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Voting matters

for the technical issues of STV

The Electoral Reform Society

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Editorial

Issue 9 of *Voting matters* contained two articles on the vexed question of ordering candidates when preferential voting is used. In the first article here Joe Otten returns to this question in the light of some problems noted in the previous ‘solutions’.

In the next two articles, David Hill questions the suggestions made in two different articles that appeared in Issue 11. As often happens in this area, a suggestion which seems fine initially, may have subtle difficulties — at least as far as people other than the author are concerned!

My own article for this issue considers the effect of numerical accuracy of STV when using the Meek algorithm. Unlike the hand-counting rules, the algorithm itself does not define the accuracy that should be used, although omitting this information is the convention with numerical algorithms.

Bob Jones questions what one wants from an electoral system and considers the use of Decision Analysis to make sense of the conflicting requirements. Readers are invited to make their own contribution. The editor hopes that, given sufficient response, a further article might be appropriate which should provide a view from the entire readership of *Voting matters*.

A major article is provided by Simon Gazeley in which a new algorithm is proposed for a computer-based STV count. As is to be expected with such an algorithm, it will take a significant effort to validate. No doubt, if a program is produced to implement it, some ambiguities will be noted. Given an implementation, then comparisons should be straightforward. It appears that the algorithm is essentially more complex than, say Meek — but does that matter?

David Hill provides a third article which is surely a warning to all who advocate STV. We have no ‘standard’ for STV and in some Australian elections, the rules do not appear to give the benefits which one would expect.

Recently, an Internet group has been formed on STV. As editor, I will keep a watching brief on this, both to report material in *Voting matters* and also to encourage others to write articles. As is usual with Internet traffic, it is rather informal and not suitable for direct publication.

A combined issue for Volume 1 of *Voting matters* has been prepared. Unfortunately, it is not economic to print it, but it is available from me in the electronic format PDF which can be printed easily on most modern computers.

Brian Wichmann.

Ordered List Selection revisited

J Otten

Joe Otten is the author of a Windows program for the current ERS STV rules.

1. Problems with methods advocated in *Voting Matters* 9

I was struck by a comment by Hill[1] that Rosenstiel's alternative method to the use of constraints violated the principle that later preferences should not be allowed to count against earlier ones (I will refer to this as ‘the principle’ in this paper). This was because the method involved running repeated counts on the same vote profile, and thus a later preference may have its effect in one count when the fate of earlier preferences was still to be decided in a later count.

I realised that the same criticism could be levelled at the method I advocated for selecting an ordered list[2] (in this case of candidates for a party to offer at a European Parliament election conducted using a list system). It could also be levelled at the similar system proposed by Rosenstiel[3]. In each case multiple counts were used, and the result of one count could affect the result of another — by the use of a constraint in my case, or by overriding it in the other.

Example 1:

AC	2
AD	10
BC	10
C	8
DC	6

This gives the following results:

Vacancies	Results
1	C
2	AC
3	ABC

Both methods give the Result: CABD.

Suppose Rosenstiel's method was used, and those voting BC changed their vote to BDC, example 1 gives

Vacancies	Results
1	C
2	AC
3	ABD

Now, C gets last place, and B and D are tied for third. The tie is broken by looking at first preferences, so D is third. Then there is a similar tie for second between A and B, so B is second. Result ABDC. Voters have improved the position of B by changing later preferences.

My method would still give the order CABD with example 2, but would violate the principle given a similar example.

Wichmann[4] suggests using the Meek keep factor for determining the ordering, and this case is not so immediately obvious, since only one count is held. The Meek algorithm does not allow later preferences to influence whether earlier preferences may be elected. However later preferences may affect the size of the keep factor for elected candidates, and so if this is used to order the candidates, the principle is violated. Electing 3, this gives ABCD in example 1 and ABDC in example 2.

2. Using the Orange Book method

The Orange Book simply suggests that if an order is required, the order in which candidates are elected during the count should be used. This seems, on face value, to be inadequate for selecting a long list of candidates, since the contest for the significant top places would be rather similar to a First Past the Post election, with a few candidates above the quota being given positions dependent only on the numbers of first preferences received. Newland himself, author of the early editions of the Orange Book, indicates in his *Comparative Electoral Systems* why he thinks this method is wrong, advocating a top-down method.

The method appears to rest on the assumption that it is the determination of the whole membership of the list that is the primary purpose of the election. That is not the case. The purpose is that however many seats the party wins, the people thus elected are those who were selected by an STV ballot with the appropriate number of vacancies. Thus in the Green Party, where no more than 1 seat was won in any region, the top of the list should be the AV winner (as indeed they were, since the Green Party used a top-down method.) The Liberal Democrats won 2 seats in some regions, so there the appropriate selection would be that of the top two candidates by an STV election with 2 vacancies.

The problem is that the number of seats that a party will win is unknown at the time of selection. However, it may be reasonable to guess at that number. The order of election (orange book method) would give the order of the candidates elected in the selection ballot, and the reverse order of exclusion could determine the order of later candidates. If a party wins 1 more or fewer seats, the distortion might not be that great.

This does not seem entirely satisfactory, but I cannot see how better can be done without abandoning the principle.

3. Abandoning the Principle

A great many articles in *Voting matters* have discussed the principle. Some have suggested that it might be relaxed, for example to allow Borda scores to be used to break ties[5]. Personally I think the only strong argument against Condorcet style election rules is that they violate the principle. Therefore if the principle must be lost, we may as well look at later preferences more freely and use an election rule more in keeping with Condorcet principles, and do better than any of the methods advocated in *Voting matters* 9.

It seems to me that a great many voters would welcome a substantial benefit to a second or third preference at the expense of a small risk to a first preference. STV does seem to rest on the assumption that the strength of a voter's support for their first preference is such that other considerations are overridden. While I don't think this assumption is true for very many voters (except perhaps for die-hard party loyalists), it is right for STV to make it. It is right because it makes the task of voting much easier. The voter does not need to assess how his or her use of later preferences might affect the fate of an earlier one. The principle encourages voters to indicate their true preferences.

