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Voting matters

for the technical issues of STV

The Electoral Reform Society

Issue 14 December 2001

Editorial

Readers will no doubt be pleased to know that New Zealand has passed legislation to use STV in area Health Board elections and also in some local elections. Some may be surprised that the legislation specifies the use of the Meek algorithm and hence means that a computer count will be undertaken. Although these elections will not be until 2004, work is in progress to ensure that appropriate software is available and fully meets the requirements. I hope that developments can reported via *Voting matters*.

In a separate move, the Republic of Ireland is considering the use of computers to undertake its counts, although in this case, the rules are those in the Irish constitution which were designed for a manual count.

In the first article in this issue Simon Gazeley reports a means of undertaking a manual count which avoids the need to elect candidates with less that the quota of votes. Comments on the logic of this proposal or its feasibility would be welcome.

In the second article, I report on the observed differences in those elected with the current ERS rules compared with the Meek algorithm — somewhat topical in view of the New Zealand decision (although it was motivated by preparing an election data-based for publication on the McDougall Trust CD-ROM).

In the third article, David Chapman makes a proposal for electing one candidate which is described as *preferential approval voting*. The counting method seems straightforward to undertake manually and yet claims some of the benefits of the more complex algorithms.

In the last article Bob Jones reports on the questionnaire which was circulated with Issue 12. Unfortunately, the number of responses was rather small and hence it is difficult to deduce much from the replies. The Decision Analysis table that Bob produced can be recommended as a means of encouraging people to think more deeply about the issues involved.

McDougall Trust CD-ROM

The CD-ROM, mentioned in the last editorial, should be available early in 2002. Hence if you have material that would be suitable, or know the source of such material, please let me know. Election data from the UK, Ireland and Malta will be included.

The CD will contain an acknowledgement to the many referees would have aided in this publication and especially to Dr David Hill who has proof-read all 14 issues.

Brian Wichmann.

STV with Symmetric Completion

Simon Gazeley

Meek's[1] formula for STV differs from manual systems in significant ways which have been explained by Hill[2]. These differences make Meek more acceptable to many than manual STV, but it means that a computer is necessary for any but the very simplest Meek counts. I believe it is possible to improve manual STV without either losing the ability to do it manually, or introducing some unintended unacceptable effect. The current ERS rules[3] are taken as a starting point in formulating the changes proposed, and will be referred to as N-B.

When a candidate has a surplus, N-B transfers the "parcel" of votes which gave rise to that surplus — ie, the votes which that candidate received most recently. Note that the ballot-papers will all be of the same value, which can be 1.0 or less. The papers in the parcel are sub-divided into transferable votes (those on which a subsequent preference has been expressed for a candidate who is not yet elected or eliminated), and non-transferable (those on which all the candidates for whom a preference has been expressed are either elected or eliminated). If the total of transferable votes at their present value is less than or equal to the surplus, they are all transferred at that value to the voters' next preferences, and sufficient of the non-transferable votes are left with the elected candidate to preserve that candidate's quota with no surplus; any non-transferable votes over and above the quota are put to the non-transferable pile. If the total of transferable votes is greater than the surplus, a new value is calculated for each transferable vote such that when all of them are transferred at that value, their total value is equal to the surplus, and the elected candidate is left with the quota.

This procedure in effect shares out the non-transferable votes among the continuing candidates in the proportions of the transferable votes, and can give a result which I consider perverse. Consider the following count for two seats, adapted from one devised by David Hill:

Case 1	
A	60
AB	60
CD	51
DC	9

The quota is 60, so A gets the first seat. N-B ignores the 60 voters who expressed no preference after A. It transfers the 60 AB votes at full value to B, who now gets the other seat. On the other hand, Meek transfers all the votes credited to A, in this case at a value of 0.5. Thus B gets 30 of the AB votes, while 30 of the A votes go to non-transferable. The new total of effective votes is now 150, making the new quota 50. C, with 51 votes, has attained this new quota and gets the second seat.

Now suppose that the 60 A voters had in fact expressed second preferences, three for C, the rest for B. Votes would be:

Case 2	
AB	117
AC	3
CD	51
DC	9

In Case 2, the N-B count is identical to the Meek count. A gets the first seat, but this time all the votes credited to A are transferred at a value of 0.5, leaving A with 60. B gets 58.5 of the transferred votes and C gets 1.5, increasing C's total to 52.5. Now, nobody other than A has the quota, so we eliminate D. C's total of votes now goes up to 61.5, more than the quota, so C gets the second seat. Comparing Cases 1 and 2, we see that the additional 57 votes on which the second preference is for B are counteracted under N-B by just three voters whose second preference is for C.

Owing to the habit of many voters of not casting preferences for all candidates, the total number of votes credited to candidates tends to decline as the count proceeds. This is countered in some rules by requiring the voters to cast preferences for all candidates, forcing them to register preferences they do not feel and perhaps cannot justify. This means that in N-B counts, the final candidates to be elected often have less than a quota. As the quota is higher in these cases than it needs to be, the opportunity is lost to transfer as many surplus votes as could have been transferred if the quota had been lower from the beginning but still attainable by only as many candidates as there are seats. In a Meek count, the quota is recalculated at every stage to take account of the votes which become non-transferable and all surpluses over each successive value of the quota are transferred. Thus, the only criterion for election in a Meek count is attainment of the quota.

It is reasonable to presume that a voter who does not rank all the candidates is indifferent to the fates of the candidates left unranked, and therefore does not wish the vote to favour any of the unranked candidates over the others. As the example above clearly shows, N-B can give second and subsequent preferences more votes than the voters are presumed to have intended them to receive. Note that the A voters have no right to feel aggrieved; if they had wanted to cast further preferences, they were perfectly entitled to do so. However, the CD voters are certainly entitled to protest that the 60 A votes were treated by N-B in effect as AB votes, thus denying the second seat to C.

In a manual count, the option of reducing the quota as in Meek is not available, as the count would have to be restarted at every change of the quota. The other option is to share among the continuing candidates the votes which would otherwise have been non-transferable, treating them as if they had in fact been cast as equal lowest preferences for the candidates concerned. Following Woodall[4], I shall call this “symmetric completion”. To those who are against symmetric completion on the grounds that it is never justified to award any part of a vote to a candidate for whom no preference has been expressed, my response is that symmetric completion treats all short votes alike and does not give too much weight to surplus votes on transfer. In both these respects, it is superior in my view to N-B.

