for STV. The greatest difficulty with Meek STV is that it cannot be counted by hand. The most obvious simplification to Meek STV to make it hand countable is to not allow vote transfers to elected candidates. With this modification, Meek STV becomes very similar to BC-STV.

The Gregory method is another well-known method for counting STV elections, which has been used for more than a century. As described above, for secondary surpluses with the Gregory method only the last received batch of votes is considered. Some may regard this as unfair since the last batch of votes may be quite different from previous batches of votes.[4] Intuitively, it seems desirable to change the transfer of secondary surpluses so that all of the candidate's votes are considered and not just the last batch. With this modification, the Gregory method becomes very similar to BC-STV.

Farrell and McAllister used the term "weighted inclusive Gregory method" to refer to rules like the BC-STV rules, and the drafters of the BC-STV rules also used this terminology.[3] While this terminology is perhaps descriptively correct, I find it misleading in that it overstates the relationship between the BC-STV and Gregory methods. Using only the last batch of votes in transferring secondary surpluses is a distinctive feature of the Gregory method. Without last-batch transfers, the similarity with the Gregory method is mostly lost. The BC-STV rules could also be described as "hand-countable Meek" or "Meek without transfers to elected candidates." A more accurate description of the BC-STV rules is simply "inclusive STV."

[2] For the sake of simplicity, I am assuming that each of the votes has a valid next choice.

[3] Farrell and McAllister appear to have coined this terminology.[4]

4 Corrections to the BC-STV Rules

Several people have pointed out ambiguities and errors in the BC-STV rules. I believe that they are all straightforward to address, and I will briefly do so.

First, the BC-STV rules necessarily entail computations with fractions. The rules do not say if these computations are to be performed exactly or through precisely-specified rounding techniques. While this is an important detail, it is one that can easily be resolved. In my implementation of the BC-STV rules, I round to eight decimal places to approximate an exact solution [6].

Second, there is one clear error in the rules, but this error has a simple and obvious fix. In the appendix, the underlined text has been added to fix this error.

Third, in two places, the rules need to be generalized. First, in part 8 of "Counting procedure rules," the rules acknowledge that it is possible for one candidate to be elected with less than a quota of votes. In reality, it is possible that multiple candidates could be elected with less than a quota of votes. One possible correction would be to delete the second sentence in part 8 and replace it with the following: "When the total number of elected and remaining candidates is equal to the number of members to be elected, then all the remaining candidates are elected even if they have less than a quota of votes." Second, part 3 of "Provisions for tied votes" explains how a tie between two candidates is to be broken, and this needs to be generalized to break a tie among three or more candidates.

Fourth, the BC-STV rules do not precisely specify how to transfer surplus votes. Suppose that two candidates have a surplus on the first count, that after transferring the largest first-count surplus a third candidate is elected, that after transferring the second first-count surplus a fourth candidate is elected, and that the fourth winner has a larger surplus than the third. The rules do not indicate which of the two remaining surpluses is to be transferred first. One could choose the largest surplus (that of the fourth winner) or the earliest surplus (that of the the third winner). In accordance with common practice, I chose to always transfer the largest surplus.

5 Advantages and Disadvantages of the BC-STV Rules

I see four advantages of the BC-STV rules: (1) the rules are very simple, (2) votes can be counted by hand, (3) the rules employ the inclusive philosophy, and (4) the rules avoid the unfairness of transferring only the last batch for secondary surpluses. Only the fourth advantage requires more explanation. Consider candidate B, described above. He received 90 first place votes and later received 20 votes that had been transfered as part of candidate A's surplus. It is quite possible that the latter 20 papers represent quite different views than the first 90 papers, yet only the latter 20 papers have further effect. This hardly seems fair to the 90 voters who ranked B first. Farrell and McAllister cite such a dispute arising from an Australian election where the Gregory method was used.[4]

I see one main disadvantage of BC-STV rules. The outcome of the count is not continuous in the sense that changing only one vote can dramatically affect the outcome. For example, consider the following two sets of ballots for electing three candidates:

Set 1	Set 2
4501 ABC	4500 ABC
2499 BD	2500 BD
1200 C	1200 C
1800 D	1800 D

The quota is 2500, and the two sets of ballots differ by just one vote. I now count these ballots using BC-STV rules.

With Set 1, candidate A is elected and has a surplus of 2001 votes. Since candidate B is second on all of these ballots and candidate B has less than a quota, candidate B receives all of these 2001 votes. Now B has a total of 4500 votes and a surplus of 2000 votes. For these 4500 votes, 2001 rank C next (the ballots transfered from A) and 2499 rank D next. Thus,

$$\frac{2000}{4500}\mathsf{X}2001 = 889.3$$

ballots of the surplus go to candidate C, and

$$\frac{2000}{4500}\mathsf{X}2499 = 1110.7$$

ballots of the surplus go to candidate D. Candidate D is elected with 2910.7 votes and candidate C loses with 2089.3 votes.