Nonetheless, if the price of the principle is reducing a contest to near equivalence to First Past the Post, I believe that price is too high. I suggest the next question is how may we reap the benefits of the information the principle denied us. In the one vacancy election, systems which violate the principle may benefit by being able to guarantee the election of the Condorcet winner if there is one. I seek now to generalise this benefit to the election of an ordered list.

4. Generalising Condorcet principles to multiple vacancies

Hill[6] describes the complexity that can arise when trying to generalise the concept of a Condorcet top-tier to a multiple vacancy election. However, if we are considering a list selection then we are not simply looking for a subset of all the candidates, but adding them one at a time to a list. This simplifies the problem somewhat. Also for the purposes of simplicity I shall refer to Condorcet to mean any single-winner rule satisfying the Condorcet Criterion. The manner in which cycles should be resolved is not a significant concern here; nor is whether Meek or ERS97 rules are used, although computer counting will be necessary.

The method which follows builds an ordered list from the top down. It, like Condorcet, does not use exclusions at all, but considers at every stage, all possible pairs of candidates for the next position to see if one beats all the others. Like STV, votes are retained by elected candidates so they have less or no influence on later positions.

The top position is elected by Condorcet (call this candidate P).

For every pair, X and Y of other candidates, we must determine which is preferred to the other for the second place. We calculate the result of an STV election between P, X and Y for 2 places (other candidates being withdrawn). This calculation determines whether X is preferred to Y or vice versa. We read off the support for X and Y after any surplus for P has been redistributed and this completes one element in the Condorcet result square. (Normally it is only of interest which of X or Y is elected in this election. However the magnitude of the difference in support will be relevant if a cycle-breaking method needs to be employed.) The calculation is repeated for all other pairs of candidates, not including P (or at least for as many pairs as are necessary to determine the winner). Call the candidate thus elected to position 2 Q.

We need to repeat this exercise for position 3, 4, 5, etc, and we now have more than one elected candidate. Each time we perform an STV count including all the elected candidates, PQR..., and a pair of unelected candidates X and Y, and no others, giving one element of the Condorcet result square as before. We then repeat this for every pair of unelected candidates, and add our new Condorcet-style winner to the list.

Applied to Example 1, the result tables look like this:

(+ values imply row candidate beats column candidate)

Condorcet (6 AV counts between 2 candidates)

	A	B	C	D
A		+2	-12	+6
B	-2		-6	-6
C	+12	+6		+4
D	-6	+6	-4	

Position 2: (3 STV counts with 3 candidates, C and two others)

AvB: C has a surplus of 2, which is non-transferable - A 12, B 10

AvD: C has a surplus of 6, which is non-transferable - A 12, D 6

BvD: C has no surplus - B 10, D 16

	A	B	D
A		+2	+6
B	-2		-6
D	-6	+6	

A is elected to position 2

Position 3: (1 STV count with all candidates)

BvD: A has a surplus of 3, which goes 0.5 to C and 2.5 to D - B 10, D 8.5

	B	D
B		+1.5
D	-1.5	

B is elected to position 3

Result: CABD

Changing the 10 votes from BC to BDC as before (example 2) creates a cycle:

Position 1:

	A	B	C	D
A		+2	-12	-4
B	-2		-6	-6
C	+12	+6		-16
D	+4	+6	+16	

D is the Condorcet Winner and is elected to position 1.

Position 2:

AvB: - A 12, B 10

AvC: - A 12, C 8 (D is guarded, so A is not elected)

BvC: D has a surplus of 4 which goes to C (strictly 3.96 with ERS97) - B 10, C 14

	A	B	C
A		+2	+4
B	-2		-4
C	-4	+4	

A is elected to position 2

Position 3:

BvC: - B 10, C 8.5 (C and D are guarded, so B is not elected)

	B	C
B		+1.5
C	-1.5	

A is elected to position 3.

Result: DABC

D and C have swapped places, as is reasonable given the change of votes from BC to BDC.

Instead of using a usual cycle-breaking rule, an alternative would be to combine the election for the position in question with the following one, elect two, and then go back to the first, where there are now only 2 candidates to choose from, so there can be no cycle. (This would be a normal STV election for the top two. Alternatively we could consider every possible triple, but this may lead to further cycles.)

This procedure is a synthesis of STV and Condorcet. At each position a Condorcet-winner is added to the list, once votes cast for already-elected candidates have been discounted (reduced in value) in the manner of STV. It is not vulnerable to the exclusion of potential winners with few first preferences.

It could also form the basis for a synthesis of STV and Condorcet for unordered elections, although this would be a solution looking for a problem as regular STV is available here. Seeking to elect *n* candidates we could apply the STV rule to every subset of *n*+1 of the candidates and see which *n* were able to beat off any individual challenger. As Hill[6] says, the subset of *n* with this property may not exist, or may not be unique. However the generalised Condorcet method above could be adapted in such cases to arbitrate between competing sets of candidates, or to provide a result where there appears to be none.

5. Summary of examples

	Ex 1	Ex 2
Repeated count rules:		
Rosenstiel /Bottom Up Overriding (R):	CABD	ABDC
Otten /Top Down Constrained (O):	CABD	CABD
Top Down Overriding (TDO):	CABD	CABD
Bottom Up Constrained (BUC):	CABD	ABDC
One count rules:		
Wichmann Meek (2 places) (WM2):	CABD	CABD
Wichmann Meek (3 places) (WM3):	ABCD	ABDC
Orange Book (1 place) (OB1):	CABD	CABD
Orange Book (2 places) (OB2):	ACBD	ACBD
Orange Book (3 places) (OB3):	ABCD	ABDC
Generalised Condorcet rule:		
Generalised Condorcet (GC):	CABD	DABC

I have not described the last two repeated count rules — they are hybrids of the Rosenstiel and Otten rules, which might be called Bottom Up Overriding and Top Down Constrained respectively. It is worth noting that BUC, like GC, does not use exclusions, (candidates already allocated to lower positions are withdrawn before the start of the next count) but with different results.