With symmetric completion, the numbers of votes credited to the continuing candidates will usually be greater than they would have been under N-B, especially at the later stages. This means that there will be a tendency for more surpluses to be available for transfer, and therefore for more voters' preferences to be taken into account. Applying symmetric completion to Case 1 above, we get at the first stage

```
A        120
C         51
D          9
```

The quota is 60, and A is elected. A's votes are all transferred at a value of 0.5 to next preferences: the 60 AB votes go to B, who now has (60 × 0.5) = 30 votes, and the 60 A votes go equally to B, C, and D, who each get (20 × 0.5) = 10 votes. Votes are now:

```
A        60
B        40
C        61
D        19
```

and C gets the second seat.

Implementing STV with symmetric completion (STV-SC) would entail some changes to the N-B procedure. This is best illustrated by an example. Six candidates are contesting three seats, with votes:

```
A          59
AEFB       66
B         172
BCAE       12
C         112
CABD       86
D          11
DFEA      195
E          33
EDCF      148
F          21
FBDC       85
         ====
         1000
```

The quota is 250. As no candidate has the quota, F, with fewest votes, is eliminated. As in N-B, the 85 FBDC votes are transferred to B. Although STV-SC puts the 21 F ballot-papers to the non-transferable pile, it does not put the 21 F votes to non-transferable, as all votes in STV-SC are transferred. Instead, we call these 21 votes on which no further preferences are expressed “dividend votes”, because they are divided equally among the continuing candidates, in this case 21/5=4.2 to each. The number of dividend votes is calculated as the difference between the total of votes currently credited to candidates and the original total of valid ballot-papers; a running total is kept against each candidate's name of the number of dividend votes (s)he has received, and the stage at which they were gained. Effective votes at stage 2 are:

```
A        129.20
B        273.20
C        202.20
D        210.20
E        185.20
```

Now, the sum of A's votes and B's surplus is less than the votes credited to E, the candidate in last-but-one place. Under N-B rules, and therefore under STV-SC rules, the transfer of B's surplus is deferred, and we eliminate A at once. The 66 AEFB votes go to E, the 59 A papers to non-transferable. The total of votes credited to the candidates is now 936.80; the 63.2 dividend votes are awarded equally to C, D, and E, 21.06 to each. Votes are now:

```
B        273.20
C        223.26
D        231.26
E        272.26
```

We now transfer B's surplus, as that is the larger. The most recent parcel received by B contains the 85 transferred FBDC votes, plus A's share of the 21 dividend F votes, making 89.2 in all. We now transfer the 85 FBDC votes to D and the 4.2 F votes to C and D @ 23.2/89.2=0.26. As this boosts D's total above the quota, we end the count.

The only criterion for election in STV-SC, as in Meek, is attainment of the quota. To cater for rounding errors in transferred votes, the number of dividend votes is recalculated at each stage as the difference between the original total (in this case, 1000) and the total of the votes credited to candidates after all transferable votes have been transferred; the number of dividend votes awarded to each continuing candidate is truncated if necessary to two decimal places. As the total of the votes credited to the candidates is the same after each stage as it was after the previous one (except perhaps for rounding error), surpluses can arise at any point, giving the voters concerned a greater opportunity than under conventional N-B to influence the subsequent course of the election.

Should symmetric completion be imported into Meek? The answer is emphatically ***no***. Woodall[4], using an example provided by David Hill, has shown that quota reduction in Meek is preferable to symmetric completion, even though Meek himself was equivocal on the point. The purpose of this paper has been to show that, given the practical constraints of a manual count, symmetric completion can deal with a problem that may arise in N-B without in general substituting one that is as bad or worse.

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3. Robert A. Newland & Frank S. Britton, How to Conduct an Election by the Single Transferable Vote, Electoral Reform Society, 3rd Edition, 1997.

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Do the differences matter?

Brian Wichmann

Introduction

In preparing material for a CD-ROM which contains ballot data[1], I have revised and extended the data which makes it feasible to undertake meaningful comparisons between the different STV counting rules.

It is naturally regrettable that the counting rules do indeed produce different results, that is, elect different candidates. This is to be expected, especially when comparing the Meek algorithm with the hand counting rules. Approximations must be made to provide a feasible manual process, so if it is required that a witnessed count be undertaken (and hence the moving of ballot papers between piles for each candidate) then a manual counting rule is required.

Unfortunately, real election data is hard to collect due to the confidentiality that usually applies to such data. However, a computer program has been written to produce such data anonymously by a random process which would not invalidate statistical tests on the anonymous data. This has resulted in a few more data sets from which a comparison can be made.

The two counting algorithms being compared here are Meek[2] and ERS 97[3].

Data selection and comparison

The total election data contains many examples used to test counting software which is not representative of real ballot data. However, 188 ballot sets have been identified as appropriate in three classes, as follows:

R001-R060. Data from real elections. This includes a few in which a random selection has been made from the total in the real election.

M001-M091. This data has been constructed from result sheets in such a way as to reflect real ballot data. In particular, the ones constructed from elections in the Irish Republic has been adjusted to reflect the observed transfers between the parties.

S001-S019, S021-S038. This set is constructed from data such as the Eurovision Song Contest, in which preferential voting could have been applied.

When a count is conducted, if a random choice has to be made, it is hard to conclude that a real difference has occurred. In fact, 29 of the above elections produced a different result, but in 10 of these a random choice was made and hence we ignore these.

We are therefore left with 19 differences out of 188 elections, ie 10.1% different. (I could have omitted those for electing one person, but I did not. These are mainly the third class above in which no difference was observed.)

Case	Votes	Candidates	Seats	Non-transferables (Meek-ERS97)/Votes	Difference
M005	27,757	7	3	-0.23%	1 (0.57%)
M010	38,410	9	4	1.83%	1 (0.71%)
M019	29,193	13	4	-1.06%	1 (*)
M028	44,454	13	5	-0.76%	1 (0.05%)
M051	39,991	10	4	0.13%	1 (0.45%)
M059	35,038	11	4	-0.13%	1 (0.58%)
M060	25,553	9	3	-0.96%	1 (0.33%)
M066	24,825	9	3	-1.79%	1 (*)
M070	44,914	13	5	-0.54%	1 (0.03%)
M073	36,407	8	4	0.16%	1 (1.01%)
M078	27,881	8	3	-0.07%	1 (0.38%)
R004	42	10	5	0.12%	1 (2.50%)
R005	58	8	7	3.79%	1 (0.40%)
R033	211	14	7	-2.61%	2
R040	257	20	15	0.07%	1 (*)
R045	2,908	12	5	5.95%	1 (0.83%)
R046	853	10	9	13.69%	1 (0.09%)
R048	944	29	10	0.04%	1 (0.15%)
R059	1,147	10	6	-0.40%	1 (0.03%)

In the table, the last entry records the number of seats whose occupancy changed and, in brackets, the number of votes less than the quota which the Meek algorithm recorded against the candidate which ERS97 elected (expressed as a percentage of the total number of votes). Hence for M005, the last remaining candidate which the Meek algorithm did not eliminate was the one elected by ERS97 and had 6358.85 votes against a quota of 6517.76 (6517.76-6358.85=158.91 votes = 0.57% of 27,757). The star indicates that the remaining candidate in the Meek count was not the one elected by ERS97 and hence the two counts diverged at an earlier point — not just the last stage. Of course, in the one case in which two seats differed, it is not possible to provide a simple numerical difference.

