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# Voting matters 

## for the technical issues of STV

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## Editorial

The year 2002 has seen significant advances with the technology of STV, from opposite sides of the world.

In the Republic of Ireland, plans for the introduction of electronic voting (in the polling booth, not at this stage, via the Internet) have advanced to a key stage. Suitable technology has been developed for the polling stations and software has been written to undertake the count. ERBS was contracted to test the counting software to ensure it adhered to the rules which are identical to the hand-counting ones. On the 17th May, the Dáil elections were held in which three constituencies were handled electronically as an experiment, while the others were handled by the traditional manual means. The software validation was completed in time under the direction of Joe Wadsworth using a program for the Irish rules written by Joe Otten and with the editor running over 400 tests, some specially written for the occasion. I am glad to report that the counting went smoothly on the day.

The Irish election data for the three constituencies (Meath, Dublin North and Dublin West) was placed on the Internet with the full results of the count. To my knowledge, this is the first time over 2,000 STV votes (ie, the full set of preferences given by each voter) has been made publicly available. It is now possible to analyse this data. It is immediately clear, even by a manual inspection that many final preferences are in ballot paper order.

The developments with STV in New Zealand have been continuing throughout 2002 and are reported in the final article in this issue by Stephen Todd.

Other articles in this issue includes a note by Peter Dean showing how the actual administration of STV has changed over the years in Tasmania (even without the impact of computers). David Hill also considers a disturbing example of changes to the preferences on ballot papers which are not visible to the traditional rules.

Eivind Stensholt presents a rather technical article about the implementation of Meek STV rules when equality of preference is permitted. (Does the observed ballot-paper ordering with the Irish election indicate that equality of preference should be allowed?)

The remaining article is a short one by myself about the vexed question of proportionality.

## Welcome to the McDougall Trust

This issue is the last one under the ERS banner. Following discussions between ERS and the Trust, Voting matters is being transferred to the Trust for publication for the time being. At this point, no significant changes are envisaged.

Brian Wichmann.

## STV in Tasmania

P Dean

Peter Dean has been involved with ERS for many years.
In his article in Voting matters ${ }^{1}$, Philip Kestelman raises the issue of positional voting bias. In Tasmania, there has been a continuous process of changing some details of the STV voting system to make it fairer. The problem of positional voting bias was addressed in 1979 and first used in 1980.

A summary of STV in Tasmania from Newman ${ }^{2}$ is as follows:

1897 First experimental use of STV.
1903 Women given the vote.
1909 First state-wide election by STV.
1917 By-elections and vacancies filled by a recount of the original ballots. First used in 1922.

1921 Women allowed to stand as candidates.
1922 Deposit lost if less than $20 \%$ of the quota if excluded or at the end of the count.

1930 Compulsory vote, previously $63-67 \%$ turnout, up to $82 \%$ in 1928.

1941 Grouping by party labels.
1954 Parliamentary term reduced from 5 to 3 years.
1955 Speaker to be chosen from party with the lower statewide vote.

1957 Assembly of 35 instead of 30 to overcome potential deadlock.

1972 Term changed to 5 years, and 4 years thereafter.
1973 Voters required to make 7 choices instead of 3 . Previously $90 \%$ of electors restricted their choice to a single party. Franchise reduced to 18.

1976 Draw for ballot position, and position within party list.

1980 first use of rotated ballot. The printer must issue equal numbers of papers showing different names in the favoured position, starting with the first name alphabetically. Thus with a columnar ballot paper 2,8 , 3 and 7 members in the 4 columns, 16 different printings are made.

A 1957 Select committee reported that it provided the Tasmanian elector with a wider freedom of choice, and a
more effective vote than any other method of Parliamentary election in the world.

## References

1. P Kestelman. Positional Voting Bias Revised. Voting matters, Issue 15, pp 2-5. June 2002.
2. Terry Newman. Hare-Clark in Tasmania. Joint Library Committee of the Parliament of Tasmania, Parliament House. Hobart. 1992.

# Implementing a suggestion of Meek's 

E Stensholt<br>Eivind Stensholt is from the Norwegian School of Economics and Business Administration

## Introduction

In preferential elections voters are often assumed to have linear rankings, i.e. they rank all candidates without ties. Here the topic is STV elections where only a "complete order" is required, which means that a voter must give each candidate a rank, but may declare equal preference. Hence in a 10 -candidate election a voter V may rank

## PQ(ABCDE)RST

which, in Hill's notation ${ }^{1}$, means that A, B, C, D, E share third to seventh rank.

At an iterative step in an algorithm for Meek's method a candidate P has a certain current retention factor: 1-p, which is a positive number less than or equal to 1 . Voter V starts on top of his list, offers P his full vote, for which 1-p is retained and offers $\mathrm{Q} p$ votes, has $p(1-q)$ retained and has $w=p q$ votes when coming to the set of equal preferences $\{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}\}$.

Meek ${ }^{2}$ suggested to count as if there were $5!=120$ "minivoters", each with a weight of $w / 120$ votes, with one minivoter for each possible way to split up the $\{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$, $\mathrm{E}\}$ into 5 singleton classes. With $n$ candidates ranked equal, there are $n$ ! possible linear rankings, and the work soon becomes too much even for computers if each minivoter is considered separately. However, the counting can be systematized, so that the necessary work grows as $n^{2}$. Thus there need not be a "combinatorial explosion", but the algorithm does not otherwise relate to Hill's discussion of how to cope with equality of preference.

## A count with five candidates equal

One minivoter ranks ABCDE , and contributes
$(1-a) w / 120, \quad a(1-b) w / 120, \quad a b(1-c) w / 120, \quad a b c(1-d) w / 120$, $\operatorname{abcd}(1-e) w / 120$
to $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$, and E , respectively. Each minivoter keeps weight abcdew/120, and hence voter V keeps abcdew to influence the ranking of $\mathrm{R}, \mathrm{S}$, and T .