What are the best results? CABD seems to be a clear favourite for example 1. With example 2, the elementary conflict is that if the electorate were to be represented by one person, the best person (from an AV point of view) would be C, and if it were to be three, the best people would be A, B and D. Rules which take greater care over the top end of the list (O, TDO, WM2, OB1) therefore place C highly and those which concentrate on the bottom (WM3, R, BUC, OB3) place C low. Notably WM3 and OB3 place C low even in example 1.

We have, it seems, not entirely escaped from the consideration in point 2, of needing to know what position on the list is the crucial one. If it is believed that a particular position on the list, say 4th, is the key one, an STV count for 4 winners could be followed by BUC to fill the top 3 and O to fill the positions from 5 down (or R and TDO respectively).

As to be expected GC succeeds in finding the Condorcet winner D in Example 2, who is not found by any of the other methods. Obviously this is an example of my choosing, and I have no doubt that other examples may show GC generating inferior results.

6. Conclusions

I have described three broad approaches to ordered list selection, all of which are unsatisfactory. The methods used by the Green Party and Liberal Democrats violate the principle, but fail to take advantage of the information this releases. The Generalised Condorcet method uses this information but also violates the principle. The orange book method, used as described here, may lead to a severely distorted result if the guess is wrong.

While the methods described in 1, appear for the moment to be the most practical solution to the question of ordering, the fact that counts for differing numbers of candidates frequently produce inconsistent results undermines their credibility.

A significant source of these inconsistencies is changes in early exclusions or the order of exclusions and in which parcel of papers elects a candidate, resulting from the higher or lower quota. (Meek should be less vulnerable to two of these effects.) While my generalised Condorcet method conceals any comparable inconsistencies that might be present, the fact that it eliminates exclusions altogether, means that it should be robust against exclusion-related effects.

The disadvantages are greater complexity and probably a more frequent violation of the principle that later preferences should not count against earlier ones. It will also require considerably more computer time than the alternatives, which may be an issue with a very large election, particularly if Meek is used. It would not be desirable to adopt a rule that then had to be abandoned for very large elections.

I do not at this point advocate that a generalised Condorcet method is adopted. However, I think the idea has its merits, and I do believe the question of ordering demands further consideration. While a single rule may not be appropriate for all circumstances, it should be possible to narrow the field somewhat from that in section 5.

References

1. I D Hill. STV with Constraints, *Voting Matters* 9, pp2-4, 1998.

2. J Otten. Ordered List Selection *Voting Matters* 9, pp8-9, 1998.

3. C Rosenstiel. Producing a Party List using STV. *Voting Matters* 9, pp7-8, 1998.

4. B A Wichmann. Editorial Comment on 3, *Voting matters*, 9, p8, 1998.

5. Lord Kitchener, Tie-Breaking in STV, *Voting Matters* 11, pp5-6, 2000.

6. I D Hill, Trying to find a winning set of candidates. *Voting matters*, 4, p3. 1995.

Tie-breaking in STV

I D Hill

Earl Kitchener[1] puts forward a scheme for using Borda scores for tie-breaking within STV. In general Borda scores are not a sensible way of conducting elections, but for this one purpose it will seem preferable to many people, to use something that takes note of the wishes of the voters, rather than a resort to randomness. The question is whether any such scheme would cause more trouble than it is worth.

We need to remember that ties rarely occur except in the case of very small elections, but it is just those very small ones where voters can see what is happening, and where the effect of later preferences upsetting earlier ones may be most troublesome.

In the real case quoted by Kitchener, there were 4 candidates for 1 seat. The 4 candidates were also the voters but not everyone voted for themselves. The votes were

ABC	1
BAD	1
ACDB	1
BCA	1

giving an AB tie for first place whether judged by Alternative Vote or by Condorcet. Using Borda scores as tie-breaker, A is elected, but this is solely because of a third preference for A against a fourth preference for B.

Now Voter 2 has a right to be cross about that. He put A as second choice meaning, according to all the best explanations of STV, "If B is out of the running, then I wish to support A" but B was not out of the running at that point.

Suppose there were the same set-up the following year. Voter 2 is likely to decide to plump because putting in a second preference the previous year was to his disadvantage. But Voter 1 may realise this and decide that he must plump too to counteract Voter 2's plumping — then Voters 3 and 4 will need to think about their strategies.

Whether anyone decides to plump or not is not really the issue. What matters is that tactical considerations have been allowed in, where STV (in its AV version in this case) is supposed to be free of them.

It may seem a pity to decide it at random, but such looking at the votes only decides it on the grounds that Voter 3 preferred D to B whereas Voter 4 preferred A to D. Is that really relevant when D is clearly out of it anyway?

My own conclusion is that to look at later votes in such circumstances, by Borda scores or any other method, is not a good thing to do, but I recognise that it is a matter of judgement, not of a clear right and wrong.

Reference

1. Earl Kitchener, Tie-breaking in STV. *Voting matters*, Issue 11, p5, 2000

Mixing X-voting and preference voting

I.D. Hill

Hugh Warren[1] puts forward a plan to incorporate X-voting into an STV election, so that those who prefer it are not forced into STV against their will. The aim is very sensible but, as he says, the voters must "be assured that it is being done in a fair way". As Hamlet said: "ay, there's the rub". Is it possible to find a way that actually is fair and, equally necessary, will be accepted as fair by those concerned?

The Warren suggestion is to treat Xs as equal first preferences, treating each X as worth 1/*m* where there are *m* places to be filled. Now suppose, as he does, that *m* = 10. If two voters each plump for a single candidate, one using an X and the other using a 1 in marking the paper, would it be regarded as fair for the second of those to be treated as worth 10 times as much as the first? Surely not.