It can be seen from the table that the differences are significant and large in some cases. In five cases (M070, R004, R005, R046 and R048) the differences are small and perhaps could be regarded as acceptable. The total number of seats in these 19 elections is 106 with 20 differences and hence a discrepancy in those elected of 18.8%, or 2.1% difference if all the elections are considered.

The difference in the handling of non-transferables between the two algorithms is a matter of controversy. To indicate whether the number of non-transferables is a factor, the difference that the two algorithms give in the number of non-transferables is expressed as a percentage of the total votes. In the case of R046, ERS97 has a very much lower number of non-transferables which surely has a key effect on the result. However, in general, the pattern is not so clear.

It could be that the method of constructing the Mddd data (first class above) produces results which would not be typical of real elections. However, the table clearly shows that the Rddd (real elections, second class) examples show similar differences.

Conclusions

I conclude that unless it is essential to have a manual, witnessed count, the Meek rules should be used for STV counting. The approximations introduced to enable a manual count produces too many differences for the hand counting rules to be used otherwise.

Any of the data upon which this paper is based can be provided to interested parties.

References

1. See Editorial, *Voting matters*, Issue 13. April 2001.

2 I D Hill, B A Wichmann and D R Woodall. Algorithm 123 — Single Transferable Vote by Meek's method. *Computer Journal*. Vol 30, pp277-281, 1987.

3. R A Newland and F S Britton. How to conduct an election by the Single Transferable Vote. ERS 3rd Edition. 1997.

Preferential Approval Voting

D E Chapman

Dr David Chapman, who has carried our research on the electoral system at the Universities of Lancaster and Virginia, is currently Director of the Democracy Design Forum, a consultancy on electoral systems.

Introduction

This paper puts forward a new method for electing, by use of preferential voting, a candidate to fill a single seat. It is proposed as an improvement on the normally used single-seat electoral systems such as Plurality (as used for the Westminster Parliament), Second Ballot (previously used in France) or Alternative Vote (used in Australia). The new system is similar in its working to Approval Voting (the system proposed in 1982 by Brams and Fishburn[1]). However, it achieves this effect by means of *preferential* voting instead of the simple X voting of the latter system. It is therefore called *Preferential Approval Voting*, or PAV for short.

The advantage claimed for PAV is one of equity, that as compared with other systems, it gives candidates and parties a stronger incentive to be *equally responsive* to the different sections of the electorate. Also, PAV appears to be a highly practicable method of election. It is not complicated to count, having about the same level of complication as the Alternative Vote, and it could easily be counted by hand, not needing to be counted by computer, however large is the number of candidates.

PAV can best be explained by means of its relation to Approval Voting. The procedure of Approval Voting is simply this: the electors vote (non-preferentially) for as many candidates as they like, for one or for more than one, and the candidate who gets most votes is elected. PAV simulates this procedure by use of preferential voting (that is, voting where the elector votes by marking the candidates in order of preference, 1 for a first preference, 2 for a second preference, and so on, for as many candidates as he wishes).

Now under Approval Voting, the voter will always vote for the candidate whom he most prefers. But under what circumstances will he vote further down his preference ordering, voting in addition for his next-preferred candidate, or for several of the next-preferred candidates? It seems likely that he will do so if he expects that a candidate whom he very much less prefers has some chance of being elected, and if he thinks that voting for the next-preferred candidate or candidates will reduce this chance. For example, a voter whose first preference is Labour, second is Liberal Democrat, and third is Conservative, will always vote for the Labour candidate, and might vote for the Liberal Democrat in addition, if he thinks that the Conservative has a significant chance of winning.

PAV approximately simulates this voting behaviour, by use of the preference orderings provided by the voters. Thus PAV always counts the voter as voting for his first-preferred candidate. PAV counts him as voting for his *next*-preferred candidate *when the latter is preferred to the leading candidate, that one who so far in the counting has obtained most votes*. In other words, this leading candidate is treated as one who has a significant chance of being elected, and therefore voters are assumed to vote for the candidates they prefer to him.

The rules of PAV

Here are the full rules of PAV. The electors vote by putting the candidates in order of preference. “Points” are assigned to candidates, according to the preferences for them, and the candidate with most points is elected. For this purpose, the counting of the votes proceeds in stages, as follows.

The first stage. In respect of each ballot paper, a point is given to the candidate marked as first preference on that paper. The points of each candidate are counted, and the *leading candidate* is found (that is, the candidate who has most points). If there is a tie between two or more candidates, one of them is selected by lot to be the leading candidate.

Any further stage. Those ballot papers are considered, in respect of which a point has *not* so far been given to the leading candidate of the *previous* stage. In respect of each such ballot paper, a point is given to the candidate next-preferred to the last candidate to receive a point, *provided this next-preferred candidate is preferred to the leading candidate of the previous stage*. The leading candidate (who will possibly be a new one) is then found, that is, the candidate who has obtained most points up to and including the current stage.

These further stages are repeated, each one giving more points to the candidates, until the final stage is reached, at which none of the electors' next preferred candidates is preferred to the leading candidate, so that no candidate is entitled to receive any further point. At this final stage, the candidate who has most points is elected.

It will be seen that the method of counting the votes for PAV, is somewhat similar to that for the Alternative Vote. Under both PAV and AV, the first stage is to count the first preferences on all ballot papers. In each later stage, the next preferences are counted on a limited number of the ballot papers, until the winning candidate is found.

A preferential system which bears some resemblance to PAV is that of Descending Acquiescing Coalitions (DAC). DAC is a new preferential election method for filling a single seat, which was recently proposed by Woodall[2,3], as an improvement on the Alternative Vote (which is discussed more fully below). DAC resembles PAV in that both can be regarded as a preferential simulation of Approval Voting. However, Woodall[2] admits DAC is “much more complicated than [the Alternative Vote]”, and would be likely to require a computer to carry out the counting. Thus it is clear that PAV will be much simpler than DAC (see below).

The effects of PAV

In order to illustrate the working of PAV, and to demonstrate the properties of the system, let us consider some numerical examples. We first consider Election 1, where the electors' preferences are single-peaked, that is, preferences are based on some dimension (such as that of left-to-right positions in policy), on which each voter has his own most-preferred point, and on which he prefers any other point less, the further it is from his most-preferred point.