What is the total contribution from the 120 minivoters to candidate E ? The contribution has 5 parts:

24 minivoters have E as number 1: $24(1-e) w / 120$
24 minivoters have E as number 2: $\quad 6(a+b+c+d)(1-e) w / 120$
24 minivoters have E as number 3:
$4(a b+a c+a d+b c+b d+c d)(1-e) w / 120$
24 minivoters have E as number 4:
$6(b c d+a c d+a b d+a b c)(1-e) w / 120$
24 minivoters have E as number 5: $24(a b c d)(1-e) w / 120$.
The total contribution from V to E is therefore

```
[1/5 + (a+b+c+d)/20 + (ab+ac+ad+bc+bd+cd)/30 +
    (bcd+acd+abd+abc)/20 + (abcd)/5](1-e)w
```

An efficient algorithm is possible because the factors that depend on $a, b, c$, and $d$ are easily calculated as the coefficients in a polynomial:

$$
\begin{aligned}
\mathrm{Q}(\mathrm{E}, x)=(x+a) & (x+b)(x+c)(x+d)= \\
& x^{4}+(a+b+c+d) x^{3}+ \\
& (a b+a c+a d+b c+b d+c d) x^{2}+ \\
& (b c d+a c d+a b d+a b c) x+a b c d .
\end{aligned}
$$

How much computational effort is involved in calculating $\mathrm{Q}(\mathrm{E}, x)$ ? Writing

$$
\begin{aligned}
\mathrm{Q}(\mathrm{E}, x)= & {\left[x^{3}+(a+b+c) x^{2}+(b c+c a+a b) x+a b c\right](x+d) } \\
= & {\left[x^{4}+(a+b+c) x^{3}+(b c+c a+a b) x^{2}+(a b c) x\right] } \\
& +\left[d x^{3}+(a+b+c) d x^{2}+(b c+c a+a b) d x+(a b c) d\right],
\end{aligned}
$$

we see that the factor $(x+d)$ involves first 3 multiplications of two real numbers with $d$ as a factor and then 3 additions of two real numbers to get the coefficients of $x^{3}, x^{2}$, and $x$. Multiplying $(x+a)(x+b)$ needs one multiplication and one addition, and $(x+a)(x+b)(x+c)$ is calculated with two more of each. Hence $\mathrm{Q}(\mathrm{E}, x)$ requires $1+2+3=6$ multiplications and $1+2+3=6$ additions. Moreover, the contribution formula contains 6 multiplications, 4 additions, and 1 subtraction.

## The general case

In general, consider $n$ candidates, $\mathrm{C} 1, \ldots, \mathrm{C} n$, with retention factors $1-p(1), \ldots, 1-p(n)$. Consider the polynomials
$\mathrm{Q}(\mathrm{Ci}, x)=[x+p(1)][x+p(2)] \ldots .[x+p(n)] /[x+p(i)]$

$$
=\mathrm{B}(0) x^{n-1}+\mathrm{B}(1) x^{n-2}+\mathrm{B}(2) x^{n-3}+\ldots .+\mathrm{B}(n-1)
$$

for $i$ from 1 to $n$. Clearly $\mathrm{B}(0)=1$ while the other $\mathrm{B}(k)$ depend on $i$. They are the elementary symmetric polynomials in the $p(j)$ where $j \neq i$. The multiplication of $n-1$ factors of type $[x+p(j)]$ involves $1+2+3+\ldots+(n-2)=(n-1)(n-2) / 2$ multiplications of two real numbers and equally many additions.

Suppose the candidates $\mathrm{C} 1, \ldots, \mathrm{C} n$ form an equal preference set for voter V, who has weight $w$ left after contributing to the higher ranked candidates. The contribution from V to candidate $\mathrm{C} i$, i.e. the votes to $\mathrm{C} i$ from $n!$ minivoters, is given by the contribution formula $\operatorname{Rev}(i)=$

$$
\begin{gathered}
{[\mathrm{K}(n-1,0) \mathrm{B}(0)+\mathrm{K}(n-1,1) \mathrm{B}(1)+\ldots+\mathrm{K}(n-1, t) \mathrm{B}(t)+\ldots+} \\
\mathrm{K}(n-1, n-1) \mathrm{B}(n-1)][1-p(i)] w
\end{gathered}
$$

where the $\mathrm{K}(n-1, t)$ are determined as follows: There are $n$ ! minivoters, with weight $w /(n!)$ each. Among them, $(n-1)$ ! have candidate $\mathrm{C} i$ as number $t+1$. The $t$ candidates ranked ahead of $\mathrm{C} i$ can be permuted in $t$ ! ways. The $n-t-1$ candidates ranked after $\mathrm{C} i$ can be permuted in ( $n-t-1$ )! ways. Thus $t!(n-t-1)$ ! of the ( $n-1$ )! minivoters have the same $t$ candidates ahead of $\mathrm{C} i$ and they offer the same support to candidate C . The total revenue collected by $\mathrm{C} i$ from these ( $n-1$ )! minivoters is $t!(n-t-1)!\mathrm{B}(t)[1-p(i)] w /(n!)$. Thus $\mathrm{K}(n-1, t)=t!(n-t-1)!/(n!)$, i.e.
$\mathrm{K}(n, t)=t!(n-t)!/((n+1)!)$.
For the use of the contribution formula, it is practical to tabulate the coefficients $\mathrm{K}(n-1, t)$.

If each $\mathrm{Q}(\mathrm{Ci}, x)$ is calculated as a product with $n-1$ factors, $i$ from 1 to $n$, the total requirement is $n(n-1)(n-2) / 2$ multiplications of two real numbers and $n(n-1)(n-2) / 2$ additions. Thus the work grows with the third power of $n$. Here we leave out the $n+1$ multiplications and $n-1$ additions and 1 subtraction that must be performed each time the contribution formula is used.