In an editorial footnote, Brian Wichmann suggests an alternative formulation, treating each X as worth 1/*n* where *n* is the number of Xs marked on the paper. That would solve the above difficulty, but only at the expense of introducing a new one.

Suppose two candidates get X-votes only, one getting 20 Xs each of value 0.5, because those voters used two Xs each, the other getting 40 Xs each of value 0.2, because those voters used five Xs each. The first then has a total vote value of 10, the second a total vote value of 8. So if one of the two is elected it will be the one getting 20 Xs, not the one getting 40 Xs. Would X-voters regard that as fair? I am quite sure that they would not. It is just this sort of situation that I presume that the Warren formulation was carefully designed to avoid.

Is there any way of doing it that everyone would think fair in all cases? I doubt it.

Reference

1. C H E Warren. Incorporating X-voting into Preference voting by STV. *Voting matters*, Issue 11, p2. 2000.

The computational accuracy using the Meek algorithm

B A Wichmann

Introduction

The Meek algorithm[1] is specified without regard to the accuracy of the computation (with the exception of the convergence criterion, which is not relevant to this paper). The formulation in Pascal uses the type **real** which is traditionally floating point, but this could have varying accuracy or even be replaced by a rational arithmetic package of unbounded precision. A natural question to ask is what computational accuracy is required to ensure that the 'correct' candidates are elected, ie, the same candidates as if infinite precision was used. We demonstrate by examples, that there are cases in which very high precision is required.

An example

If a candidate A has first preference votes which only just exceed the quota, then those who have given A as their first preference will have only a small fraction of their vote passed on to their subsequent preference. Moreover, if most of A's subsequent preferences are for B (say) and just one for C, then the fraction going to C can be made smaller still.

The above leads to the following example in which 3 seats are to be filled:

333	AX
333	AY
333	AZ
333	BX
333	BY
333	BZ
667	X
667	Y
667	Z
1	ABX
1	ABY
2	BAX

The total number of votes is 4003, which gives an initial quota of 1000.75. Since A and B each have 1001 first preference votes, there is a surplus to transfer after their election of a quarter of a vote. This implies that the weight associated with A and B is roughly (1−1/4000). This further implies that the vote ABX makes a contribution to X of roughly 1/16,000,000th of a vote.

After the election of A and B, one of X, Y or Z must be eliminated. In the cases above, it is clear this should be Z, since that candidate has no contribution from the last three votes, but X and Y do. However, if the implementation of Meek only recorded millionths of a vote, then the last three candidates would be regarded as equal, in which case, a tie-break would occur.

For this test, we are only concerned as to what happens at the third stage. If a tie-break occurs, we know that the implementation does not have the accuracy necessary to compute the same result that would arise from infinite accuracy.

The above example illustrates that the accuracy required to give the same result as with infinite precision is unbounded even with six candidates (since we can just use more votes to increase the accuracy needed). However, the same technique can be employed with more candidates to increase the accuracy without increasing the number of votes. For instance, with 69 candidates and less than 1,000 votes, one can produce an example requiring 127 decimal places! The full details of this are available from the author.

Conclusions

There are somewhat bizarre voting patterns in which the accuracy required by the Meek algorithm is high, if the same result is to be obtained as that which would result from infinite precision.

One cannot expect the accuracy provided by an actual implementation to be high enough to guarantee the same result as that from infinite precision. (The highest available accuracy that is easily provided on a modern computer is 17 decimal places.)

The examples used here involved only the first two stages of a count. However, an important property of the Meek algorithm is that there is no accumulation of rounding error from one stage to the next, since the state is just the (discrete) record of those elected and eliminated. The weights are not really relevant since they only provide a starting point for the next iterative step.

One could gauge the impact of computational accuracy if one knew the rate at which ties arose which are *not* due to an algebraic tie. If such a computational tie arose with my database of around 370 elections, then it should be detected. In work which involved comparing two implementations of Meek(using all these 370 elections), it is likely that one implementation would report a tie-break when the other implementation did not. Such an occurrence did not arise.

Hence the overall conclusion is that the accuracy of the existing implementation of 64-bits is sufficient in practice, but not theoretically if the requirement is to produce the same result as that given by infinite precision.

Reference

1. I D Hill, B A Wichmann and D R Woodall. Algorithm 123 — Single Transferable Vote by Meek's method. *Computer Journal*. 1986.

A Comparison of Electoral Systems using Decision Analysis

H G Jones

Bob Jones is a retired mathematician and former secretary Derbyshire Electoral Reform Group (DERG)

Introduction

Decision Analysis is a method by which comparisons between different courses of action may be evaluated in order to obtain a desired end product. In the field of electoral reform the end product is the best electoral system, and the means of evaluating different systems is by comparing how well they measure up to desirable features of such systems.

The idea of applying Decision Analysis to electoral systems was first suggested by Tony Cooper, chairman of DERG,in the late 1980s and initially the performance of a system

	Feature											
System	PRO-R	PRO-N	CHO-P	ONECM	EASV	EASC	EASBC	EW&E	LOC	PLOC	Total	Ranking
FPTP(SM)	3(9)	4(12)	0(0)	10(20)	10(20)	10(10)	2(2)	2(6)	10(30)	4(8)	55(117)	
AV(SM)	4(12)	5(15)	0(0)	10(20)	9(18)	9(9)	2(2)	2(6)	10(30)	6(12)	57(124)	
PL(MM))	10(30)	10(30)	0(0)	10(20)	10(20)	7(7)	10(10)	5(15)	0(0)	0(0)	62(132)	
PLRC(MM)	10(30)	10(30)	0(0)	10(20)	10(20)	8(8)	10(10)	5(15)	2(6)	3(6)	68(145)	
PLRO(MM)	10(30)	10(30)	5(10)	10(20)	9(18)	7(7)	10(10)	7(21)	2(6)	4(8)	74(160)	3
STV(MM)	8(24)	9(27)	10(20)	10(20)	8(16)	7(7)	10(10)	10(30)	9(27)	10(20)	91(201)	1
AMS(HY)	9(27)	10(30)	0(0)	5(10)	9(18)	9(9)	7(7)	7(21)	8(24)	3(6)	67(152)	4
AV+(HY)	7(21)	8(24)	5(10)	5(10)	8(16)	7(7)	5(5)	5(15)	9(27)	7(14)	66(149)	5
AV+50(HY)	10(30)	10(30)	5(10)	5(10)	8(16)	7(7)	7(7)	7(21)	8(24)	7(14)	74(169)	2