(The notation used to describe the election is explained as follows. The first lines show the voters' preference listings of the candidates. Thus in the top line, 35 voters rank L first, C second, and R third. The subscripts against some of the candidates in a preference listing, show in what stage points are given to the candidate. Thus in the third line, 16 points are given to C in the first stage, and 16 points are given to R in the second stage. After the preference listings, each column shows the total points which have been obtained by each candidate by the specified stage. Thus by stage 2, L has obtained 35 points, C 65, and R 49. The greatest total of points, that of C, is shown in underlined, C being the leading candidate at stage 2.)

Election 1

35 $L_1C\ R$
16 $C_1L\ R$
16 C_1R_2L
33 R_1C_2L

	Stage 1	Stage 2	Stage 3
L	$\underline{35}$	35	35
C	32	$\underline{65}$	$\underline{65}$
R	33	49	49

In stage 1, each candidate gets one point for each first preference. L is the leading candidate, getting most points. In stage 2, candidate C (who is the next preference of the 33 first-preference supporters of R, and who is preferred by them to L, the leading candidate of the previous stage) therefore gets 33 more points. Similarly, R gets 16 more points, by being preferred to L by 16 first-preference supporters of C. C, now having most points, becomes the new leading candidate. In stage 3, none of the next-preferred candidates is preferred to C, the leading candidate of the previous stage, and so no candidate gets any more points. Thus C, having most points in the final stage, is elected.

We can use the results of Election 1 to illustrate how PAV deals with *incomplete preference listings*, that is, ballot papers which do not express a preference for all the candidates. It makes no difference whether or not a last preference is expressed by the voter. For example, if the 33 voters voting RCL voted RC instead, this would not alter the result, since we would still know, for stage 2, that they preferred C to L, the leading candidate, so that C would still get 33 extra points. However, it *does* make a difference if a *non-last* preference is not expressed. For example, if the 33 voters voted just R, that is, first preference for R, with no preference given for any other candidate, then C would get no extra points in stage 2, since no preference for C over L would have been expressed.

But let us return to the original results of Election 1 as shown above. In this situation of single-peaked preferences, PAV has elected the centre candidate in the left-to-right dimension. This candidate elected by PAV is also the so-called Condorcet winner, that is, the candidate who beats each other candidate, always being preferred to the other candidate by a majority of voters. (C is preferred over L by 65 voters to 35 and over R by 67 to 33.) Note that PAV achieves this result (that is, of electing the centre candidate or Condorcet winner) despite the fact that C has fewest first preferences, which would prevent C from being elected under the Alternative Vote, that form of preferential system which is most commonly used for electing to one seat.

However, if PAV is actually in use for a series of elections, then it is unlikely that the electors' preferences between the candidates will remain single-peaked. For candidates L and R will surely come to realise that under PAV, their respective extremist positions are going to lose them election after election, and so they will adjust their appeals to give themselves a better chance of winning. Thus L will appeal to the supporters of R, to persuade more of them to change their preference listing to RLC instead of RCL, and R will appeal to supporters of L to get them to change to LRC. The pattern of the electors' preferences will then no longer be single-peaked, but will tend towards what might be called a symmetrical pattern, where there is about the same number of voters with each possible preference listing (that is, in this case, one-sixth LCR, one-sixth LRC, and so on). Thus a typical election might be something like Election 2.

Election 2

18	L_1C_4R
17	L_1R_3C
17	C_1L_4R
15	C_1R_2L
17	R_1C_2L
16	R_1L_3C

	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5
L	35	35	51	68	68
C	32	49	49	67	67
R	33	48	65	65	65

Thus by broadening their appeal, L and R have got more points, and L has succeeded in getting elected. L now gets second preferences, not only from first-preference supporters of C as before, but also from the first-preference supporters of R, and similarly R now gets second preferences from the first-preference supporters of L. This illustrates how PAV gives a candidate or party the incentive to appeal to, and to be responsive to, all sections of electors.

Election 2 can be used to illustrate the general strategy by which a candidate will seek to win under PAV. A candidate wins by getting a point from the most voters. A candidate C gets a point from any one voter V either if C gets V's first preference, or otherwise *if C is preferred by V to that one of the leading candidates who is least preferred by V*. Thus in Election 2, L gets a point not only from the 18 LCRs and 17 LRCs, but also from the 17 CLRs and the 16 RLCs.

This has implications for a candidate's general strategy. He will be primarily concerned to persuade voters to prefer him over their least preferred leading candidate. Once they do this, he will not seek to persuade them to give him a still higher preference (that is, a first preference in Election 2), since this will tend to be difficult to achieve, and in any case *it will not bring him any more points*. Thus when there are three leading candidates, as in Election 2, each one will direct his appeal primarily at those electors who have tended to give him last preference, and in general, each candidate will be seeking to get second preferences rather than first preferences.

Further properties of PAV

PAV has the same property as does the Alternative Vote, and also DAC, that a candidate who gets an absolute majority of first preferences is necessarily elected. This can be simply shown as follows. Suppose A has the first preferences of more than half the voters. Thus A is the leading candidate at the first stage, with a point from more than half the voters. At the second stage, the best that any other candidate can do is to get a point from every voter who did not vote first preference for A, that is, he must get points from less than half the voters. Thus A, with a point from more than half the voters, must be the leading candidate at the second stage. By a similar argument, A must be the leading candidate at the next stage, and at any stage after that. Thus A must be elected.

However, PAV is *unlike* the Alternative Vote in that the candidate with fewest first preferences can be elected, as was the case in the single-peaked example of Election 1 above. Indeed, PAV can enable a candidate to get elected who has very few or even no first preferences. A *non*-single-peaked example of this, which might well occur occasionally in practice, is Election 3. Here a candidate C, who has few first preferences, gets more points than either A or B (each of whom have close to half the first preferences) by persuading many of As and Bs first-preference supporters to give C their *second* preferences.

Election 3

32	A_1C_3B
16	A_1B_4C
15	B_1A_4C
32	B_1C_2A
2	C_1B_2A
3	C_1A_3B

	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5
A	$\underline{48}$	48	51	66	66
B	47	$\underline{49}$	49	65	65
C	5	37	$\underline{69}$	$\underline{69}$	$\underline{69}$

This lack of the need for first preferences under PAV, can be expected to reduce the entry barrier against new candidates. For it is likely to be easier to gain second preferences than first preferences, thus making it easier under PAV for a new candidate to compete successfully with already established candidates, than it would be under the Alternative Vote, or in particular under Plurality. Thus under PAV, at least when it has been in use for some time, it is likely that few candidates will obtain a majority of first preferences, and that the most usual situation in each constituency will be for there to be three strong candidates (or perhaps sometimes more than three) in not very unequal competition. In other words, it is likely that under PAV, there will be a tendency towards a symmetrical situation like that shown in Election 2.