However, with $n>5$ one may reduce the work by first calculating $\mathrm{Q}(x)=$

$$
\begin{aligned}
& {[x+p(1)][x+p(2)] \ldots[x+p(n)]=} \\
& \\
& \quad \mathrm{A}(0) x^{n}+\mathrm{A}(1) x^{n-1}+\mathrm{A}(2) x^{n-2}+\ldots+\mathrm{A}(n)
\end{aligned}
$$

by means of $n(n-1) / 2$ multiplications and $n(n-1) / 2$ additions, and then for each $i$ perform the division with $[x+p(i)]$ :

$$
\begin{aligned}
& \mathrm{A}(0) x^{n}+\mathrm{A}(1) x^{n-1}+\mathrm{A}(2) x^{n-2}+\ldots+\mathrm{A}(n)= \\
& \quad\left[\mathrm{B}(0) x^{n-1}+\mathrm{B}(1) x^{n-2}+\mathrm{B}(2) x^{n-3}+\ldots+\mathrm{B}(n-1)\right][p(i)+x]
\end{aligned}
$$

leads to $\mathrm{A}(0)=\mathrm{B}(0)=1$ and

$$
\begin{aligned}
& \mathrm{A}(1)=\mathrm{B}(0) p(i)+\mathrm{B}(1), \\
& \mathrm{A}(2)=\mathrm{B}(1) p(i)+\mathrm{B}(2), \ldots, \\
& \mathrm{A}(n-1)=\mathrm{B}(n-2) p(i)+\mathrm{B}(n-1) .
\end{aligned}
$$

Hence $\mathrm{Q}(\mathrm{Ci}, x)$ is calculated as follows:
$\mathrm{B}(1)=\mathrm{A}(1)-\mathrm{B}(0) p(i)$,
$\mathrm{B}(2)=\mathrm{A}(2)-\mathrm{B}(1) p(i), \ldots$,
$\mathrm{B}(n-1)=\mathrm{A}(n-1)-\mathrm{B}(n-2) p(i)$.

The division with $[x+p(i)]$ requires $n-1$ multiplications with $p(i)$ as a factor and $n-1$ subtractions. All the divisions for $i$ from 1 to $n$ require $n(n-1)$ multiplications and $n(n-1)$ subtractions. Thus it is enough to perform $3 n(n-1) / 2$ multiplications and $3 n(n-1) / 2$ additions/subtractions instead of $n(n-1)(n-2) / 2$ of each.

There are of course also $n(n+1)$ multiplications and $n^{2}$ additions/subtractions associated with the use of the contribution formula for n candidates, and so we arrive at $n(5 n-1) / 2$ multiplications and $n(5 n-3) / 2$ additions/ subtractions.

Further small savings are obviously possible, e.g. by keeping $\mathrm{Q}(\mathrm{C} n, x)$ as an intermediate result from the calculation of $\mathrm{Q}(x)$ instead of dividing $\mathrm{Q}(x)$ by $[x+p(n)]$, but they do perhaps not justify the extra programming.

## A program for calculating the contributions

Here is a Maple routine for calculating the contribution from a voter with weight 1 to each candidate in an equal preference set of $n$ candidates $1,2, \ldots, n$, with given retention factors. The total number of candidates is denoted by C .

Set $\mathrm{n}=$ number of candidates ranked equally by the voter:
> n: =9;

$$
\mathrm{n}:=9
$$

Set $\mathrm{p}(\mathrm{i})$ for candidates $1,2, \ldots$, n , so that $1-\mathrm{p}(\mathrm{i})$ is the current retention factor for candidate i.
> for i from 1 to n do $\mathrm{p}(\mathrm{i}):=0.5+0.04 *$ i; od;

$$
\begin{aligned}
& \mathrm{p}(1):=0.54 \\
& \mathrm{p}(2):=0.58 \\
& \mathrm{p}(3):=0.62 \\
& \mathrm{p}(4):=0.66 \\
& \mathrm{p}(5):=0.70 \\
& \mathrm{p}(6):=0.74 \\
& \mathrm{p}(7):=0.78 \\
& \mathrm{p}(8):=0.82 \\
& \mathrm{p}(9):=0.86
\end{aligned}
$$

As an example we use these equidistant values for the $\mathrm{p}(\mathrm{i})$.

The routine consists of a "preparation" and two instructions. The preparation is used only once per run of the election program. It sets the coefficients $\mathrm{K}(\mathrm{i}, \mathrm{j})=\mathrm{j}!(\mathrm{i}-\mathrm{j})!/$ (i+1)! by first calculating the binomial coefficients " i choose - j " $=\mathrm{i}!/(\mathrm{j}!(\mathrm{i}-\mathrm{j})!)$.

Preparation. Set the table of constants. Let $C$ be the total number of candidates:
$>\mathrm{C}:=20$ : for i from 0 to $\mathrm{C}-1$ do $\mathrm{K}(\mathrm{i}, 0):=1.0$; od:
for j from 1 to $\mathrm{C}-1$ do $\mathrm{K}(0, \mathrm{j}):=0.0$; od:
for i from 1 to $\mathrm{C}-1$ do for j from 1 to $\mathrm{C}-1$ do
$\mathrm{K}(\mathrm{i}, \mathrm{j}):=\mathrm{K}(\mathrm{i}-1, \mathrm{j}-1)+\mathrm{K}(\mathrm{i}-1, \mathrm{j})$; od: od:
for i from 1 to $\mathrm{C}-1$ do for j from 0 to i do
$\mathrm{K}(\mathrm{i}, \mathrm{j}):=1.0 /((\mathrm{i}+1) * \mathrm{~K}(\mathrm{i}, \mathrm{j}))$; od: od:
Instruction 1. Calculate the polynomial of degree n:
$>\mathrm{A}(0):=1.0: \mathrm{B}(0):=1.0$ : for j from 1 to n do $\mathrm{A}(\mathrm{j}):=0.0$; od: for j from 1 to n do for i from 0 to $\mathrm{j}-1$ do $\mathrm{A}(\mathrm{j}-\mathrm{i}):=\mathrm{A}(\mathrm{j}-\mathrm{i}-1) * \mathrm{p}(\mathrm{j})+\mathrm{A}(\mathrm{j}-\mathrm{i}) ;$ od; od;