Now consider Set 2. Candidate A is elected and has a surplus of 2000 votes. Since candidate B is also elected, A's surplus of 2000 votes goes directly to candidate C. Thus, candidate C wins with 3200 votes and candidate D loses with 1800 votes. Although there is only one different ballot in these two sets, the outcome differs by more than 1000 votes.

In comparison, with all of the other STV counting methods mentioned in this paper, there is no such discontinuity with these two sets of ballots. For example, let us count the two sets of ballots with the Gregory method. With Set 1, A's surplus of 2001 votes goes to candidate B. B now has a surplus of 2000 votes. Only votes from the last batch are further transfered, so 2000 votes are now transfered to candidate C who wins with 3200 votes. With Set 2, A's surplus of 2000 votes goes directly to candidate C who again wins with 3200 votes. Here, the change in one ballot produced a similarly small change in the outcome.

6 Conclusions

In considering the relative merits of BC-STV and Gregory methods, there is no clear winner. With the Gregory method, one can argue that it is unfair to use only the last batch of received votes in transferring secondary surpluses. With BC-STV, the outcome is not necessarily continuous with small changes in the ballots. The clear solution to this conundrum is to use Meek STV, assuming that computer counts are possible, which does not suffer from either of these disadvantages.

7 References

[1] Making Every Vote Count. The Case for Electoral Reform in British Columbia. Technical Report. December 2004. Available via McDougall web site.

[2] Douglas J. Amy, *Behind the Ballot Box* (2000).

[3] Center for Voting and Democracy. A Sample Statute for Choice Voting. Available via McDougall web site.

[4] David M. Farrell and Ian McAllister, *The 1983 Change in Surplus Vote Transfer Procedures for the Australian Senate and its Consequences for the Single Transferable Vote*, Australian Journal of Political Science 38:479-491 (2003).

[5] Dave Robinson. Courageous STV. Description available via McDougall web site.

[6] OpenSTV – Software for computing the Single Transferable Vote. Available at http://stv.sourceforge.net.

Appendix: The Recommended BC-STV Electoral System

[Author's note: James Gilmour pointed out a small but important error in the counting rules. This has been fixed with the addition of the underlined text. I have also corrected the incorrect numbering in the section "Provisions for tied votes."]

This section describes the recommended BC-STV system. It provides guidelines to be used in drafting a new election act and in making changes to the current Electoral Boundaries Commission Act.

In addition to choosing an electoral system that incorporates its basic values, the Citizens' Assembly on Electoral Reform wanted a system that is open to public scrutiny and whose results can be reviewed and validated. Consequently, BC-STV is designed to use paper ballots which are available for recount, if required.

General

1. BC-STV is a system of proportional representation by the single transferable vote (STV) method.

2. The members of the Legislative Assembly of British Columbia will be elected from multi-member electoral districts.

3. The number of members in each district will vary from two (2) to seven (7). Given that achieving proportional electoral outcomes is a primary reason for recommending BC-STV, using larger rather than smaller numbers of members per district should always be preferred when drawing district boundaries. While some very sparsely populated areas may require districts with as few as two members, the principle of proportionality dictates that, in the most densely populated urban areas, districts should be created at the upper end of the range.

4. The "Droop quota" will be the formula for calculating the number of votes required by a candidate for election in a district. The quota formula is:

$$\left(\frac{\begin{array}{c}\text{total number of valid} \\ \text{ballots cast in the district}\end{array}}{1 + \begin{array}{c}\text{number of members} \\ \text{to be elected}\end{array}} \right) + 1$$

Fractions are ignored.

5. The method of distributing surplus votes from those candidates with more than the minimum number of votes needed to be elected will be the "Weighted Inclusive Gregory method" (see below, as well as Appendix: Glossary [Author's note: the Glossary is not included.]).

The ballot paper

1. The ballot paper will display the names of all the candidates contesting seats for a district. The names will be grouped according to party affiliation.

2. Candidates who do not indicate a party affiliation, and candidates who do not indicate that they are running as an independent, will be grouped together.

3. Parties with only one candidate, and each candidate running as an independent, will each have their own group.

4. Groupings with more than one candidate in a district will have the rank order of the candidates' names rotated at random so that each candidate has an equal chance of being placed in every position within the grouping.

5. The rank order of groupings appearing on the ballot will be rotated at random so that each grouping has an equal chance of being placed in every position on the ballot paper.

6. The ballot paper will not provide the option of voting for all the candidates of one group by marking a party box (this is the so called "above the line" option used in some Australian elections).

Valid ballots

1. Voters will indicate their preference for the candidates listed on the ballot paper by putting the numbers 1, 2, 3, 4, etc. next to candidates' names.

2. A ballot paper must include a first preference for the ballot to be counted as a valid ballot. The number of subsequent preferences marked on the ballot is at the discretion of the voter.

3. In the case of a ballot paper with gaps or repetitions in the sequence of numbers beyond a first preference, the preferences are valid up to the break in the sequence.