In all the examples given above, Elections 1, 2 and 3, there were only three candidates competing. How then will PAV operate, if there is a larger number of candidates? The same procedure will be followed, that of sorting and counting the next preferences stage by stage, until that stage is reached, where no next-preferred candidate is preferred to the leading candidate, and thus no candidate is entitled to receive any further points. Because there are more candidates, there will of course be more next preferences to sort and to count. But the extra counting need not be in proportion to the number of extra candidates. The reason for this is that on any one ballot paper, only the top preferences need to be counted, down to the preference for the candidate who is one preference step above that one of the "leading candidates" whom the voter least prefers. It is likely that the extra candidates will be given a very low preference (or no preference) by most of the voters, and that because of this their preferences for them will not need to be counted.

Election 4 is given below, as an example of a four-candidate election. Election 4 is assumed to be a re-run of Election 2, in which one party, the party which previously ran L as its candidate, now runs two candidates L and M, one a woman and one a man, in order to give the electors a wider choice. Electors are assumed to put L and M in the same position in their preference listings as they put L in Election 2.

Election 4

10	$L_1\ M_2\ C\ R$
8	$M_1\ L_2\ C\ R$
9	$L_1\ M_2\ R\ C$
8	$M_1\ L_2\ R\ C$
9	$C_1\ L_2\ M\ R$
8	$C_1\ M_2\ L\ R$
8	$C_1\ R_3\ L_4\ M$
7	$C_1\ R_3\ M_5\ L$
9	$R_1\ C_3\ L_4\ M$
8	$R_1\ C_3\ M_5\ L$
9	$R_1\ L_4\ M\ C$
7	$R_1\ M_3\ L\ C$

	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5	Stage 6
L	19	$\underline{44}$	44	$\underline{70}$	$\underline{70}$	$\underline{70}$
M	16	43	$\underline{50}$	50	65	65
C	32	32	49	49	49	49
R	$\underline{33}$	33	48	48	48	48

PAV and Condorcet

Another question of some interest is how PAV differs from Condorcet, the well-known method of electing to a single seat by means of preferential voting. Under Condorcet, A beats B if there are more voters who prefer A to B than those who prefer B to A. But under PAV, A beats B if there are more voters who give A a first preference, or otherwise prefer A to a "leading candidate", than those who give B a first preference, or prefer B to a leading candidate. Thus an important difference between the two systems, is that under Condorcet, a voter supports either A or B, but cannot support both; whereas under PAV, it will often be the case that the same voter supports both A and B, preferring A to a leading candidate, and also preferring B to a leading candidate. Not surprisingly, PAV is in this respect similar to normal Approval Voting, where any one voter can vote (in this case with an "X") for both A and B.

But how far does PAV tend to elect the Condorcet winner (CW)? The CW was elected in Election 1, where preferences were single-peaked, and also in the more likely preference situation of Election 2 (L, the PAV winner, being preferred over C by 51 voters to 49 and over R by 52 to 48). However, in Election 3, where C, the PAV winner, got most of his votes from second preferences, the CW was *not* elected, the CW being candidate A (who was preferred over B by 51 voters to 49, and over C by 63 to 37). It thus appears that in practice, in the preference situations most likely to occur, PAV has a very high probability of electing the CW, but that it might not elect the CW in some unusual situations, where the PAV winner obtains an especially high proportion of his points from lower preferences.

PAV and Descending Acquiescing Coalitions (DAC)

It is of especial interest to compare PAV with DAC, which is another new single-seat preferential system which can be regarded as a preferential simulation of Approval Voting. The rules of DAC can be explained as follows.

A voter is said to *acquiesce* to a set of candidates if there is no candidate outside the set whom he prefers to any candidate in the set. (In other words, in respect of any pair of candidates, one in the set and one outside the set, he always *either* prefers the candidate in the set, *or* expresses no preference between them.) The set of all those voters who acquiesce to the candidates A and B is referred to as the *coalition* acquiescing to A and to B, or as {A, B}. For example, if there are only three candidates A, B and C, then {A, B}will be all those voters voting as follows: ABC, AB, BAC, BA, A or B.

That candidate is elected who obtains the acquiescence of a greater number of voters than any other candidate. This is determined as follows. A candidate A is said to *beat* a candidate B if the greatest coalition acquiescing to A and *not* acquiescing to B, is greater than the greatest coalition acquiescing to B and not acquiescing to A. That candidate is elected who beats each other candidate.

This can be illustrated by the following two examples, taken from Woodall[2] .

Election 5 (Election 3 of Woodall)

11	AB
7	B
12	C

This produces acquiescing coalitions as follows, in descending order of size.

{A, B, C}	30
{B, C}	19
{A, B}	18
{A, C}	12
{C}	12
{A}	11
{B}	7

B beats A, because {B, C} > {A, C}. B beats C, because {A, B}> {A, C}. Thus B is elected.

Election 6 (Election 4 of Woodall)

5	ADCB
5	BCAD
8	CADB
4	DABC
8	DBCA

This produces a set of the greatest acquiescing coalitions as follows.

{A, B, C, D}	30
{A, B, C}	13
{D}	12
{A, D}	9
{A, C}	8
{B, C, D}	8
{B, D}	8
{C}	8

A beats B, because{A, D} > {B, C, D}. A beats C, because {A, D}> {B, C, D}. A beats D, because{A, B, C}> {D}. Thus A is elected.

Let us now compare DAC with PAV. Under DAC, A beats B if more voters are in the greatest coalition acquiescing to A and *not* acquiescing to B, than are in the greatest coalition acquiescing to B and not acquiescing to A. Under PAV, A beats B if there are more voters who give A a first preference, or otherwise prefer A to a “leading candidate”, than those who give B a first preference, or prefer B to a leading candidate.

DAC is like PAV, and *un*like the Alternative Vote, in that it does not require a candidate to get first-preference votes in order to get elected, and so it can elect the candidate with fewest first preferences (as it does in Election 5). The two systems DAC and PAV are similar to each other, and to Approval Voting, in that each of them can give value to one or more of the highest non-first preferences of an elector, and in that if it does, the value of a non-first preference is the same as that of a first. DAC can thus be regarded as a preferential simulation of Approval Voting, as can PAV.

PAV and lack of monotonicity

A system is non-monotonic if it is possible under it for a candidate who gets more voting support, to lose the election as a consequence. The ten monotonicity properties, that is, ways in which a system can be monotonic or not, are analysed in Woodall[2,3]. Elections 7 to 9 below, show PAV to be non-monotonic in at least two of these ways.

Election 7

10	A_1 B_2
9	B_1
2	C_1 B
9	C_1
8	D_1 A_2

	Stage 1	Stage 2	Stage 3
A	10	18	18
B	9	$\underline{19}$	$\underline{19}$
C	$\underline{11}$	11	11
D	8	8	8

Thus B is elected.