Instruction 2. Calculate the polynomial of degree n-1 for candidate $s$ and simultaneously set $\operatorname{Rev}(\mathrm{s})=$ the revenue for candidate $\mathrm{s}, \mathrm{s}=1,2, \ldots, \mathrm{n}$ :
> for s from 1 to n do $\operatorname{Pr}:=\mathrm{K}(\mathrm{n}-1,0): \mathrm{q}:=\mathrm{p}(\mathrm{s})$ :
for j from 1 to $\mathrm{n}-1$ do $\mathrm{B}(\mathrm{j}) \quad:=\mathrm{A}(\mathrm{j})-\mathrm{B}(\mathrm{j}-1) * \mathrm{q}$; $\operatorname{Pr}:=\operatorname{Pr}+\mathrm{B}(\mathrm{j}) * \mathrm{~K}(\mathrm{n}-1, \mathrm{j}) ;$ od: $\operatorname{Rev}(\mathrm{s}):=\operatorname{Pr}^{*}(1-\mathrm{q}) ;$ od:

Another instruction shows the revenue $\operatorname{Rev}(\mathrm{s})$ collected by candidate s from all n! "minivoters" :
$>$ for s from 1 to n do $\operatorname{Rev}(\mathrm{s}):=\operatorname{Rev}(\mathrm{s})$; od;

$$
\begin{aligned}
& \operatorname{Rev}(1):=.171708815169 \\
& \operatorname{Rev}(2):=.154311932284 \\
& \operatorname{Rev}(3):=.137512907077 \\
& \operatorname{Rev}(4):=.121258700936 \\
& \operatorname{Rev}(5):=.105503965732 \\
& \operatorname{Rev}(6):=.0902095328389 \\
& \operatorname{Rev}(7):=.0753412681397 \\
& \operatorname{Rev}(8):=.0608691895186 \\
& \operatorname{Rev}(9):=.0467667763417
\end{aligned}
$$

These contributions sum to 0.963483088037 .
The voter keeps $\mathrm{p}(1) \mathrm{p}(2) \ldots \mathrm{p}(\mathrm{n})=0.036516911963$.

## What happens in the example above?

Consider 9 candidates sharing ranks 1 to 9 in a vote, and assume the retention factors are as above. The preparation has calculated a table including $(\mathrm{K}(8,0), \ldots, \mathrm{K}(8,8))=$
( $0.1111111111,0.01388888889,0.003968253968$, $0.001984126984,0.001587301587,0.001984126984$, $0.003968253968,0.01388888889,0.1111111111)$

With the $\mathrm{p}(\mathrm{i})$ above, instruction 1 gets the polynomial of degree $\mathrm{n}=9$,

$$
\begin{aligned}
& \mathrm{Q}(\mathrm{x})=[\mathrm{x}+0.54][\mathrm{x}+0.58][\mathrm{x}+0.62][\mathrm{x}+0.66][\mathrm{x}+0.70][\mathrm{x}+0.77] \\
& \quad[\mathrm{x}+0.78][\mathrm{x}+0.82][\mathrm{x}+0.86]= \\
& 1 \mathrm{x}^{9}+6.30 \mathrm{x}^{8}+17.5920 \mathrm{x}^{7}+28.576800 \mathrm{x}^{6}+29.75937888 \\
& \mathrm{x}^{5}+20.60302608 \mathrm{x}^{4}+9.482569153 \mathrm{x}^{3}+2.797730344 \mathrm{x}^{2}+ \\
& 0.4801360978 \mathrm{x}+0.03651691196 .
\end{aligned}
$$

Then for $\mathrm{s}=9$, instruction 2 gets $\mathrm{Q}(\mathrm{C} 9, \mathrm{x})=\mathrm{Q}(\mathrm{x}) /[\mathrm{x}+0.86]=$
 $x^{4}+7.93158876 x^{3}+2.661402819 x^{2}+0.508923920 x+$ 0.0424615266 ,
and at the same time it calculates the contribution from the voter with weight 1 to candidate 9 :
$[1 \times 0.1111111111+5.44 \times 0.01388888889+12.9136 \times$ $0.003968253968+17.471104 \times 0.001984126984+$ $14.73422944 \times 0.001587301587+7.93158876 \times$ $0.001984126984+2.661402819 \times 0.003968253968+$ $0.508923920 \times 0.01388888889+0.0424615266 \times$ $0.1111111111] \times(1-0.86)$
$=0.3340484026 \times(1-0.86)=0.04676677636$.

## Acknowledgement

The author is grateful to two referees for suggestions that have made the presentation clearer.

## References

1. I D Hill, Difficulties with equality of preference, Voting matters, Issue 13, April 2001, pp8-9.
2. B L Meek, Une nouvelle approche du scrutin transferable (fin), Mathématiques et sciences humaines, 9 no. 29 pp 33-39. 1970
3. B L Meek, A new approach to the Single Transferable Vote, Voting matters, Issue 1, March 1994, pp1-11.

## What would a different method have done?

I D Hill

Following an election, the question is often raised of what the result would have been had a different electoral method been used. In general, no reply can be given to this question not only because sufficiently detailed information is not available on the votes, but also because voters can be expected to behave differently if a different system is used.