against each feature was evaluated as *excellent*, *good*, *fairly good* and *poor*. More recently the evaluation has been carried out numerically with scores being given up to a maximum of 10.

As well as this scoring procedure, it was realised that certain features were of greater importance than others, and weighting factors (WF) were therefore applied to each feature. For example, proportionality is considered to be very important and is thus given a WF of 3 , the relevant feature score being multiplied by WF. On the other hand ease of counting is not of great importance as the returning officer and his or her staff will have been trained to deal with the relevant system. In this case the weighting factor (WF) is taken as 1.

Notation for systems

1. **Single Member Constituencies**

FPTP(SM): First-past-the-post.

AV(SM): Alternative Vote.

2. **Multi-Member Constituencies**

PL(MM): Party List based on the whole country (as in Israel).

PLRC(MM): Party List based upon regions using a closed list.

PLRO(MM): Party List based on regions with an open list.

STV(MM): Single Transferable Vote.

3. **Hybrid Systems**

AMS(HY): Additional Member System as used in Germany and in differing forms for the Scottish Parliament and Welsh Assembly.

AV+(HY): AV(SM) with a top-up as proposed by Lord Jenkins for Westminster.

AV+50(HY): Similar to AV+(HY) but having equal numbers of local and regional members.

Notation for Features

PRO-R: How proportional is the result within a region? (A region is visualised as, say, ten adjacent single-member constituencies).

PRO-N: How proportional is the total election result?

CHO-P: Is there a choice within a party as well as across party lines?

ONECM: Is there one class of elected members?

EASV: How easy is the system for the voter?

EASC: How easy is it to conduct the count?

EASBC: Does the system ease the task of determining constituency boundaries?

EW&E: Does the system encourage women and persons of ethnic minorities to stand for election?

LOC: How closely is the elected member linked to his or her constituency?

PLOC: How easily can a voter contact an elected member of their own political persuasion?

My Decision Table

1. **Weighting factors**

The weighting factors I have chosen for the features above are:

WF=3 for PRO-R, PRO-N, EW&E, LOC.

WF=2 for ONECM, EASV, PLOC, CHO-P.

WF=1 for EASC, EASBC.

2. **Decision Table**

The figures in parentheses are obtained by multiplying the score (out of 10) by the weighting factor WF, thus obtaining a weighted score. The total (weighted) score is the sum of the weighted scores for each feature of a system. The figures presented in the table gives my own judgement of the features for each system.

Conclusions

On this basis STV appears to be the best system. This, however, is something I have believed for the last 20 years or so. Maybe I have been subconsciously biased!

The scoring and weighting reflects my personal opinions and feelings. Small differences in scoring and, particularly in WFs, can easily change the above conclusions and I would be grateful for other opinions.

STV with Elimination by Electability Scores

Simon Gazeley

1. Introduction

It is widely thought among students of electoral reform that a candidate in a single-seat election who can beat every other in Condorcet pairwise comparisons is the most representative possible of the expressed views of that electorate. This proposition can be disputed, but for present purposes I shall regard it as axiomatic. The Condorcet principle can be extended to cover elections for n seats when $n>1$; one way of achieving this is to conduct mini-elections by STV to select n out of every possible set of $n+1$ candidates, and to elect the set of n candidates that wins the largest number of these mini-elections.

There are two problems with this extended form of Condorcet. One is that, when two or more seats are being contested, it is not practicable for any but the smallest elections: 15 candidates contesting 5 seats would give rise to 5005 contests; 27 candidates standing for the 15 seats on the Council of the Electoral Reform Society would give rise to 13,037,895 contests. Confronted with the result sheet of such an election, the electorate would find it difficult to understand how the winning candidates won and, perhaps more importantly, how the losing candidates lost. The other problem is that there could be more than one set of n candidates (whether $n>1$ or $n=1$) which gain the equal greatest number of victories. We would have to provide some kind of tie-breaker.

I believe that we can achieve the effect of Condorcet for one or more seats without these practical difficulties. Indeed, David Hill[1] has suggested one such scheme which selects sets of n candidates and tests each set against the other candidates one at a time. He admits that his scheme can elect a candidate other than the Condorcet winner in an election for one seat: I believe that the system propounded here will always elect the Condorcet winner, if there is one.

2. A Brief Digression on Proportionality

Woodall[2] has proved that no system can be devised which has all the following properties:

1. Increased support, for a candidate who would otherwise have been elected, should not prevent their election.

2. a. Later preferences should not count against earlier preferences.

   b. Later preferences should not count towards earlier preferences.

3. If no second preferences are expressed, and there is a candidate who has more first-preference votes that any other candidate, then that candidate should be elected.

4. If the number of ballots marked X first, Y second, plus the number marked Y first, X second, is more than half the number of ballots, then at least one of X and Y should be elected.

Given that preferential voting is desirable, few would consider any system which lacks either of properties 3 or 4 to be acceptable. Woodall later[3] extended 4, dubbing it the “Droop Proportionality Criterion” (DPC), which he stated thus:

If, for some whole numbers K and L satisfying 0<K≤L, more than K Droop quotas of voters put the same L candidates (not necessarily in the same order) as the top L candidates in their preference listings, then at least K of those L candidates should be elected.