Now suppose that in Election 8, the two voters who voted CB in Election 7, change to voting BC instead. The stages of the count will then be as shown below, and A will be elected. Thus by moving up the preference listing of these two voters, B will have lost the election.

Election 8

10	A_1 B
9	B_1
2	B_1 C_3
9	C_1
8	D_1 A_2

	Stage 1	Stage 2	Stage 3	Stage 4
A	10	$\underline{18}$	$\underline{18}$	$\underline{18}$
B	$\underline{11}$	11	11	11
C	9	9	11	11
D	8	8	8	8

Alternatively, suppose that in Election 9, the profile is as in Election 7, except that three new voters enter the election, and vote first preference for B, so that the second line in the election profile is 12 B instead of 9 B.

Election 9

10	A_1 B
12	B_1
2	C_1 B_3
9	C_1
8	D_1 A_2

	Stage 1	Stage 2	Stage 3	Stage 4
A	10	$\underline{18}$	$\underline{18}$	$\underline{18}$
B	$\underline{12}$	12	14	14
C	11	11	11	11
D	8	8	8	8

Thus A is elected. Again, B has lost the election, this time by getting more voters to vote for him.

It should be pointed out that the Alternative Vote is also non-monotonic, whether more or less so than PAV I am unable to determine. DAC, on the other hand, was designed to satisfy as many monotonicity properties as possible, and in fact satisfies eight out of ten of them.

How far, then, would this lack of monotonicity in PAV be a problem not just in theory, but in actual practice in real elections? The main objective of PAV is to give each candidate the incentive to be responsive to each section of electors. Thus the important question is, how far will lack of monotonicity interfere with this incentive? Will a candidate (such as B in Elections 7 to 9 above) ever have the incentive to *displease* the electors, so that they give him a lower preference, or so that fewer of them vote for him?

This seems unlikely, for two reasons. First, a non-monotonic profile of votes such as those of Elections 7 to 9 seems itself unlikely when candidates are competing strongly, not only for first preferences, but for second and third preferences as well. Then the profile tends towards a more symmetrical pattern such as that shown in Election 2 above, which would be monotonic. Second, in order for the candidate to be provided with this negative incentive, he must be able to predict that the overall profile of votes at the next election will be such as to produce this non-monotonicity, and furthermore that his own votes will be in that presumably narrow range where he will benefit from losing votes. In the absence of this prescience, the candidate will have the incentive to respond positively to the electors, in the expectation that nearly always it will be beneficial for him to get more votes rather than fewer of them. Thus it seems unlikely that this lack of monotonicity will affect the candidates' incentives, or will be of practical importance.

Strategic voting

It is well known that any non-probabilistic method of election provides the opportunity, in some situation or other, for electors to engage in strategic voting. What form then will this strategic voting take, under PAV? It appears that the most likely strategy will be for the voter to give a *truncated preference listing*. For example, if it is expected that either A or B will get most points, and that both will get considerably more than C, then some of the ABCs (that is, electors whose preferences are A first, B second, C third) might adopt the strategy of voting only a first preference for A, and giving no preference for the other candidates (and similarly some BACs might vote only a first preference for B). Thus by not giving any votes to B, the ABCs make it more likely that A, their first preference, will be elected.

The other systems similar to PAV are liable to strategy in a similar way. Thus under normal (non-preferential) Approval Voting, a similar strategy is very likely to be used—ABCs voting only for A and BACs voting only for B, when the election is expected to be a two-horse race between A and B. Under DAC, a preferential system with some similarity to PAV, this same strategy of the truncated preference listing is likely to be used (according to Woodall[4]).

How far, then, is it a problem, that there is this opportunity for strategy under PAV? The strategy will be used in constituencies where two of the candidates are clearly stronger than the others, and it is expected that the winner will be one or other of them. But in constituencies where there are *three* or more strong candidates, and it is unclear which of them is going to get most points, the electors will tend *not* to vote strategically, but to express fully their preferences between these candidates.

However, there are reasons to expect that any constituency will tend to move from the former situation towards the latter, that is, from one with two strong candidates to one with three or more. Firstly, as it was shown above, PAV does not require a candidate to get many first preferences in order to win, and so it presents relatively little entry barrier to an effective new candidate. Secondly, when there are two strong candidates, let us say A and B, and a weaker candidate C, the strategic voting which this situation encourages actually benefits C. For some ABCs will vote only first preference for A, which will reduce Bs votes, and some BACs will vote only first preference for B, thus reducing As votes. This reduces the number of first or second preferences which C needs to get, to approach about the same number of votes as A or B, making it easier for C to become a third strong candidate. It will then be uncertain which of the three candidates is going to get most votes, and strategic voting will become unlikely.

Thus in conclusion, it seems that the tendency in any constituency is towards a situation where there are three (or perhaps more than three) strong candidates, each with some chance of winning. To the extent that this situation occurs, the truncation strategy will tend not to be used, and voters will express fully their preferences for the candidates.

An evaluation of PAV

In the view of this paper, the main objective of an electoral system is to provide the elected candidates, and the parties to which they belong, with the incentive *to respond to the needs of the electors*; and to respond not just to a part of the electorate, even a majority part, but to respond equitably to *each section* of electors, each possible minority. How far then does PAV provide the incentive to this equitable all-round responsiveness?

To answer this question, let us consider the examples of Elections 1 and 2 above. In Election 1, candidates L and R fail to respond to all sections of electors, L not responding to the right-wing electors, and so getting a last preference from them, and R not responding to the left-wing electors. Consequently, they lose points, and neither of them has any prospect of getting elected.

However, in Election 2, each of them has broadened his appeal to include the whole electorate, L responding to right-wing electors, and R to left-wingers. L now gets second preferences, not only from centre electors as before, but also from right-wing electors, and similarly R gets second preferences from left-wingers. Thus by broadening their appeal, L and R get more points, and L succeeds in getting elected. This illustrates how PAV gives each candidate the incentive to respond to each section of electors.

Note that in Election 2, the situation between all three candidates is *symmetrical* in the sense that any two candidates compete with each other for the second preferences of the third candidate's first-preference supporters. Thus L and R compete for the second preferences of centre electors (just as they did in Election 1). But now L competes with C for right-wingers' second preferences, and similarly R competes with C for left-wingers' second preferences. Any one candidate thus needs to be responsive to the first-preference supporters of any other candidate, in order to compete with the third candidate for their second preferences. For example, L needs to be responsive to centre electors to compete with R, and to right-wing electors to compete with C. Thus PAV gives each candidate the incentive to be responsive to each section of the electorate.