In comparing one STV system with another, however, rather than totally different systems, it seems unlikely that there would be very much difference in how voters behave, and a reasonable reply is possible provided that the full voting pattern is divulged. It is very welcome that it has been divulged for the three constituencies counted by computer in the recent general election in Eire. Such openness is to be commended. Too often, though, the full voting pattern is regarded as confidential, and the only information is a result sheet, which is quite insufficient for the purpose.

As an example, the question might be whether the result of the 2002 ERS Council election would have been different had the Meek system been used. Working solely from the result sheet (the only information available) I have constructed a voting pattern in which some votes have the character \# inserted within their preferences. Before running such data on a computer the \# characters have to be replaced, either by a number representing a candidate, or by a space which is then ignored by the STV program.

If the \# characters are all replaced by a space, and ERS97 rules used, the actual result sheet is reproduced. If Meek rules are used the same candidates are elected, following a similar order of events.

However, if the \# characters are all replaced by the number that represents any one of the defeated candidates, and ERS97 rules used, the same result sheet appears, identical in every particular, but if Meek rules are used, that defeated candidate is elected, at the expense, of course, of one of those who was actually successful.

There is no suggestion that this artificial voting pattern is anything like the true one. I am absolutely sure that it is not, but it is somewhat remarkable that it is possible to devise such a voting pattern with no effect at all on the ERS97 result sheet. The fact that it is possible shows the extent to which the information available is totally inadequate to answer the question. I believe it to be impossible to do the reverse, leaving the Meek result unchanged while varying the ERS97 result.

The artificial voting pattern can be supplied on request.

## What sort of proportionality?

I D Hill

In pure mathematics proportionality is a well-defined concept, but that is because we can always go into fractions whenever necessary. For proportionality within voting systems we are restricted to whole numbers in those elected for each party (using 'party" in the general sense of any relevant grouping of the candidates, not only in the sense of a formal political party). Under such circumstances it is in many cases not at all
easy to say whether one result is more nearly proportional than another. This is particularly so where some parties, quite correctly, get zero seats, while none get zero votes.

I agree with Philip Kestelman ${ }^{1}$ that none of the measures that he discusses is perfect. I agree also that the comparative answers that they produce are so similar that, if using any, we might as well settle on one of them. But as I have said before ${ }^{2}$ they are all fundamentally flawed in basing their calculations on first-preference votes only, and this can be very misleading, particularly where there is a substantial amount of cross-party voting for successive preferences.

However there is an additional point to be considered, even where first preferences do give full information on party popularity, there being no cross-party voting at all. Under such circumstances it could be the rule that if $n$ is the minimum value, across parties, of votes per seat, then any party with at least $n$ votes must get at least 1 seat, any party with at least $2 n$ votes must get at least 2 seats, any party with at least $3 n$ votes must get at least 3 seats, and so on. Given the restriction to whole numbers, and that some parties may get zero seats, what could be more proportional than that? Yet none of the measures that Kestelman considers meets that rule.

For simplicity, consider the case of only 2 parties and only 2 seats to be filled. Suppose the votes are 70 for party A and 30 for party B. We can at once rule out the option of giving both seats to party B , but is it better to give both to A or one to each?

Suppose we allot them as 1 to each. Then $n=30 / 1$ so party A with more than $2 n$ votes must get at least 2 seats and the rule is violated. Suppose we allot them as both to party A. Then $n=70 / 2$ and the rule is satisfied for party B does not reach 35 to be worth a seat. Yet every one of the measures that Kestelman considers says that 1 to each is a better answer than both to party A. To my mind that shows all those measures to be unsatisfactory. I regret that I do not know of a better alternative, but to do without a measure is preferable to using a defective one.

If anyone doubts that both to party A is the better answer, let them assume that there had been only 3 candidates and votes 36 A1 A2, 34 A2 A1, 30 B. The measures all say that to elect A1 and B, or even A2 and B, is preferable to A1 and A 2 , which is surely nonsense.

However, I am grateful to Philip Kestelman for the suggestion that we might, perhaps, say that to elect A1 and B is more party-representative, while to elect A1 and A2 is more candidate-representative. There might be something in that.

## References

1. P Kestelman, Quantifying representativity. Voting matters, Issue 10, 7-10. 1999.
2. I D Hill, Measuring proportionality. Voting matters, Issue 8, 7-8. 1997.

## Proportionality Revisited

B A Wichmann

## Introduction

The issue of proportionality in the last article ${ }^{1}$, raised two problems in my mind which are addressed here.

## A flaw

Consider the hypothetical case of an STV election in the UK, in which there is a United Kingdom Independence Party (UKIP) candidate together with a Tory candidate. A Tory voter who is on the Europhobic wing of the party could well decide to give his/her first preference to the UKIP candidate. On the other hand, if the Tory candidate was also Europhobic, then the voter would surely place his/ her first preference with the Tory. In other words, the first preference votes for the Tory and UKIP cannot reasonably be analysed in isolation.

Of course, this issue is not specific to the Tory party - the same problem could arise with a Socialist Party candidate standing against a New or Old Labour candidate.

I conclude from this that an analysis of party support based upon first preferences alone is doomed to failure.

## Granularity

In this section, we set aside the flaw noted above, and analyse the issue of proportionality from just one point of view: the granularity imposed by the size of the constituencies. If a constituency elects 4 members, then it is clear that strict proportionality could only be obtained if each party had a multiple of $25 \%$ of the first preference votes. Obviously, there will always be a mismatch between the first preference votes and the proportion of candidates elected.