A voter who puts those L candidates (in any order) as the top candidates in order of preference is said to be “strongly committed” to that set of L candidates. We will refer to a set of candidates to whom one set of voters is strongly committed as a “DPC set”.

Under any of the rules in current use, the elimination of candidates in an STV election makes votes available to other candidates in the DPC sets to which they belong. No candidate who has a quota at the relevant stage is eliminated, and, with insignificant exceptions, eliminations are made one at a time. This ensures that the result of an STV count is consistent with the Droop Proportionality Criterion. STV with Elimination by Electability Scores (STV(EES)) shares this characteristic.

3. The aim of STV(EES)

Conventional STV (whether by Meek's method[4] or one of the manual methods) is directed towards identifying with as little ado as possible the candidates who should get the seats: election takes precedence over elimination. The problem with this approach is that only as many of each voter's preferences are examined as are necessary to award the quota to sufficient candidates within the rules. For example, the second and subsequent preferences of the voters whose first preference was cast for the eventual runner-up are not even examined.

On the other hand, the aim of STV(EES) is to identify those candidates who certainly should not be elected. It does so by taking account of all the preferences of every voter; in some circumstances, this feature will cause the system to fail on Woodall's second property. To identify candidates for elimination, it calculates “electability scores” (see below) for the candidates: as new electability scores are calculated at successive stages, these form the basis for the elimination of candidates one by one until only sufficient are left to fill the available seats. These remaining candidates are elected.

STV(EES) differs in another way from conventional STV. As we are identifying candidates for elimination, not election, we do not have to use the Droop quota, and in fact its use can lead to perverse results. Instead, we calculate the “threshold”, which any of the other candidates must be able to attain in order to survive.

4. How STV(EES) works

STV(EES) is based on Meek's method, the most significant feature of which in this context is that votes are transferred in strict order of the voter's preference, regardless of whether the receiving candidates already have a quota of votes or not. In STV(EES), all candidates start as “contending” candidates. We then calculate the “electability score” (see below) of each contending candidate in turn, and candidates are withdrawn on the basis of those electability scores.

A stage of STV(EES) culminates in the withdrawal, either temporary or permanent, of a candidate. It consists of two substages: the first establishes the threshold of votes which a candidate must be capable of achieving in order to survive; the second is to test whether the candidates who start with less than the threshold can in fact achieve it. At the end of the second sub-stage, one of these candidates is withdrawn from contention. This withdrawal takes one of two forms: the candidate is either “eliminated”, which means that (s)he takes no further part in the count, and is treated from that point on as though (s)he had withdrawn before it started; or is “temporarily excluded”, which means that (s)he is withdrawn for the time being, but comes back in after the next elimination.

Before explaining how to calculate electability scores, we must define the “retention factor”, which Meek calls the “proportion retained”. In a Meek count, a point will be reached when a candidate has more than the quota. Clearly, that candidate should get less of the incoming votes in the next iteration of the count than were credited this time; and in successive iterations, the proportion of each incoming vote that stays with that candidate will diminish. The tendency will be for each new total of votes credited to that candidate to be closer to the quota than the last. To achieve this, an incoming whole vote or fraction of a vote is multiplied by an amount m where $0<m<1$; the result of this multiplication is the fraction of that vote which is credited to that candidate. This amount m is known as the retention factor. Retention factors start with a value of 1.0, and those for the candidates with more than the quota are re-calculated at every iteration; thus retention factors will diminish as the count progresses. The Droop quota is also re-calculated at every iteration on the basis of the votes credited to candidates, ignoring those which have become non-transferable.

In an STV(EES) election, the first sub-stage of each stage is the calculation of the threshold. It does this by calculating the mean of the votes of the n candidates who have the most votes. Surpluses over the mean are transferred, then a new mean is calculated. This process of distributing the votes, calculating the mean, and transferring surpluses is repeated until the first n candidates have the same number of votes. The top n candidates are then known collectively as the “probables”, and their common total of votes is the threshold (T). The value of T remains fixed throughout the second substage, which is the calculation of the contending candidates' electability scores. Let C be the contending candidate whose electability score we are calculating (the “candidate under test”), and let all the contending candidates other than C have a common retention factor of c. C's own retention factor remains at 1.0. In successive iterations, c and the retention factors of the probables are recalculated until the votes credited to all the probables are equal to the threshold and C either has the threshold or has less than the threshold while no other contending candidate has any votes at all. At this point, c is declared to be C's electability score. The electability scores of the remaining contending candidates are calculated in like fashion. The smaller C's electability score, the greater the number of votes that have had to be transferred from contending candidates other than C in order to ensure that C and the probables get their thresholds.

If the votes credited to the candidate under test and the contending candidates have a collective total of less than *T*, this indicates that the probables had a Droop quota of votes each when that candidate's electability score was being calculated. In that case, that contending candidate is eliminated, and all the non-eliminated candidates are re-classified as contending. On the other hand, if all the contending candidates' electability scores are at least 0.0, the one with the highest electability score is temporarily excluded, and only the existing probables are re-classified as contending. The new set of contending candidates proceeds to the next stage.

Stage succeeds stage until there are only *n* candidates who have not been eliminated, and those final candidates are elected. Note that at any stage when there are only *n*+1 "active" candidates (ie, candidates who have not been eliminated or temporarily excluded), one of them is certain to be eliminated. We therefore know that candidates will be eliminated until only *n* active candidates survive; thus an STV(EES) election must come to an end.