Another way of understanding the incentives provided by PAV is as follows. In the likely situation where there are three candidates competing, and each becomes a leading candidate at some stage in the counting, a candidate receives one point for each first preference and one point for each second preference. Thus (assuming all voters express their second preferences), a candidate needs to get either a first-preference or a second-preference vote from *at least two-thirds of the voters* in order to get elected. He is not likely to achieve this, in competition with two other candidates also trying to do the same thing, unless he appeals to each section of electors. Thus the candidate has the incentive to respond to each section of the electorate.

Furthermore, a first preference is worth no more than a second preference--both are worth only one point. Thus there will be no need for a candidate to appeal to a given section of electors any more strongly than is necessary to get second preferences from it, and no reason to give the section any specially favourable treatment, in order to obtain from it a higher proportion of first preferences. This is clearly a factor making for the candidates' *more equal* responsiveness to each section.

It is interesting to compare the situation under PAV as described above, with that under the Alternative Vote. Here, in order to get elected, a candidate needs to obtain the support not of two-thirds of the voters, but of only one-half. Thus he is likely to appeal less widely. Further, each candidate must strive for first preferences, since the candidate with fewest first preferences will be excluded. This seems likely to create an incentive for a candidate to favour some sections of electors over others, in order to get first preferences from them.

To illustrate this, let us consider an example with three candidates, A, B and C, where it is expected that C will be excluded, and that it will be a close finish between A and B. Each of A and B will have his core supporters, to whom he is strongly responsive, in order to obtain first preferences from them. Also, each of A and B will be strongly responsive to those voters giving first preference to C, in order to compete with the other candidate for these voters' second preferences. But A will tend to be unresponsive to the core supporters of B, because of the difficulty of persuading them to switch from first preference for B to first preference for A. Similarly, B will tend to be unresponsive to the core supporters of A. Thus the Alternative Vote, by forcing candidates to strive for first preferences, makes for their unequal responsiveness to the different sections of electors. In comparison, PAV, which makes no requirement for first preferences, will give candidates the incentive to respond more equally to each different section of electors.

PAV in the UK

If PAV were introduced in the UK for the Westminster Parliament, the present single-member constituencies would be retained. The only difference for the electors would be that they would vote by putting candidates in order of preference, instead of X-voting for only one candidate.

What then would be the effect on the parties' shares of seats? The present Plurality system, which essentially gives a seat to the candidate with most first preferences, discriminates strongly against the Liberal Democrats, who have third most first preferences. However, under PAV, they would be likely to get many more seats than now, since there seems no reason why they should not get about as many second preferences as either of the other two major parties. Thus it seems likely that the three major parties would be more equal in their seats than they are now, and that no one party would get a majority; so that a coalition government would need to be formed, by some two of them.

The point of most interest, and the main advantage claimed for the new system, is that it would give parties the incentive to change their policies to be more inclusive, more equitably responsive to the different sections of the electorate. For example, the Conservative Party currently tends to be unresponsive to strong Labour supporters, since under the present Plurality system few of them could be persuaded to switch to voting for the Conservatives. But under PAV, the Conservative Party would become more responsive to them, in order to compete with the Liberal Democrats for their second preferences. Similarly, the Conservatives would become more responsive to strong Liberal Democrat supporters, in order to compete for their second preferences with Labour. Thus the three major parties would tend to converge in policy, towards a policy more equally responsive to each section of electors; and as a result of this convergence, a coalition government formed by any two of them would be likely to be stable, and acceptable to all sections of the electorate.

Other uses of PAV

PAV could be used with advantage, instead of the Two-Ballot System, for the election by popular vote of individual office-holders, such as the president of France, the president of Russia, or the prime minister of Israel. The advantage of PAV for this purpose, can be explained as follows. Under the Two-Ballot System, the usual rule is that if there are more than two candidates on the first ballot, and no-one gets a majority of the votes, then the two strongest candidates go forward to the second ballot, where one of them must get a majority. Thus a moderate or centre candidate, who is widely acceptable to the electorate, and who could win in the second ballot if he got there, may well fail to get elected, because he gets too few votes on the first ballot. But as was explained above, under PAV there is no requirement to get first preferences (corresponding to first-ballot votes in the Two-Ballot System), and a candidate can be elected just as well by second as by first preferences. Thus this moderate or centre candidate, with few first preferences but many second preferences, is likely to get elected under PAV, where he would not be elected under the Two-Ballot System.

For similar reasons, it might be desirable to use PAV for purposes such as the following: the election of a president, or of a chairman, by the members of a legislature; the election of the party leader by the party membership, or by the party's MPs.

PAV could also be used for a multi-option referendum, to enable the electorate to choose one option out of three or more. This can be justified as follows.

In the usual type of referendum, electors choose between two options, these options being some proposed action, let us say A, and the status quo S. Proposers will be concerned to find an A which will get a majority over S, and in doing so they may come up with an A which is very harmful to the minority, while perhaps only marginally beneficial to many people in the majority. Thus the two-option referendum might lead to very unequal treatment of different sections of the electorate, and to division and conflict.

However, if a PAV-using multi-option referendum is introduced, a compromise option C is likely to be proposed, one which is better than A for S preferrers and some A preferrers, and better than S for other A preferrers. Thus there will be three options on the ballot paper, A, S and C, for the electors to place in order of preference. Since C will have many second preferences, it is likely that C will be adopted. This illustrates how a PAV-using multi-option referendum tends to improve the outcome, reducing the risk that any section of the electors will be severely harmed.

References

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Decision Analysis — Responses to a Questionnaire

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Introduction

An article describing the application of Decision Analysis to choice of "best" electoral system was given in Issue 12 of *Voting matters*. Readers were invited to complete their own version of the Analysis Table supplied. The present article gives an analysis of the responses received.

Not surprisingly, in view of the readership of *Voting matters* nearly all favoured STV. It was therefore decided to invite a wider population to respond. This was just before the General Election on June 7th and candidates from the local "Jenkins AV+" area were contacted. The area consists of the present constituencies of Cheltenham, Gloucester, Tewkesbury, Stroud, Cotswold, and Forest of Dean. Responses from some 20 candidates was sparse so other political and non-political people were contacted.

A total of 14 responses was received.

Method of averaging

For each FEATURE (of a voting system) the average value from respondents was evaluated. These features are plotted in Figure 1 in the order giving the most liked feature first. In that order, the features are:

PRO-N: How proportional is the national result?

EASYV: How easy is the system for the voter to use?

PRO-R: How proportional is the result within a region? (A region is visualised as, say, 10 of the present neighbouring constituencies.)

LOC: Local link — How closely are MPs linked to an area?

EW&E: Does the system encourage women and people from ethnic minorities to stand for election?

CHO-MP: Is there a choice within a party as well as across party lines?

PLOC: How easily can constituents contact an MP of their preferred political persuasion?

ONECMP: Is there one class of MP? (Some systems have regional as well as local MPs)

EASYC: How easy is the process of counting?