As an example, we consider the 1997 Irish General election ${ }^{2}$. The 166 seats for the Dáil are from 41 constituencies having 3,4 or 5 seats each. In this analysis, we consider three categories for the first preference votes: those of Fianna Fáil (FF), those for Fine Gael (FG) and the others. It can reasonably be said that the 'others' does not represent a party, but if strict proportionality is obtained for FF and FG, then the others as a single group will also be
represented proportionally. We return to this problem later.
Kestelman ${ }^{3}$ considers several measures of proportionality. Here, we consider some of those measures as applied to each individual constituency and compare this with the actual result. The measures used here are the Loosemore-Hanby Index, Gallagher Index of Disproportionality, Sainte-Laguë Index and the Farina Index (all taken from the above paper).

Given a specific index, then one can determine the number of seats for each party which would give the closest fit with respect to that index. In fact, all the indices give the same result with one exception: the Sainte-Laguë Index gives a different result for the Dublin Central constituency. Ignoring this isolated value we have the table as follows:

| Constituency | Actual | Best | Fit $(\%)$ | Comparison |
| :--- | ---: | ---: | ---: | :---: |
| Carlow-Kilkenny | $(2,2,1)$ | $(2,2,1)$ | 13.998 | $=$ |
| Cavan-Monaghan | $(2,2,1)$ | $(2,2,1)$ | 8.850 | $=$ |
| Clare | $(3,1,0)$ | $(2,1,1)$ | 7.452 | FF to Other |
| Cork East | $(2,2,0)$ | $(2,1,1)$ | 16.773 | FG to Other |
| Cork North-Central | $(3,2,0)$ | $(2,1,2)$ | 12.473 | Two changes |
| Cork North-West | $(2,1,0)$ | $(2,1,0)$ | 24.912 | $=$ |
| Cork South-Central | $(3,2,0)$ | $(2,2,1)$ | 11.923 | FF to Other |
| Cork South-West | $(1,2,0)$ | $(1,1,1)$ | 20.608 | FG to Other |
| Donegal North-East | $(2,0,1)$ | $(1,1,1)$ | 17.801 | FF to FG |
| Donegal South-West | $(1,1,1)$ | $(1,1,1)$ | 12.710 | $=$ |
| Dublin Central | $(2,1,1)$ | $(2,0,2)$ | 17.771 | FG to Other |
| Dublin North | $(2,1,1)$ | $(1,1,2)$ | 16.756 | FF to Other |
| Dublin North-Central | $(2,1,1)$ | $(2,1,1)$ | 4.487 | $=$ |
| Dublin North-East | $(2,1,1)$ | $(2,1,1)$ | 19.113 | $=$ |
| Dublin North-West | $(2,0,2)$ | $(2,1,1)$ | 15.808 | FG to Other |
| Dublin South | $(2,2,1)$ | $(2,1,2)$ | 11.999 | FG to Other |
| Dublin South-Central | $(2,1,1)$ | $(1,1,2)$ | 13.301 | FF to Other |
| Dublin South-East | $(1,1,2)$ | $(1,1,2)$ | 4.042 | $=$ |
| Dublin South-West | $(2,1,2)$ | $(1,1,3)$ | 12.192 | FF to Other |
| Dublin West | $(2,1,1)$ | $(1,1,2)$ | 11.492 | FF to Other |
| Dun Laoghaire | $(2,2,1)$ | $(1,2,2)$ | 11.226 | FF to Other |
| Galway East | $(2,2,0)$ | $(2,1,1)$ | 7.923 | FG to Other |
| Galway West | $(2,1,2)$ | $(2,1,2)$ | 10.324 | $=$ |
| Kerry North | $(1,1,1)$ | $(1,1,1)$ | 19.729 | $=$ |
| Kerry South | $(1,0,2)$ | $(1,0,2)$ | 18.479 | $=$ |
| Kildare North | $(1,1,1)$ | $(1,1,1)$ | 9.214 | $=$ |
| Kildare South | $(1,1,1)$ | $(1,1,1)$ | 8.464 | $=$ |
| Laoighis-Offaly | $(3,2,0)$ | $(3,1,1)$ | 13.281 | FG to Other |
| Limerick East | $(2,1,2)$ | $(2,1,2)$ | 9.015 | $=$ |
| Limerick West | $(1,2,0)$ | $(1,1,1)$ | 4.945 | FG to Other |
| Longford-Roscommon | $(2,2,0)$ | $(2,1,1)$ | 15.181 | FG to Other |
| Louth | $(2,1,1)$ | $(2,1,1)$ | 12.575 | $=$ |
| Mayo | $(2,3,0)$ | $(2,3,0)$ | 14.288 | $=$ |
| Meath | $(3,2,0)$ | $(2,2,1)$ | 3.803 | FF to Other |
| Sligo-Leitrim | $(2,2,0)$ | $(2,1,1)$ | 15.211 | FG to Other |
| Tipperary North | $(2,0,1)$ | $(1,0,2)$ | 24.890 | FF to Other |
| Tipperary South | $(1,1,1)$ | $(1,1,1)$ | 11.361 | $=$ |
| Waterford | $(2,1,1)$ | $(1,1,2)$ | 14.951 | FF to Other |
| Westmeath | $(1,1,1)$ | $(1,1,1)$ | 15.218 | $=$ |
| Wexford | $(2,1,2)$ | $(1,1,3)$ | 13.758 | FF to Other |
| Wicklow |  |  |  |  |
|  | $(2,2,1)$ | 3.036 | $=$ |  |
|  |  |  |  | $=$ |

The content of the table is best explained by taking an entry: say Waterford, with 4 seats. The Actual and Best entries give the seats in the order (FF, FG, Other). The Best entry is
computed according to all the indices apart from the isolated result already noted. The Fit\% figures are calculated from the formula:

Fit\% $=\sqrt{ }\left(\sum(\mathrm{S} \%-\mathrm{V} \%)^{2}\right)$, which is related to the Gallagher index.

The last column gives the comparison between the actual and best entries in seats. For Waterford, a single change in the actual result by a FF seat becoming an Other seat would produce the 'best' result.