STV(EES) aims to identify a set of *n* candidates which can score at least as many victories in Condorcet mini-elections as every other. This means, for every eliminated candidate X, that there must be no set of *n* candidates including X which can score more victories in Condorcet mini-elections than every set of *n* not including X. We know at any given stage that every probable is better supported at that stage than X, and that every temporarily excluded candidate was better supported than X at the time of their temporary exclusion. Any DPC set to which X belongs has more members than can be elected by the number of voters that support it, and every other member of that DPC set is better supported than X. We can therefore be confident, though not certain, that there is no set of *n* candidates including X that can score more victories in Condorcet mini-elections than every set of *n* not including X. We can, however, state with certainty that in a count for one seat, the Condorcet winner (if there is one) will win. This is because, by definition, the Condorcet winner will win a contest with any one other candidate: and since no candidate is eliminated unless *n* other candidates have a Droop quota of votes each, the Condorcet winner cannot be eliminated.

5. An Illustration

Six candidates are contesting two seats, and votes are:

```
ABCDEF 3670
CBAEFD 3436
DEFABC 1936
EFDBCA 1039
FDECAB 1919
       =====
       12000
```

After sub-stage 1.1, A and C are probables, and the threshold (the number of votes held by both A and C when transfers are complete) is 3436. At sub-stage 1.2, electability scores are:

```
B 0.1319
D 0.4125
E 0.2860
F 0.3478
```

This means that if D, E, and F had a common retention factor of 0.1319, A, B, and C would have 3436 votes each when surpluses have been transferred; if B, E, and F had a common retention factor of 0.4125, A, C, and D would have 3436 votes each when surpluses have been transferred, and so on. As D has the largest electability score at this stage, we act on the presumption that D has a better chance of being elected than B, E, or F, and so we ensure by temporary exclusion that D does not run the risk of being eliminated at substage 1.2. Note that this presumption is like the presumption of innocence in a criminal trial: the process tests it and may very well overturn it.

At substage 2.1, effective votes are:

```
ABCEF 3670
CBAEF 3436
EFABC 1936
EFBCA 1039
FECAB 1919
      =====
      12000
```

Again, A and C are probables and the threshold is 3436. At 0.7608, E's electability score is higher than B's or F's, so E is temporarily excluded at substage 2.2. Effective votes are now:

```
ABCF 3670
CBAF 3436
FABC 1936
FBCA 1039
FCAB 1919
     =====
     12000
```

At substage 3.1, A and F are probables, and the threshold is 4016.9493, more than the current Droop quota. As neither B nor C can get that many votes if the other is temporarily withdrawn, we can eliminate both. D and E are now reclassified as contending, making effective votes:

```
ADEF 3670
AEFD 3436
DEFA 1936
EFDA 1039
FDEA 1919
     =====
     12000
```

At substage 4.1, A and D are probables, and the threshold is 3696.7554. At substage 4.2, E's electability score is 0.4741 and F's is 0.3385, so E is temporarily excluded. Active votes are now:

```
ADF 3670
AFD 3436
DFA 1936
FDA 1039
FDA 1919
    =====
    12000
```

At substage 5.1, A and F are probables, and the threshold is 4309.9757, more than the current Droop quota. D cannot get that many votes and therefore is eliminated. E now comes back in, and active votes are:

```
AEF 3670
AEF 3436
EFA 1936
EFA 1039
FEA 1919
    =====
    12000
```

At substage 6.1, the threshold is 5040.5, more than the current Droop quota, and A and E are probables. There is no prospect that F can attain the threshold, so we eliminate F. A and E are the only active candidates left, so they are elected.

6. Discussion

The example above is unusual in that there are two discrete DPC sets, ABC and DEF, supported respectively by 7106 and 4894 voters. The result is consistent with the Droop Proportionality Criterion in that each set contributes one winning candidate. In fact, an exhaustive Condorcet count produces a three-way tie for first place between AD, AE, and AF. This results from a paradox whereby AD wins the ADE contest, AE wins the AEF contest, and AF wins the ADF contest. Any of these outcomes is as valid as either of the others. It is noteworthy that STV(EES) does not “hang up” on a Condorcet paradox.

If there are too few DPC sets with sufficient support to “soak up” all *n* seats being contested, can the system still produce a reasonable outcome? Let there be 4 candidates contesting 2 seats with votes:

```
ABCD 41
BCDA 30
CDAB 25
DABC 24
     ===
     120
```

The results of an exhaustive Condorcet count are:

Contest Winners

```
ABC AB
ABD AD
ACD AC
BCD BC
```

We have a paradox in that AB wins the ABC contest, but AD wins the ABD contest and AC wins the ACD contest; there is also a four-way tie. As A starts with a quota of first preferences, A must be one of the winning candidates, but which of the other three should take the second seat?

Under STV(EES), A and B are probables, and the initial threshold is 35.5. At stage 1, the electability scores of C and D are respectively 0.5625 and 0.54, so C is temporarily excluded. At stage 2, A and D win the ABD contest, and B is eliminated. At stage 3, A, C, and D remain in the contest, so A and C are elected.

How can the elimination of B and D be justified? Part of the answer is that D was in only one winning set in the exhaustive Condorcet count, whereas the other candidates were in at least two. But is there any objective reason why B rather than C should be eliminated? Here we must confess that the system may be said to be perverse: 95 voters prefer B to C, but only 25 prefer C to B. In defence of this outcome, we can say that set AC is one of the joint Condorcet winners, so it meets the aim of STV(EES); and that when a tie is the result of a paradox, it will be arbitrary to some extent. But I would still have preferred AB to be the winning set in this case.

I submit that STV(EES) will in most cases (perhaps all) give a result that is compatible with an exhaustive Condorcet count: and that even if it does not, the result will still be defensible.

Acknowledgement

I am grateful to David Hill, Hugh Warren, and Brian Wichmann, whose queries, comments, suggestions, and advice have made this paper very much better than it would otherwise have been. My deep gratitude is due also to the independent referee, who not only made many suggestions but also pointed out a fatal flaw in an earlier version of the system.

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2. D R Woodall, An impossibility theorem for electoral systems, *Discrete Mathematics,* **66,** (1987) 209-211.