EASYBC: How easy is the task of the Boundary Commission?

STAB: Stability of government. STAB really asks the question "Is the government likely to complete its normal period of office?" Critics of PR sometimes say it results in "weak" coalition government. This has some validity with Party Lists, particularly when based on the whole country as in Israel. Experience in Germany since 1945 with AMS, and in Eire since 1922 with STV are to the contrary.

It should be noted that the *Voting matters* article used a range of weighing factors from 0 to 3, whereas from March 2001 a range from 0 to 10 was in use. Furthermore the additional FEATURE of STAB was not considered as it did not appear in the original *Voting matters* article.

For each voting system, a similar plot is produced in Figure 2. Here the systems in reducing order of preference are:

STV: Single Transferable vote.

PLRO: Party List based upon a region and using open lists.

PLRC: Party List based upon a region and using closed lists.

AV50: Similar to AV+, but having a 50% top-up element.

PL: Party List.

AV+: The proposal made by Lord Jenkins.

AMS: Additional Member System as used in Germany since 1945 and in differing forms for the Scottish Parliament and the Welsh Assembly.

AV: Alternative vote.

FPTP: First Past the Post (as used in Westminster).

Readers who would like to fill in their own questionnaire can obtain a copy from the Editor by writing to ERS or electronically by e-mailing Brian.Wichmann@freenet.co.uk.

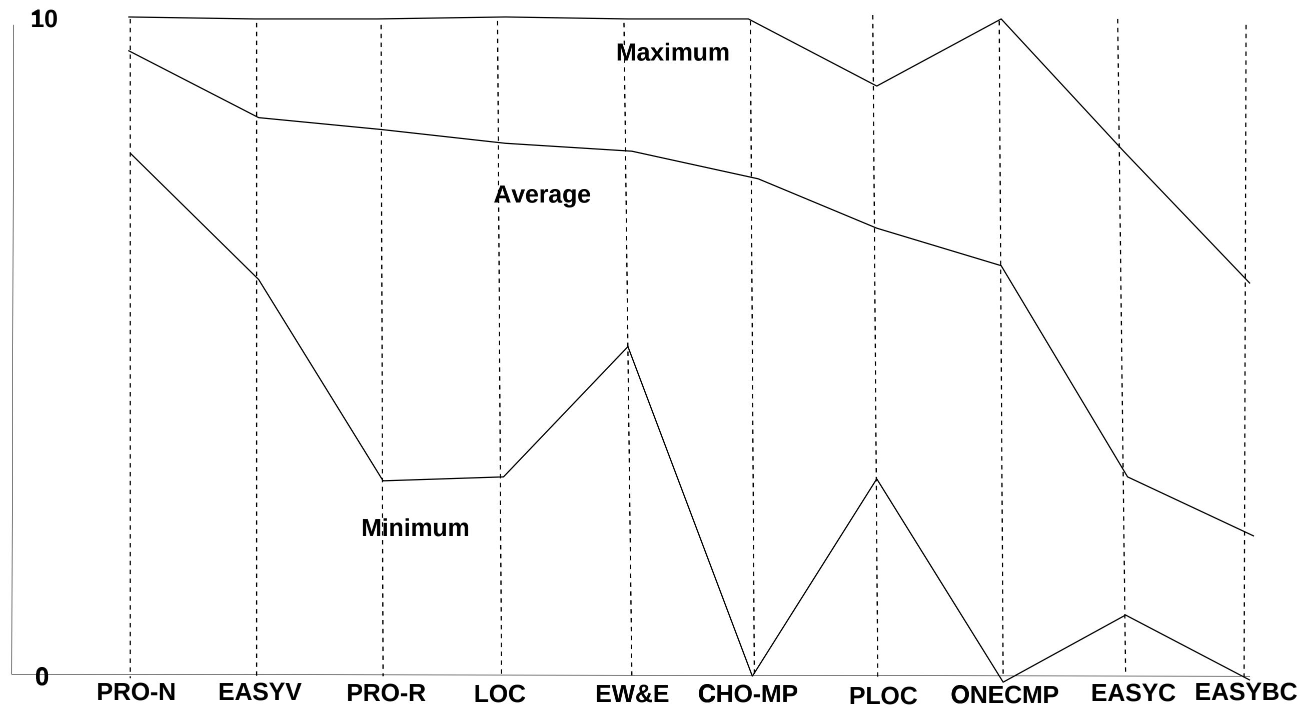

Figure 1: Features

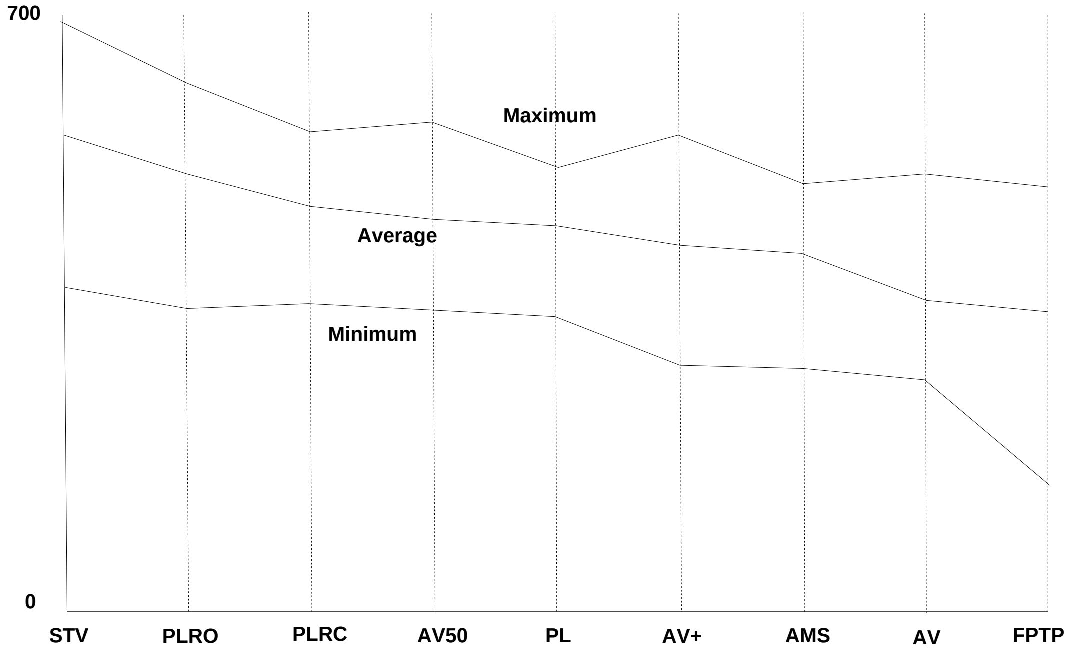

Figure 2: Systems