One can see from this result that 18 constituencies would remain unchanged if they gave the best fit to first preference proportionality. The major difference is that the two major parties have gained over the others - the best fit giving 56 seats in the Dáil for 'others' against the actual number of 35 .

Two constituencies are different from the others. In the case of Cork North-Central, a two seat change is needed from the actual result to get the best fit. The reason for this is a high level of transfers from the other candidates to the two major parties. The case of Donegal North-East is special because the difference in the actual and best does not involve an increase in the 'other' seats. The reason for this was a significant transfer from FG to FF in the actual election when an FG candidate was still available for transfers.

As would be expected, there is a wide variation in the Fit entries. Also, the Fit values decrease with increased constituency seats: an average of $15.7 \%$ for 3 -seats, $12.8 \%$ for 4 -seats and $10.7 \%$ for 5 -seats.

The under-representation of the Other group is to be expected as many of those candidates are excluded early in the count with many transfers to the major parties (as well as to nontransferables). This effect clearly indicates the dubious nature of grouping all the parties other than the major two into one.

The conclusion from this analysis seems to be that there is little loss in proportionality due to the natural granularity of the STV system. The lack of proportionality compared to the first preferences is caused by the vote transfers. There is a capital T in STV.

In addition to the above analysis of granularity, the same data reveals a very close correlation between the indices used. This is gratifying, since they are clearly supposed to be measuring the same property. However, the correlations can be represented approximately in a graph as follows in which the indices are indicated by their initials and the distance between them increases with a lack of correlation. From this it appears that the Loosemore-Hanby Index is centrally placed which reinforces Kestelman' s support for that index.

# Correlation graph $\underset{\text { GID }}{\bullet}$ <br> LHI 

SLI

## References

1. I D Hill, What sort of proportionality? Voting matters, Issue 16, pp5-6. 2003
2. 28th Dáil General Election, June 1997, Election Results and Transfer of Votes. The Stationery Office, Dublin 1998.
3. P Kestelman, Quantifying Representativity. Voting matters. Issue 10. pp7-10. March 1999.

# STV in New Zealand 

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In May 2001, the New Zealand Parliament enacted the Local Electoral Act 2001. At section 3 of the Act, it is stated that its purpose 'is to modernise the law governing the conduct of local elections and polls ..." including, to "allow diversity (through local decision-making) in relation to ... the particular electoral system to be used for local elections and polls[.]"

Section 5 of the Act defines "electoral system" as "... any of the following electoral systems that are prescribed for use at an election or poll:
(a) the system commonly known as First Past The Post:
(b) the system commonly known as Single Transferable Voting (STV) using Meek' smethod of counting votes[.]"

As a result of this legislation, New Zealand becomes the first country in the world to adopt STV by Meek' smethod for use in public elections. Indeed, although local authorities have the choice of switching to STV if they or their electors
want it, the Act, at section 150, amends the New Zealand Public Health and Disability Act 2000, to make it mandatory for the seven elected members of the country' s twenty-one district health boards to be elected by STV.

It will come as no surprise to learn that the road to STV becoming a reality in New Zealand was not an easy one. In 1994, on behalf of the Electoral Reform Coalition, I prepared a draft bill for the Deputy Leader of the Opposition (Labour Party), the Hon David Caygill, MP. After consulting the Electoral Reform Society in the UK, I incorporated the Northern Ireland rules in the relevant Schedule of the bill. Mr Caygill took the bill to a subsequent meeting of the Labour caucus, which agreed that it should be accepted as a private member' s bill.

At that point it became the responsibility of the opposition spokesperson on Local Government, Richard Northey, MP. He placed it in the fortnightly ballot of members' bills in October 1994, and it was drawn from that ballot the following April. Mr Northey introduced the bill (Local Elections (Single Transferable Vote Option) Bill) into the House of Representatives on 19 July 1995.

Ten of 78 submissions on the bill were heard by the Electoral Law select committee, in November 1995. On 31 July 1996, the committee established a subcommittee, comprising Richard Northey (Chairperson) and Hon. David Caygill, to consider the bill. Advice was received from officials in the Department of Internal Affairs, and the subcommittee reported its findings to the committee on 21 August 1996. The bill was reported back to Parliament in early September, just as Parliament was dissolved so that New Zealand' sfirst MMP election could be held (on 12 October). The bill was held over for consideration by the new Parliament.

Part of the "advice [...] received from officials" was to abandon the Northern Ireland rules on the ground that they did not treat all votes equally, particularly with regard to those votes given for successful candidates that were not in the actual parcel of votes that put a candidate up over the quota. Such inequality in the treatment of votes was seen as unfair.

Furthermore, knowing that computer technology was increasingly being used in local elections, the committee wanted counting rules that were more compatible with the use of such technology.

Unfortunately, the rules written to replace the Northern Ireland rules in the Report copy of the bill were logically unsound. The main problem was that the word "votes", as used in the rewritten rules, did not always mean the same thing. Sometimes it referred to transferable papers and other times to the value of those papers. In undertaking the rewrite, the authors overlooked the fact that, regardless of whether hand-counting rules are carried out by hand or by
computer, it is voting papers that are being transferred, sometimes at full value, sometimes at a reduced transfer value, rather than votes. A number of consequential errors arising from this and other misunderstandings, rendered the rules inoperable.

The rule pertaining to the calculation of the transfer value was a case in point. In the case of the transfer of a surplus resulting from a previous transfer of votes, the transfer value of the votes transferred [was to] be "the result of dividing the surplus by the total number of votes transferred in that previous transfer to the candidate from whom the surplus is transferred."

A transfer value is calculated by dividing the surplus by the number of transferable papers, not by the sum of the value of those papers and non-transferable papers, i.e. total votes. Under normal hand-counting rules, for example, an elected candidate may obtain the quota upon receiving a batch of 280 voting papers, each having a transfer value of 0.35 - a total of 98 votes. If this candidate now has a surplus of 60 votes and only 240 of the 280 papers last received are transferable, then they would be transferred at a transfer value of 0.25 .