3. D R Woodall, Properties of Preferential Election Rules, *Voting matters*, Issue 3 (1994), 8-15.

4. B L Meek, A New Approach to the Single Transferable Vote, *Voting matters*, Issue 1 (1994), 1-6. (Paper 1), 6-11 (Paper 2).

Annex - An Algorithm for STV(EES)

All candidates start as contending candidates with a retention factor(RF) of 1.0. They should be in random order.

The suggested procedure is as follows:

Substage 1

1. Set every active candidate's retention factor to 1.0.

2. Repeat the following procedure until *n* candidates have *T* votes each.

   a. Distribute the votes in accordance with "Distributing the Votes" below.

   b. Calculate *T*, the mean of the votes of the *n* candidates with most votes.

   c. For every candidate who has more than *T* votes, calculate a new retention factor by multiplying their present RF by *T* and dividing the result by the number of votes credited to that candidate.

3. If *n* candidates have *T* votes each, classify those *n* candidates as probables. If more than *n* candidates have *T* votes each, classify the first *n* in ranking order as probables.

Substage 2

1. Select each contending candidate in turn to be the "candidate under test" and calculate their electability scores as follows:

a. If $T>V/(n+1)$, where *V* is the total of votes credited to all the candidates, mark the candidate under test for elimination. Otherwise, set the retention factor of the contending candidates, the candidate under test, and the probables, to 1.0, then repeat the following procedure until the probables and the candidate under test have *T* votes each, or until $T>V/(n+1)$:

i. Distribute the votes in accordance with "Distributing the Votes" below.

ii. Recalculate the retention factor (RF) of any probable who has more than *T* votes by multiplying it by *T* and dividing the result by the number of votes credited to that candidate. Recalculate the common RF of the contending candidates by multiplying it by $(V-(n+1)T)/C$, where *C* is the total of votes credited to the contending candidates other than the candidate under test.

b. If $T=V/(n+1)$ and there are only $n+1$ active candidates. or if $T>V/(n+1)$ mark the candidate under test for elimination. Otherwise, set the electability score of the candidate under test to the common RF of the other contending candidates.

2. Award the probables a notional electability score of 1.0, then rank the active candidates in their present order within descending order of electability score.

3. If any contending candidate is marked for elimination, eliminate all the marked candidates, reclassify all the non-eliminated candidates as contending, and rank them in random order. Otherwise, temporarily exclude the highest-ranked contending candidate, set that candidate's RF to 0.0, and reclassify only the probables as contending candidates.

Distributing the Votes

Examine each vote in turn and:

1. Multiply the value of the vote by the retention factor of the voter's first preference. Award that amount of the vote to that candidate.

2. If any of the vote is unallocated, multiply it by the retention factor of the candidate of the voter's next preference. Award that amount of the vote to that candidate. Repeat until none of the vote is left, or until the voter's preferences are exhausted.

3. If any of the vote is left when all the candidates have had their shares, put it to non-transferable.

How to ruin STV

I D Hill

To ruin STV by turning it, in effect, into merely a party list system, the following steps may be taken:

1. Make voting compulsory so that even the laziest have to turn out;

2. Insist that votes, as given by voter-defined preferences, are not valid unless every candidate (from a long list) is given a preference number, without gaps or repetition;

3. Allow the voter the alternative option of merely ticking a party box, and take that to indicate an STV vote as specified by the chosen party;

4. Use traditional STV counting rules, so that it can be guaranteed that, if you choose your own order, either your first choice will not be elected, or if elected but not on the first count, then all your hard work entering later preferences will be totally ignored;

5. Insist that, as the party box method is optional, this is not taking anything away from the voters.

Since many voters are lazy, most can then be expected (save in very exceptional circumstances) to use the party box method, as to do anything else is a lot of work and almost certainly for no benefit. Is it unimaginable that party politicians would try to pervert STV in this way? Unfortunately not; all these things now happen in Australia, and nearly all the virtues of STV have consequently been lost.

To see the dire effects of this, consider the election of 6 Senators for New South Wales at the 1998 Federal Election, for which there were 69 candidates. In some Australian STV elections not all the candidates have to be given preference numbers, though they usually require a substantial number. In this one all 69 had to be put in strict preference order. Just imagine doing that when the alternative of merely ticking a party box was available.

Probably many voters would not be aware of the effect mentioned in item 4 above, so that may not have much effect on what happens, but it would certainly add to the frustration for anyone who did know about it.

The remarkable thing in the circumstances is not that practically everyone used the party option but that 19012 voters, or 0.51%, did not.

The whole output table is much too vast for reproduction here, but the sense of it can be derived by looking at just the party that did best, with candidates A1, A2, A3 and A4 in that order on the party ticket. The first four stages for those candidates were:

A1	1446231	−909698	536533		536533		536533
A2	2914	+908567	911481		911481	−374948	536533
A3	864	+196	1060	+11	1071	+374505	375576
A4	2551	+130	2681	+3	2684	+199	2883

Eventually A3 also was elected. It can be seen, just from this small part of the information, how the party listing is totally dominant, and crushes all individualism. In particular, note how the party's preference for A3 over A4 overwhelms the fact that A4 got three times as many first preferences as A3. In fact, after transfers, all the votes ended up pointing at the three candidates highest on the list of the above party that took three seats, the two candidates highest on the list of another party that took two seats, the candidate first on the list of a further party that took one seat, and the candidate first on the list of the runner-up party. For the candidates, it is clear that getting a high place on the party list, rather than being liked by the voters, is what matters, as with party list systems in general.

Is it wise to tell politicians that STV can be perverted like this? Given that it has already happened in Australia, it can hardly be hidden from them anyway. The important thing is to bring the facts to the attention of STV supporters, so that they know that it is something to be ready to fight against.

Editorial Note

Unfortunately, there was a very misleading typographical error in Issue11. This was on the table marked **Old rules** on page 8. The entry against candidate C should have the word 'Elected' deleted. I am sorry if this caused any confusion. A corrected version is available electronically from me.