4. If a voter puts a mark next to only one candidate's name, and that mark makes the voter's intention clear, the mark will be accepted as the expression of a single preference for that candidate and the ballot will be counted as a valid ballot.

Counting procedure rules

1. Once the total number of valid ballots is established in each multi-member district, the minimum number of votes required for a candidate to be elected is calculated using the Droop quota formula.

2. All ballots are counted and each ballot is allocated as a vote to the candidate against whose name a first preference (i.e., "1") is shown on the ballot.

3. If a candidate(s) on the first count has a number of first preference votes exactly equal to the minimum number of votes needed to be elected, then that candidate(s) is declared elected and the counted ballot papers indicating that candidate(s) as a first preference are put aside and the other preferences recorded on the ballots are not examined.

4. If a candidate on the first count gains more than the minimum number of votes needed to be elected, the candidate is declared elected, and the number of votes in excess of the number of votes needed to be elected (the surplus) is recorded. All of the elected candidate's ballots are then re-examined and assigned to candidates not yet elected according to the second preferences marked on the ballots of those who gave a first preference vote to the elected candidate. These votes are allocated according to a "transfer value." The formula for the transfer value is:

$$\frac{\text{surplus votes cast for the elected candidate}}{\text{total number of votes received by the elected candidate}}$$

5. If two or more candidates on the first count gain more than the minimum number of votes needed to be elected, all of those candidates are declared elected. The ballots of the candidate with the largest number of first preference votes will be re-examined first and assigned (at the transfer value) to candidates not

yet elected according to the second preferences marked on that candidate's ballots, or the next available preference, if the second preference candidate has already been elected. The ballots of the other elected candidate(s) will then be re-examined and their surpluses distributed in order according to the number of first preference votes each candidate received.

6. If a candidate reaches more than the minimum number of votes needed to be elected as the consequence of a transfer of votes from an elected or excluded candidate, the number of votes in excess of the number of votes needed to be elected (the surplus) will be transferred to other candidates. This transfer will be to the next available preference shown on all of this candidate's ballots. These ballots now include 1) the candidate's first preference ballots, and 2) the parcel(s) of ballots transferred to the candidate from one or more elected or excluded candidates. The transfer value for the candidate's first preference ballots is:

$$\frac{\text{surplus votes cast for the elected candidate}}{\text{total number of votes received by the elected candidate}}$$

The transfer value for each parcel of ballots transferred to the candidate from one or more elected or excluded candidates is:

$$\left(\frac{\text{surplus votes cast for the candidate}}{\text{total number of votes received by the candidate}}\right) \times \left(\text{the transfer value of the parcel of ballots received by the candidate}\right)$$

7. If no candidate has a number of votes equal to or greater than the minimum number of votes needed to be elected, the candidate with the smallest number of votes is excluded. All of that candidate's ballots–both first preference ballots and any parcel or parcels of ballots transferred from other candidates–are transferred to candidates who have not been elected or excluded according to the next available preference shown on the excluded candidate's ballots. The excluded candidate's first preference ballots are transferred to the second (or next available) preferences at full value. Ballots received from previously-elected (or excluded) candidates are transferred at the transfer value at which the ballots were received.

8. Counting continues in the described sequence: the surplus of elected candidates is assigned until no more candidates are elected, then the ballots of excluded candidates are assigned until another candidate is elected. When all but one of the candidates to be elected from the district have been elected, and only two candidates remain in the count, the candidate with the most votes is declared elected, even though the candidate may not have reached the minimum number of votes (the quota) needed to be elected.

9. If, during the transfer of preferences, a ballot paper does not indicate an available preference, the ballot is put aside as "exhausted." This can occur because:

- the voter only indicated one, or a small number of preferences;
- all the preferred candidates have already been elected or excluded; or
- there are gaps or repetitions on the ballot in the sequence of numbering preferences.

Provisions for tied votes

1. Where two or more candidates have the same number of first preference votes at the end of the first count, and this number is more than the minimum number of votes necessary for election, then the candidate whose surplus is distributed first will be decided by lot.

2. Where no candidate has a number of first preference votes equal to or greater than the number of votes necessary for election at the end of the first count, and two or more candidates have the same number of first preference votes, this number being the smallest number of first preference votes gained by any candidate, then the candidate who is excluded first will be decided by lot.

3. If, at any stage of the count other than during the first count, two candidates have the same number of votes, the candidate who is declared elected first, or who is not excluded will be:

a) the candidate with the larger number of votes in the previous or immediately next preceding count where there is a difference in the votes between the two candidates; or

b) the candidate whose name is drawn by lot, where there is no difference in the number of votes between the candidates at any preceding count.

By-elections

The single transferable vote method (preferential voting) is to be used for by-elections where a candidate is to be elected to fill a single casual vacancy in a district. The BC-STV method is to be used where candidates are to be elected to fill two or more casual vacancies in a district.