The above-mentioned rule, however, states that the transfer value shall be calculated by dividing the surplus of 60 votes by the 98 votes transferred at the previous transfer, which comes to $0.612244 \ldots$ If this transfer value ( 0.61 ?) were then applied to the 240 transferable papers (although there was nothing to say it should be), a total of 146.40 votes would be transferred instead of 60 , and the total number of nontransferable votes would be increased unnecessarily by 24.40 !

Not only was there no direction as to how many decimal places the transfer value was to be taken to, but it was very obvious that the votes would not sum to the correct totals. Something had to be done.

The Electoral Law Committee of the new Parliament called for submissions on the Report copy of the bill, to be received by 30 October 1997.

During the course of my efforts to make sense of the rewritten counting rules, I realised quite suddenly that what officials had been attempting to do, was to replace the Northern Ireland rules with Meek-equivalent rules, unaware that Meek' smethod of counting votes had already been invented, and subsequently perfected.

Consequently, in the Electoral Reform Coalition' submission, we recommended to the committee that the counting rules be replaced by Meek' method. Our efforts were all to no avail, however, with the bill being lost following a tied vote (4-4) in committee in May 1998.

That month, I set to work drafting a completely new bill, this time for opposition Green Party MP, Rod Donald, in which I
incorporated Meek' s method of counting votes. The Explanatory Note to the bill explained that Meek' smethod was a significant improvement over the various hand-counting rules, and why; that it treated all votes equally; and that a Meek count had to be carried out by computer.

The draft was completed in December 1998 and sent out to interested parties for comment. Reaction from the local government sector was generally unsupportive, but two prominent political scientists with a particular interest in local government agreed that Meek' smethod was an improvement over hand-counting rules.

The local government sector was resisting the STV option because local returning officers (now called electoral officers) were terrified at the thought of having to learn how to conduct a complicated hand-count of votes. They imagined dozens of people constantly shuffling thousands of pieces of paper from one pile to another over several days. In these cost-conscious times, when the public demands instant results, they simply didn' t want to know about it.

Although sector representatives indicated continued resistance, this new bill happened to coincide with a push by the sector to have the local electoral legislation completely rewritten and up-dated.

In June 1999, I was invited to attend a workshop on matters pertaining to the administration and conduct of local elections to give a presentation on Meek' method. Soon after, perhaps realising that their main objection to STV (fear of handcounts) need not be a relevant consideration, and that the issue of STV was not going to go away, sector representatives decided to include provision for an STV option in their list of proposed improvements to the legislation.

A year later, in July 2000, Rod Donald' bill was drawn out in the fortnightly ballot of members' bills and given its first reading. At this time, the newly-elected Labour-led government decided that seven of the 11 members of the 21 district health boards (DHBs) that it intended to set up to replace the structure put in place by the previous government, would be elected by STV.

A significant reason for this decision was to ensure that the Maori population would have the means to ensure they were represented on these boards by people they helped to elect, if that was what they wanted. The legislation stipulates that at least two of the 11 positions must be filled by Maori, so enabling Maori to elect Maori members would enable, in most cases, the four appointed positions to be filled having regard to criteria other than ethnicity.

The government, which generally relied on the Green Party for its majority, and needing the support of the Greens to ensure the Local Electoral Bill would be enacted during the first half of 2001, agreed to include provisions for local
authorities to adopt STV in that bill. In turn, Rod Donald allowed his bill to lapse in select committee.

At this point, late-July 2000, a decision needed to be made as to which of the several forms of STV would be included in the Local Electoral Bill. Relevant officials in the Department of Internal Affairs consulted well-known political scientists, and with myself, and reduced the choice to four - Tasmania' sHare-Clark rules, Northern Ireland' s "senatorial" rules, the "original" STV rules, as used in the Republic of Ireland, and Meek' s method.

In September 2000, a paper was submitted to Cabinet recommending that Meek' method be accepted as the form of STV best suited for New Zealand. Meek was 'preferred to the hand counting forms of STV because it best contributes to effective and fair representation, and public confidence and understanding of local elections."

Two factors which contributed to this recommendation being made were that writing a computer program to implement Meek' s method would be far more straightforward than if one of the forms of hand-counting rules were adopted, and because Meek' method reduces the number of "wasted" votes to an absolute minimum, and ensures all successful candidates achieve the required quota for election.

Furthermore, officials "noted that in 1996, the Electoral Law Committee proposed that Richard Northey' sSTV Option Bill be amended from the senatorial rules to a form that reflected the intent of the Meek rules, in order to remove the necessary arbitrariness generated by hand counting."

As alluded to in the first paragraph above, the Local Electoral Act provides for local authorities to resolve to change to STV, or to hold a poll on the electoral system, and also for electors to demand a poll be held on the electoral system.

In August and September 2002, eight (out of a total of 86) local authorities resolved to adopt STV to elect their councils and community boards (if any) in October 2004. A further two councils (Wellington and Whangarei) resolved to hold a poll of electors, on 30 November and 5 December, respectively. Wellington voted narrowly to adopt STV; Whangarei voted by a margin of almost 2 to 1 to retain the first-past-the-post (FPTP) system.

Since then, the Opotiki District Council, which was one of the eight local authorities to resolve to change to STV, and the Masterton District Council, which resolved to stay with FPTP, have further resolved to hold a poll of electors.

At the time of writing (January 2003), there have been 10 successful poll demands, with possibly a handful more by the end of February. All polls must be held no later than 21 May 2003, the results of which are binding on the councils
concerned for the next two triennial general elections of the country' docal authorities (9 October 2004 and 13 October 2007).

